Flipping Physics Lecture Notes:

Effects of Drag Force on Free Fall
http://www.flippingphysics.com/drag-force-free-fall.html

Let's analyze the motion of a rubber dodgeball which was dropped from the moment it rebounds off the ground, until the moment right before it strikes the ground again. Today we are going to include the drag force, so the ball is *not* in free fall. In the past we have used this equation for drag force:

$$F_D = \frac{1}{2}C_D \rho A v^2$$

However, today we are going to use a much simpler form of the drag force equation: $\vec{F}_D = -b\vec{v}$

Several things to notice about this equation:

- The drag force is a vector and its direction is opposite the direction of the velocity.
- "b" is called the *proportionality constant* and it has units of kg per second:

$$\vec{F}_D = -b\vec{v} \Rightarrow b = \frac{F_D}{v} \Rightarrow b(units) = \frac{N}{m/s} = \frac{kg \bullet m/s^2}{m/s} = \frac{kg}{s}$$

- This equation is valid when a small object is moving at slow speeds.

When do we know which equation to use?

- In order to truly determine which equation(s) is(are) valid to use for the drag force, you would need to do experiments to test the equations.
- As far as the AP exam is concerned, In the past, the College Board® told students what equation to use for drag force. In other words, do not memorize either of these equations.

Let's look at the ball as it moves upward. Notice the force of gravity and the force of drag are both down while the ball moves upward. And, as the ball moves upward, the net force is down, causing a downward acceleration, which means the speed of the ball will decrease. That means the drag force will decrease in magnitude, that means, as the ball moves upward, the magnitude of the net force on the ball will decrease, and therefore, the magnitude of the acceleration of the ball also decreases. And, at the very top of its path, we know the velocity of the ball is zero, so the drag force is zero, so, at that instant in time, the dodgeball has an acceleration equal to negative g.

As the ball moves downward, the force of gravity is still down, however, the force of drag is now up. This means, as the ball moves downward, the net force is down, so its speed increases, causing its drag force to increase, however, the magnitude of the net force on the ball decreases, therefore, the magnitude of the acceleration of the ball also decreases. That means on the way up the magnitude of the acceleration of the ball decreases to the acceleration due to gravity and on the way down the magnitude of the acceleration continues to decrease.

Notice this means, for the same magnitude displacement in the y-direction, the time while the dodgeball is moving upward should be less than the time for when the ball is moving downward. Think of it this way, the average acceleration on the way up has a larger magnitude than the average acceleration on the way down. The magnitude of the displacement in the y-direction is the same. Both have an initial or a final velocity equal to zero, therefore, a larger magnitude acceleration means a smaller change in time.

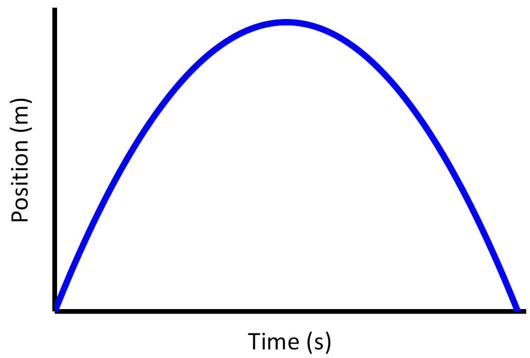
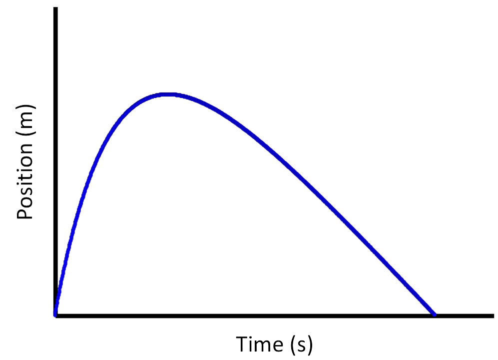
For the dodgeball in our example, the difference only amounts to roughly 6 thousandths of a second.
For a large beach ball in a similar example, the difference is only roughly 2 hundredths of a second.
Clearly it takes larger heights, therefore faster motion, and therefore larger drag forces to cause more of a difference between the change in time going upward vs. downward.

