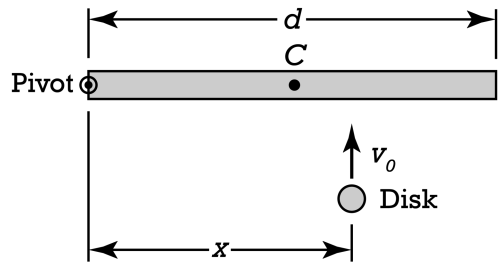
2017 #3 Free Response Question - AP Physics 1 - Exam Solution

http://www.flippingphysics.com/ap1-2017-frq3.html

This Quantitative/Qualitative Translation question also works as a part of the AP Physics C: Mechanics curriculum.

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.



Top View

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C?

____ To the left of C ____ At C _X_ To the right of C

Briefly explain your reasoning without manipulating equations.

The angular momentum of the disk before the collision is linearly proportional to x. Therefore, a larger x value will mean a larger initial angular momentum of the system. Because angular momentum is conserved about the pivot during this collision, this will mean a larger final angular momentum of the system. Because angular momentum equals rotational inertia times angular velocity, a larger final angular momentum will mean a larger final angular speed. Therefore, a larger value of x will mean a larger final angular speed. The student should slide the disk to the right of C.

Or

A larger distance x from the axis of rotation will mean a larger net torque applied by the disk on the rod. A larger net torque will mean a larger angular acceleration of the rod during collision which will mean a larger final angular velocity of the rod after the collision. The student should slide the disk to the right of C.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{disk} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a)? In other words, does this equation for ω have the expected dependence as reasoned in part (a)?

_X_ Yes ____ No Briefly explain your reasoning without deriving an equation for ω.

My qualitative reasoning in part (a) states that a larger x results in a larger ω. The student's equation also shows that a larger x results in a larger ω. Therefore, Yes, the equation does have the expected dependence.

From the Scoring Guidelines: "If 'No' is selected, the explanation may still earn full credit if an incorrect selection was made in part (a)." *In other words, if you get part (a) wrong, you can still get full credit for part (b). Please, answer every part of every question.*

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \dfrac{Ixv_0}{m_{disk}d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

A larger m_{disk} will result in a larger initial angular momentum of the disk about the pivot which will result in a larger final angular momentum of the rod-disk system which will result in a larger final angular velocity of the rod. However, according to the student's equation, the mass of the disk would be inversely proportional to the final angular velocity of the rod, which is incorrect.

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{disk}x^2$ and its angular momentum with respect to the pivot is $m_{disk}v_0x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

$$\vec{L}_i = \vec{L}_f \Rightarrow L_{ri} + L_{di} = L_{rf} + L_{df} \Rightarrow 0 + m_d v_o x = I_r\omega_{rf} + I_d\omega_{df} = I\omega_f + \left(m_d x^2\right)\omega_f = \left(I + m_d x^2\right)\omega_f$$

$$\omega_{rf} = \omega_{df} = \omega_f \;\&\; I_r = I \;\&\; I_d = m_d r_d^{\,2} = m_d x^2$$

$$\Rightarrow \omega_f = \frac{m_d v_o x}{I + m_d x^2}$$

*From the Scoring Guidelines: You gain one point "*For indicating that the initial angular momentum of the system is equal to $m_{disk}v_0x$.*" In other words, you can get one point for simply identifying that the initial angular momentum of the rod is zero and, therefore, the initial angular momentum of the system is the angular momentum of the disk only. Even if you did not know how to solve this problem, that is something you should be able to identify.*

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

_X_ Greater than ____ Less than ____ Equal to Briefly explain your reasoning.

Assuming the initial direction of the disk's motion is positive, when the disk bounces off the rod it will have a negative velocity and a negative angular momentum instead of a positive angular momentum when the disk sticks to the rod. Because angular momentum is conserved about the pivot during the collision, the final angular momentum of the rod will need to have a larger positive value in order to add up to the same initial angular momentum. Because the final angular momentum of the rod is larger, the final angular velocity of the rod is also ***greater than*** *the original final angular velocity.*

Grading note: Part (e) is worth 2 points. 1 point is for describing what happens to the rod and 1 point is for describing what happens to the disk. There are two objects in this problem. You need to describe what happens to both. Keep that in mind for future problems.