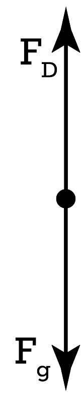
Flipping Physics Lecture Notes:

Demonstrating and Solving for Drag Coefficient

http://www.flippingphysics.com/drag-coefficient.html

We drop a steel sphere into water and it reaches its terminal velocity. Solve for the drag coefficient of the steel sphere in terms of variables. Then, determine those variables and solve for the drag coefficient.



Start by drawing the free body diagram. Force of drag is up. Force of gravity is down. By definition, at terminal velocity the acceleration of the steel sphere will equal zero. So, sum the forces and solve for drag coefficient:

$$\sum F_y = F_D - F_g = ma_y = m(0) = 0 \Rightarrow F_D = F_g \Rightarrow \frac{1}{2}C_D \rho A v_t^2 = mg \Rightarrow C_D = \frac{2mg}{\rho_{water}\pi r^2 v_t^2}$$

We need the following:

- Mass of the steel sphere
- Density of water
- Radius of steel sphere
- Terminal speed of the steel sphere

$$\Delta y = -12.6cm \;\&\; \Delta t = 0.056\sec \Rightarrow \left\|v_t\right\| = \frac{\left\|\Delta y\right\|}{\Delta t} = \frac{0.126m}{0.056\sec} = 2.25\frac{m}{s}$$

$$m = 67.8g;\; d = 2.55cm \Rightarrow r = \frac{d}{2} = \frac{0.0255m}{2} = 0.01275m;\; \rho_{water} = 1000\frac{kg}{m^3}$$

We can now solve for the drag coefficient of the steel sphere:

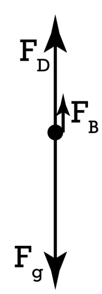
$$\Rightarrow C_D = \frac{(2)(0.0678)(9.81)}{1000\pi(0.01275)^2(2.25)^2} = 0.514510 \approx 0.51$$

Please realize that it would actually take a very long distance for the steel sphere to travel through the water until it gets close to its terminal velocity. I calculated a height to drop the sphere from such that it would be close to its terminal velocity as it went through the water.

If you drop the sphere from too high, it will be moving faster than its terminal velocity, the drag force will be greater than the force of gravity, and the sphere will be slowing down on its way to terminal velocity.

If you drop the sphere from too low, it will be moving slower than its terminal velocity, the drag force will be less than the force of gravity, and the sphere will be speeding up on its way to terminal velocity.

In addition, there is actually a force buoyancy acting upward on the sphere as well.
Including the force buoyancy in the calculations looks like this:



$$\sum F_y = F_D + F_B - F_g = m_s a_y = m(0) = 0 \Rightarrow F_D = F_g - F_B \Rightarrow \frac{1}{2}C_D \rho A v_t^2 = m_s g - m_f g = (m_s - m_f)g$$

$$F_B = m_f g \;\&\; \rho = \frac{m}{\forall} \Rightarrow m_f = \rho_f \forall_f \;\&\; C_D = \frac{2(m_s - m_f)g}{\rho_{water}\pi r^2 v_t^2} = \frac{2(m_s - \rho_f \forall_f)g}{\rho_{water}\pi r^2 v_t^2} = \frac{2\left(m_s - \rho_f\left(\frac{4}{3}\pi r^3\right)\right)g}{\rho_{water}\pi r^2 v_t^2}$$

$$\Rightarrow C_D = \frac{(2)\left(0.0678 - (1000)\left(\frac{4}{3}\pi\right)(0.01275)^3\right)(9.81)}{1000\pi(0.01275)^2(2.25)^2} = 0.448625 \approx 0.45$$