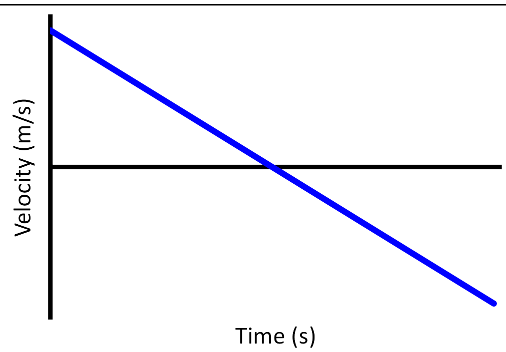
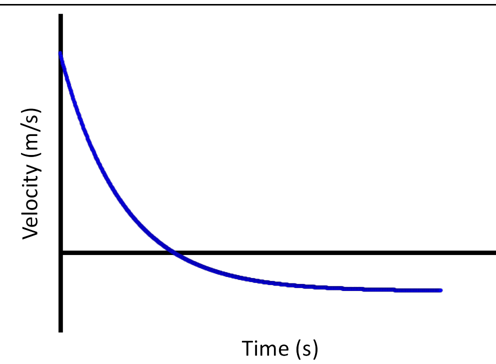
We can use Newton's Second Law to determine the terminal velocity of the ball as it moves downward.

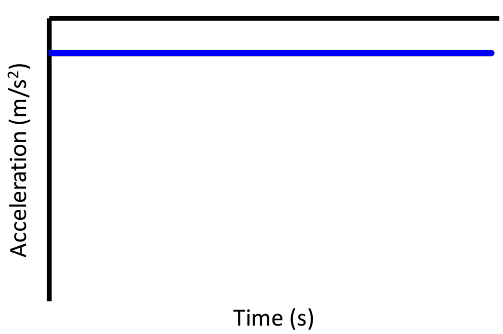
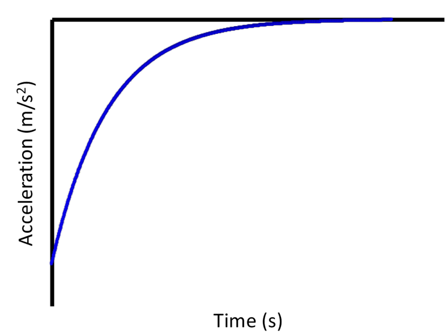
$$\sum F_y = F_D - F_g = ma_y = m(0) = 0 \Rightarrow F_D = F_g \Rightarrow bv_t = mg \Rightarrow v_t = \frac{mg}{b}$$

Please realize the ball in our demonstration never gets close to its terminal velocity.

Let's look at the graphs of position, velocity, and acceleration as a function of time for the dodgeball if it were in free fall and compare those graphs to what they look like with a force of drag. Assume the ball is thrown upward fast enough such that it will get close to terminal velocity on the way down.

Free Fall (no drag Force)	*with* Drag Force
 • Symmetrical parabola • Time going up is the same as time coming down	 • *Not* a symmetrical parabola • Time going up is the less than time coming down • Final slope of the graph is terminal velocity


• Slope = -9.81 m/s^2
• Crosses time axis at half the total time
    ○ When the ball is at the top of its path
• Initial and final velocities have the same magnitude
	
	• Initial slope has a magnitude larger than 9.81 m/s^2
	• Crosses time axis before half the total time
	    ○ When the ball is at the top of its path
	• Final slope is zero
	• Final velocity is the terminal velocity

 • Acceleration is uniform, slope equals 0 • Free fall acceleration near Earth = -9.81 m/s^2	 • Initial acceleration magnitude larger than 9.81 m/s^2 • Acceleration at the top of the path = -g • Final acceleration approaches zero