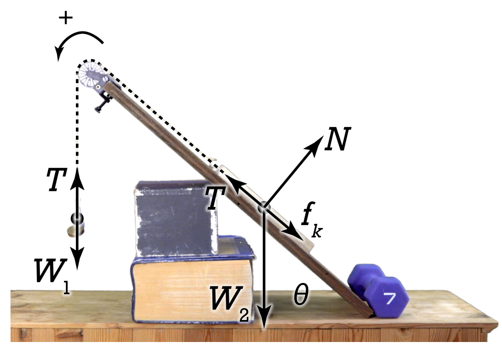
Flipping Physics Lecture Notes:

An incline, 2 free body diagrams, and a pulley.
What could be more fun?
http://www.flippingphysics.com/incline-masses-pulley.html

A 55 g mass is attached to a light string, which is placed over a frictionless, massless pulley, and attached to a 199 g block which is on a board inclined at 39.3° as shown. Assuming the block starts at rest and the μ_k between the incline and block is 0.38, how long will it take the block to move 13 cm?

Identify the 55 g mass as mass 1 and the 199 g mass as mass 2.

$m_1 = 55g;\ m_2 = 199g;\ \mu_k = 0.38;\ \theta = 39.3°;\ \Delta d_\parallel = 13cm;\ \Delta t = ?$

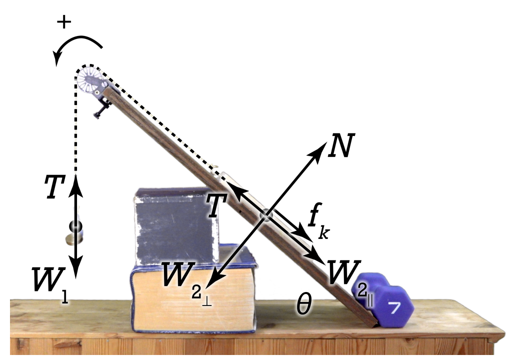


Now we draw the free body diagrams:

On mass 1 there is a weight or force of gravity acting down, let's call it weight 1, W_1. There is also a force of tension acting upward, T.

On mass 2 there is a weight or force of gravity acting down, let's call it weight 2, W_2. There is a force normal, N, acting up and perpendicular to the incline. There is also a force of tension acting up the incline, T.

Because the pulley is massless and frictionless, those two forces of tension are equal in magnitude. If the pulley had mass, the two forces of tension would not be equal in magnitude. If the pulley were not frictionless, the two forces of tension would not be equal in magnitude. Do not worry, pulleys will have mass and friction in later problems. ☺



The force of kinetic friction, f_k, is parallel to the incline and opposes motion, therefore, it is either up or down the incline. We need to predict which direction the masses will move in order to determine the direction of the force of kinetic friction. Let's predict mass 1 will move down and mass 2 will move up the incline. Therefore, the force of kinetic friction is opposite the direction mass 2 will move and is down the incline.

Break the Weight 2 or Force of Gravity 2 into its components. And add the direction we think the objects will move.

Now we can begin using Newton's Second Law and sum the forces. Whenever we sum the forces, we need to identify:

- The object (or objects) we are summing the forces on.
- The direction we are summing the forces in.

The goal with summing the forces is to solve for the acceleration of the two masses, so we can then use a uniformly accelerated motion equation to solve for the change in time.

$$\sum_{\substack{+\,direction\\ on\ both}} F = W_1 - T + T - f_k - W_{2_\parallel} = m_t a_\parallel \Rightarrow m_1 g - \mu_k N - m_2 g \sin\theta = \left(m_1 + m_2\right) a_\parallel$$

At this point notice there are two unknown variables: force normal and acceleration parallel. This equation goes in our equation holster and we move on to another net force equation.

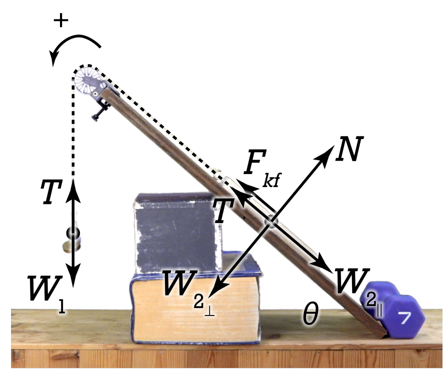
$$\sum_{\substack{\perp\ direction\\ on\ mass\ 2}} F = N - W_{2_\perp} = m_2 a_\perp = m_2(0) = 0 \Rightarrow N = W_{2_\perp} = m_2 g\cos\theta$$

We now have an equation for the force normal which we can substitute into our equation holster equation.

$$\Rightarrow m_1 g - \mu_k\left(m_2 g\cos\theta\right) - m_2 g\sin\theta = \left(m_1 + m_2\right)a_{\parallel} \Rightarrow a_{\parallel} = \frac{m_1 g - \mu_k m_2 g\cos\theta - m_2 g\sin\theta}{m_1 + m_2}$$

And now we can substitute in numbers, remembering to convert all of the masses to kilograms. Actually, technically we do not have to convert the masses to kilograms because the units for mass cancel out of the equation, however, it is still a good habit to get into. Another smiley face → ☺

$$\Rightarrow a_{\parallel} = \frac{(0.055)(9.81) - (0.38)(0.199)(9.81)\cos(39.3) - (0.199)(9.81)\sin(39.3)}{0.055 + 0.199} = -5.00390\frac{m}{s^2}$$



And we get a negative for our acceleration…

This means we predicted the incorrect direction for the masses to move. We now need to go back, change the direction of the force of kinetic friction, and redo the problem. Weeeeeee! Please realize we cannot simply "make the acceleration positive" because we have the incorrect direction for the force of kinetic friction. Here, have two smiley faces! ☺☺

$$\sum_{\substack{+\ direction\\ on\ both}} F = W_1 - T + T + f_k - W_{2_\parallel} = m_t a_{\parallel} \Rightarrow m_1 g + \mu_k N - m_2 g\sin\theta = \left(m_1 + m_2\right)a_{\parallel}$$

$$\Rightarrow m_1 g + \mu_k\left(m_2 g\cos\theta\right) - m_2 g\sin\theta = \left(m_1 + m_2\right)a_{\parallel} \Rightarrow a_{\parallel} = \frac{m_1 g + \mu_k m_2 g\cos\theta - m_2 g\sin\theta}{m_1 + m_2}$$

$$\Rightarrow a_{\parallel} = \frac{(0.055)(9.81) + (0.38)(0.199)(9.81)\cos(39.3) - (0.199)(9.81)\sin(39.3)}{0.055 + 0.199} = -0.483741\frac{m}{s^2}$$

And now we can use a uniformly accelerated motion equation to solve for the change in time:

$$\Delta d_{\parallel} = v_i\Delta t + \frac{1}{2}a\Delta t^2 \Rightarrow 2\Delta d_{\parallel} = a\Delta t^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta d_{\parallel}}{a}} = \sqrt{\frac{(2)(-0.13)}{-0.483741}} = 0.733129 \approx 0.73\sec$$