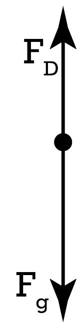
Flipping Physics Lecture Notes:

What is Terminal Velocity? How Do We Find It?

http://www.flippingphysics.com/terminal-velocity.html

If we drop an object in the vacuum you can breathe, the object is in free fall.
However, the reality is that air exists, and the object will not be in free fall.
Let's draw a free body diagram on an object after it is released, at rest, in the air.



We can sum the forces on the object in the y-direction:

$$\sum F_y = F_D - F_g = ma_y \Rightarrow \frac{1}{2}D\rho A v^2 - mg = ma_y$$

And we can solve for the acceleration of the object:

$$\Rightarrow a_y = \frac{D\rho A v^2}{2m} - g \Rightarrow \text{no air resistance means } a_y = -g$$

In other words, in the absence of air resistance, the object is in free fall.

When the object is first dropped, the velocity of the object is zero, therefore the force of drag acting on the object equals zero. At that initial point: $a_y = -g$

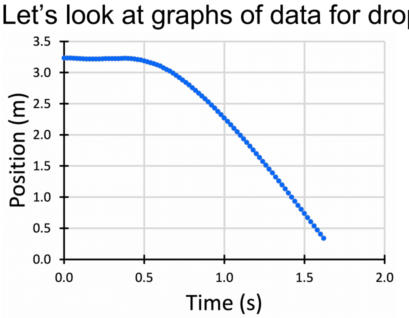
As time goes by, the velocity of the object increases in magnitude, causing the force of drag to increase in magnitude, causing the acceleration of the object to get closer and closer to zero.

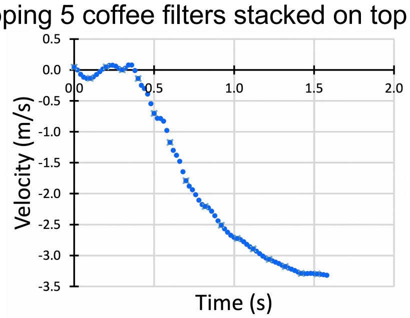
When the force of drag has increased to the point where it is equal in magnitude to the force of gravity, the acceleration of the object equals zero; the velocity of the object is now constant. This constant velocity is called the terminal velocity of the object, v_t. (Not to be confused with the tangential velocity of an object which is also v_t.) We can solve for the terminal velocity of the object by setting the acceleration of the object in the y-direction equal to zero.

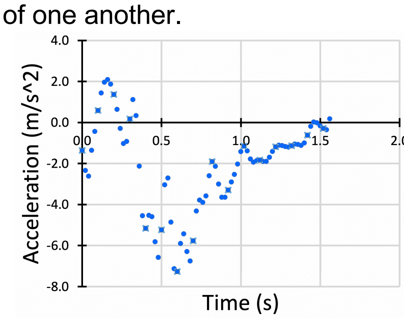
$$\Rightarrow 0 = \frac{D\rho A v_t^2}{2m} - g \Rightarrow \frac{D\rho A v_t^2}{2m} = g \Rightarrow v_t = \sqrt{\frac{2mg}{D\rho A}}$$

Please do not memorize this equation! Instead, understand how to derive it.

Let's look at graphs of data for dropping 5 coffee filters stacked on top of one another.







Looking at the position as a function of time graph, you can see the initial slope of the best fit line of the data equals zero because the coffee filters are initially at rest. After the coffee filters are dropped, the best fit line slope increases in magnitude until it reaches a magnitude of roughly 3.3 m/s, the terminal velocity of the 5 coffee filters. The velocity is negative because the coffee filters are going down. The velocity as a function of time graph starts out at zero and decreases in value to roughly 3.3 m/s as discussed previously.

One thing to notice is the increase in “noise” in the data as we go from position, to velocity, to acceleration. The position data looks quite good, the velocity data looks okay, and the acceleration data really only approximates what we discussed. The acceleration should be zero when I am holding the coffee filters at rest, however, clearly, I am not quite able to hold the coffee filters at rest and mild fluctuations in the location of the coffee filters cause the acceleration to vary quite a bit. The initial acceleration of the coffee filters should be -9.81 m/s^2, however, we measure a value closer to -7.5 m/s^2. This is likely due to the “noise” in the data. We can see that the acceleration of the coffee filters does trend toward and reach zero.

Please realize this is NOT uniformly accelerated motion.

We can use what we did before to determine the drag coefficient of the coffee filters.

$$m = 7.8g;\ \rho_{air} = 1.2\frac{kg}{m^3};\ d = 17.4cm \Rightarrow r = \frac{d}{2} = \frac{0.174m}{2} = 0.087m;\ \left\|v_t\right\| = 3.3\frac{m}{s}$$

$$\sum F_y = F_D - F_g = ma_y = m(0) = 0 \Rightarrow F_D = F_g \Rightarrow \frac{1}{2}D\rho A {v_t}^2 = mg$$

$$\Rightarrow D = \frac{2mg}{\rho_{air}\pi r^2 {v_t}^2} = \frac{(2)(0.0078)(9.81)}{(1.20)\pi(0.087)^2(3.3)^2} = 0.492489 \approx 0.49$$

And the terminal velocity of a baseball is:

$$m = 145g;\ D = 0.3;\ d = 75mm \Rightarrow r = \frac{d}{2} = \frac{0.075m}{2} = 0.0375m$$

$$v_t = \sqrt{\frac{2mg}{D\rho\pi r^2}} = \sqrt{\frac{(2)(0.145)(9.81)}{(0.3)(1.2)\pi(0.0375)^2}} = 42.2937\frac{m}{s}\left(\frac{3600s}{1hr}\right)\left(\frac{1mi}{1609m}\right) = 94.6286 \approx 90\frac{mi}{hr}$$

Just so you know, I calculated that it would take almost 13 seconds and more than 400 meters for a dropped baseball to get close to terminal velocity. I used Numerical Modeling to make those calculations. An excel file showing those calculations is linked on my webpage for this video. Numerical modeling is what we are doing next…