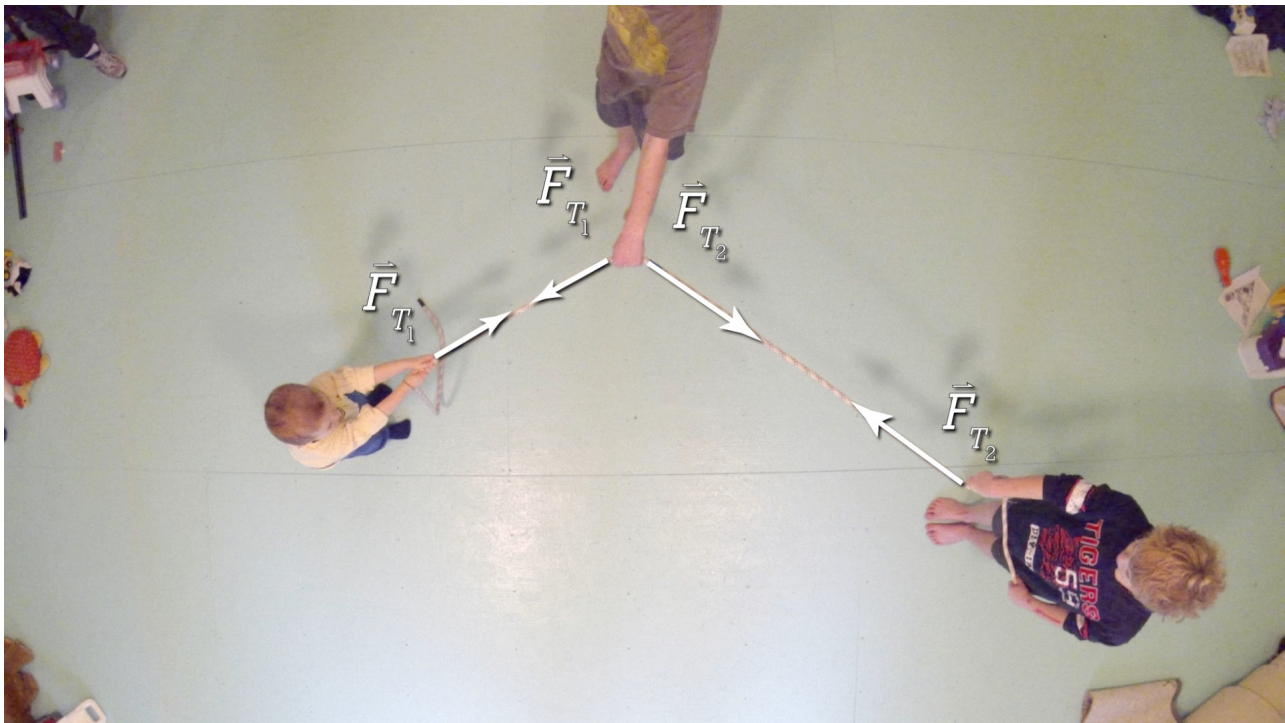


Flipping Physics Lecture Notes:
Understanding the Tension Force

The Tension Force or the Force of Tension is the force transmitted through a rope, cable, string or wire pulled taut by forces acting on both ends.

- Symbol: F_T or T [Often referred to as just “Tension.”]
- The Tension Force, like all forces, is a vector and has both magnitude and direction.
 - Direction:
 - Always a pull.
 - In the direction of the rope.
 - In opposite directions on opposite ends of the rope.
 - Magnitude: Equal in magnitude on both ends of the rope.





Flipping Physics Lecture Notes:
Introduction to Equilibrium

An object is in equilibrium if the net force acting on the object is zero. $\sum \vec{F} = 0$

In other words, if you add up all of the forces acting on an object they add up to zero.

Put one more way, all the forces acting on the object balance out or cancel one another.

Because the net force, according to Newton's Second Law, equals mass times acceleration, the acceleration of the object must be zero. Because acceleration equals change in velocity over change in time, the change in velocity of the object is zero. Therefore an object in equilibrium is either at rest or moving at a constant velocity.

$$\sum \vec{F} = 0 = m\vec{a} \Rightarrow \vec{a} = 0 = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} = 0$$

