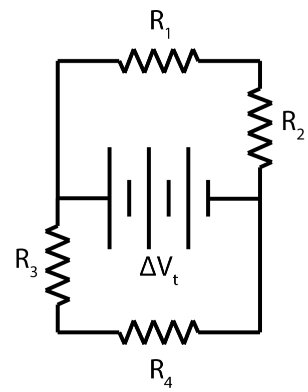
Flipping Physics Lecture Notes:

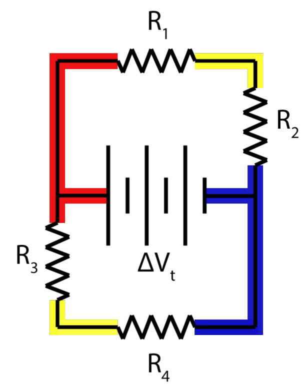
Intermediate Series and Parallel Resistor Circuit

https://www.flippingphysics.com/series-parallel-resistors-intermediate.html



The circuit shown has four 5.0 Ω resistors and a 5.0 V power source. Determine …

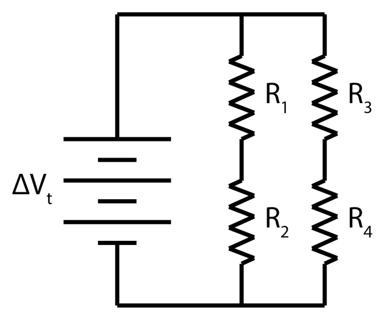
a) The equivalent resistance of all four resistors.
b) Current through each circuit element.
c) Electric potential difference across each resistor.
d) Power dissipated by each resistor.



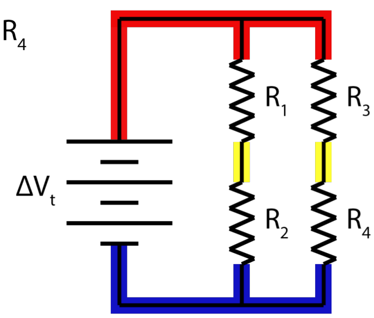
Start by color-coding the electric potential along all the wires of the circuit. From the color-coded circuit diagram, you can now see several things:

1) R_1 and R_2 are in series.
2) R_3 and R_4 are in series.
3) Those two sets of series resistors are in parallel.

In other words, we can redraw the circuit diagram like this:



In fact, it is even easier to see they are the same circuit when we color-code the electric potential of the circuit:

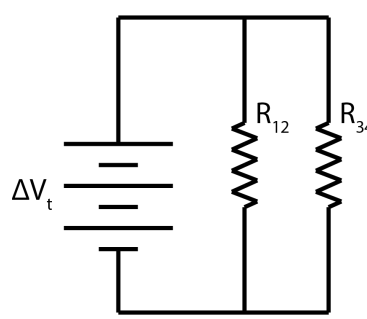


I do want to point out that the only reason we know the wires between R_1 and R_2, and R_3 and R_4 are all at the same electric potential, and therefore the same color yellow, is because all four resistors have equal resistance. If they did not have the same number of ohms, then the electric potential difference across each resistor would be different and the electric potential between the resistors would not be the same.

Now we can begin solving the problem. Let's start by determining the equivalent resistance of the resistor pairs.

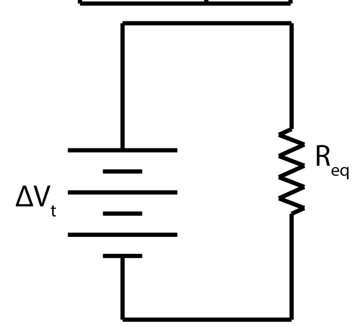
$$R_{12} = R_1 + R_2 = 5 + 5 = 10\Omega \quad \& \quad R_{34} = R_3 + R_4 = 5 + 5 = 10\Omega$$

We can replace R_1 and R_2 with equivalent resistor R_{12}.
We can replace R_3 and R_4 with equivalent resistor R_{34}.



