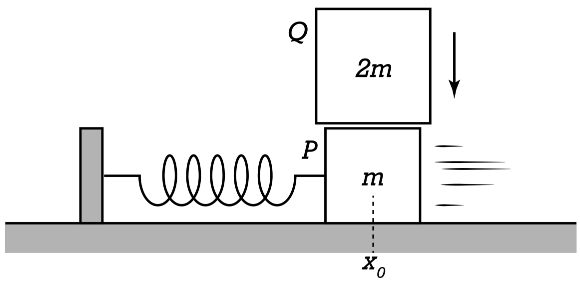
Flipping Physics Lecture Notes:

2018 #5 Free Response Question - AP Physics 1 - Exam Solution
http://www.flippingphysics.com/ap1-2018-frq5.html

This Paragraph Argument Short Answer question also works as a part of the AP Physics C: Mechanics curriculum.



Block P of mass m is on a horizontal, frictionless surface and is attached to a spring with spring constant k. The block is oscillating with period T_P and amplitude A_P about the spring's equilibrium position x_0. A second block Q of mass 2m is then dropped from rest and lands on block P at the instant it passes through the equilibrium position, as shown. Block Q immediately sticks to the top of block P, and the two-block system oscillates with period T_{PQ} and amplitude A_{PQ} .

(a) Determine the numerical value of the ratio T_{PQ}/T_P .

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T_P = 2\pi\sqrt{\frac{m_P}{k}} = 2\pi\sqrt{\frac{m}{k}} \;\&\; T_{PQ} = 2\pi\sqrt{\frac{m_{PQ}}{k}} = 2\pi\sqrt{\frac{3m}{k}} = \left(\sqrt{3}\right)\left(2\pi\sqrt{\frac{m}{k}}\right) = \left(\sqrt{3}\right)T_P \Rightarrow \frac{T_{PQ}}{T_P} = \sqrt{3}$$

(b) The figure is reproduced above. How does the amplitude of oscillation A_{PQ} of the two-block system compare with the original amplitude AP of block P alone?

__X__ $A_{PQ} < A_P$ ____ $A_{PQ} = A_P$ ____ $A_{PQ} > A_P$

In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, explain your reasoning.

Part 1: Block Q colliding with Block P:
When block Q lands on block P it is a perfectly inelastic collision and linear momentum is conserved. The mass of the system is increased; therefore, the velocity of the system must be decreased.

$$\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow \vec{p}_{Pi} + \vec{p}_{Qi} = \vec{p}_{Pf} + \vec{p}_{Qf} \Rightarrow m_P\vec{v}_{Pi} + m_Q\vec{v}_{Qi} = m_P\vec{v}_{Pf} + m_Q\vec{v}_{Qf}$$

$$\vec{v}_{Pf} = \vec{v}_{Qf} = \vec{v}_f \Rightarrow m\vec{v}_{Pi} = m\vec{v}_f + 2m\vec{v}_f = 3m\vec{v}_f \Rightarrow \vec{v}_f = \frac{1}{3}\vec{v}_{Pi} \Rightarrow \vec{v}_{PQ} = \frac{1}{3}\vec{v}_P$$

As you can see from the conservation of momentum equation during the collision, the velocity of both blocks after the collision is one-third the velocity of block P before the collision. To make this clear, I will identify the velocity of block P at point x_0 as v_P and the velocity of blocks PQ as v_{PQ} .

Part 2: Block(s) slide from equilibrium to amplitude:
Because the surface is frictionless and there are no other forces acting on the blocks-spring system, there is no external work done on the system, and mechanical energy is conserved. There is no change in gravitational potential energy of the blocks while they are moving horizontally, no matter where we put the horizontal zero line. Therefore, when the block is at x0, the equilibrium position, the only mechanical energy it has is kinetic energy, therefore, this is the location of maximum kinetic energy and that equals the total mechanical energy of the system. The decrease in total mechanical energy can be found by using the equation for kinetic energy.

$$KE_P = \frac{1}{2}m_P v_P^{\ 2} = \frac{1}{2}mv_P^{\ 2} \ \& \ KE_{PQ} = \frac{1}{2}m_{PQ}v_{PQ}^{\ 2} = \frac{1}{2}(3m)\left(\frac{1}{3}v_P\right)^2 = \frac{3}{9}\left(\frac{1}{2}mv_P^{\ 2}\right) = \frac{1}{3}KE_P$$

In other words, the addition of block Q decreases the total mechanical energy of the system to one-third its previous value with just block P. At maximum displacement from equilibrium position, or amplitude, all of the kinetic energy will be converted to elastic potential energy. $PE_{ef} = \frac{1}{2}kx_f^{\ 2} = \frac{1}{2}kA^2$ *Because the maximum kinetic energy was decreased with the addition of block Q, the maximum elastic potential energy will also be decreased and, because the spring constant does not change, the amplitude must decrease with the addition of block Q.*

Notes about grading:
This problem is unusual in that

1) Part (a) is independent from part (b). Usually the Paragraph Argument Short Answer questions are one continuous problem and the parts of the problem before the “In a clear, coherent paragraph-length …” question actually end up being a part of the paragraph answer.
2) The paragraph argument ends up needing more equations than usual.

So, please be aware that clearly some of the Paragraph Argument Short Answer questions will have independent parts and may require more equations than perhaps you are used to.