And now we have two resistors in parallel and can deterimen the equivalent resistance of those two resistors:

$$R_{eq} = \left(\frac{1}{R_{12}} + \frac{1}{R_{34}}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = \left(\frac{2}{10}\right)^{-1} = \left(\frac{1}{5}\right)^{-1} = 5.0\Omega$$



Part (a): We can replace all four resistors with one equivalent resistance of, R, or $\boxed{5.0\ \Omega.}$

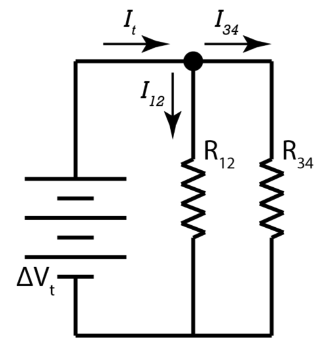
Now we can determine the current delivered by the power source to the equivalent resistor:

$$\Delta V = IR \Rightarrow I_t = \frac{\Delta V_t}{R_{eq}} = \frac{5}{5} = \boxed{1.0A}$$

So, the power source is delivering 1 coulomb of charge every second to the circuit. And we know the electric potential difference across the power source is the same magnitude as the electric potential difference across the two resistors R_{12} and R_{34}.

$$\Delta V_t = \Delta V_{12} = \Delta V_{34} = 5.0V$$

Therefore, we can determine the current through R_{12} and R_{34}.



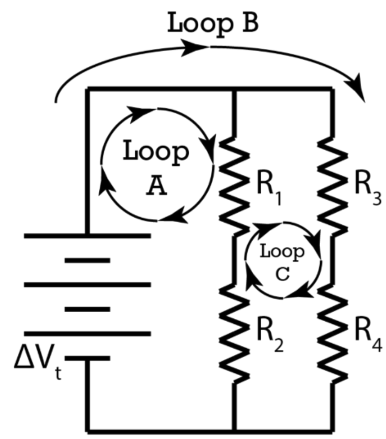
(b) $$I_{12} = \frac{\Delta V_{12}}{R_{12}} = \frac{5}{10} = \boxed{0.50A = I_1 = I_2} \; \& \; I_{34} = \frac{\Delta V_{34}}{R_{34}} = \frac{5}{10} = \boxed{0.50A = I_3 = I_4}$$

These currents make sense because we know Kirchhoff's Junction Rule states that the current going into a junction equals the current going out of a junction:

$$I_t = I_{12} + I_{34} = 0.5A + 0.5A = 1A$$

Now, we can determine the electric potential difference across each resistor:

(c) $$\boxed{\Delta V_1 = I_1 R_1 = (0.5)(5) = 2.5V = \Delta V_2 = \Delta V_3 = \Delta V_4}$$



This makes sense because we know Kirchhoff's Loop Rule states that the electric potential difference around any loop equals zero:

$$\Delta V_{loop\ A} = 0 = \Delta V_t - \Delta V_1 - \Delta V_2 = 5 - 2.5 - 2.5 = 0$$

$$\Delta V_{loop\ B} = 0 = \Delta V_t - \Delta V_3 - \Delta V_4 = 5 - 2.5 - 2.5 = 0$$

$$\Delta V_{loop\ c} = 0 = \Delta V_1 + \Delta V_2 - \Delta V_3 - \Delta V_4 = 2.5 + 2.5 - 2.5 - 2.5 = 0$$

And lastly, we can determine the rate at which energy is dissipated in each circuit element:

$$\boxed{\begin{aligned} &P_t = I_t \Delta V_t = (1)(5) = 5.0 watts \\ &P_1 = I_1^2 R_1 = (0.5)^2 (5) = 1.25 watts \approx 1.2 watts = P_2 = P_3 = P_4 \end{aligned}}$$

Which makes sense because the power added to the circuit from the power source needs to equal the power dissipated by the circuit:

$$P_t = P_1 + P_2 + P_3 + P_4 = 1.25 + 1.25 + 1.25 + 1.25 = 5.0 watts$$