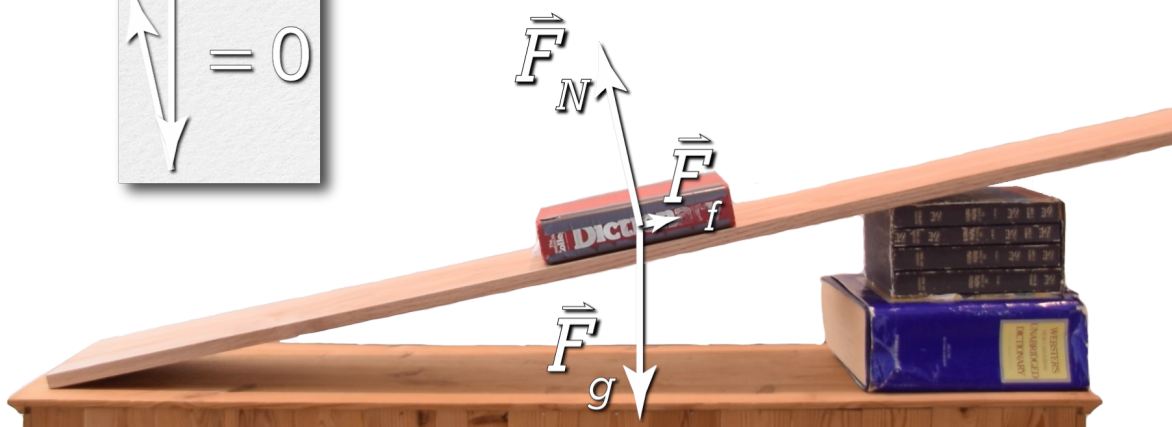
More specifically, this type of equilibrium is called *Translational* Equilibrium. This means the is non-rotational equilibrium

Examples:

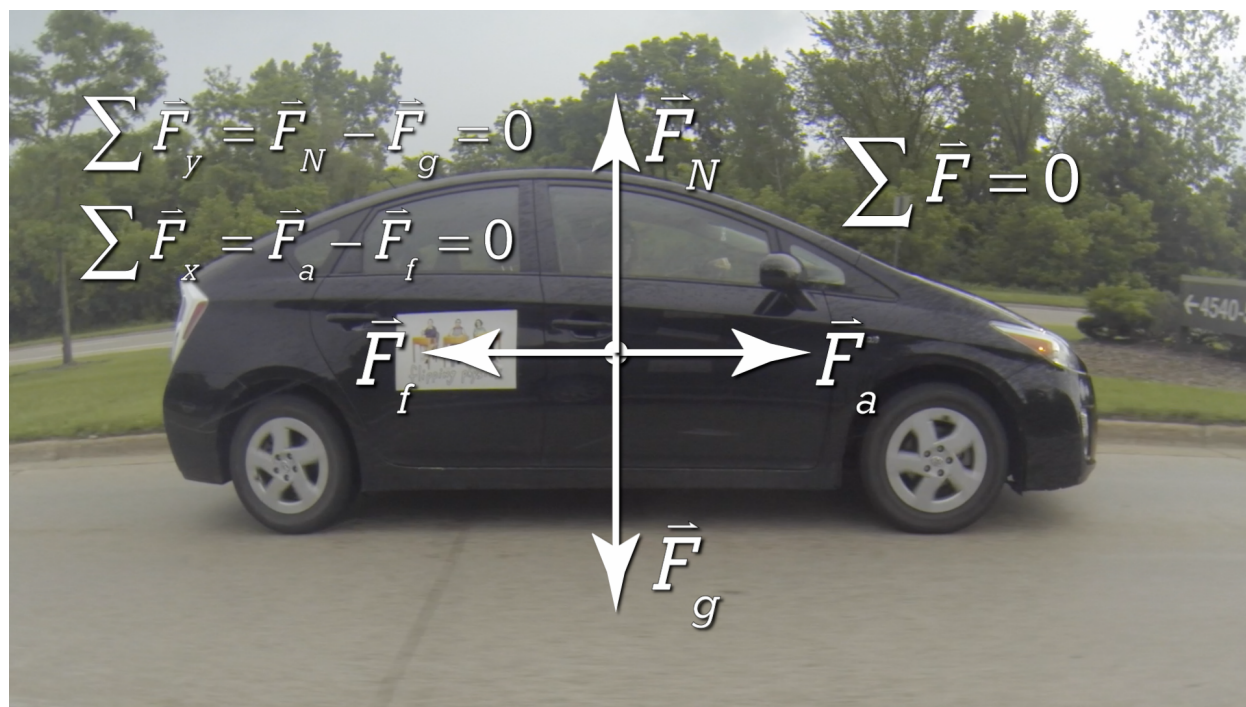
Book at rest on an incline.

$$\sum \vec{F} = \vec{F}_N + \vec{F}_g + \vec{F}_f = 0$$

$$\sum F_y = 0$$



Vehicle moving at a constant velocity



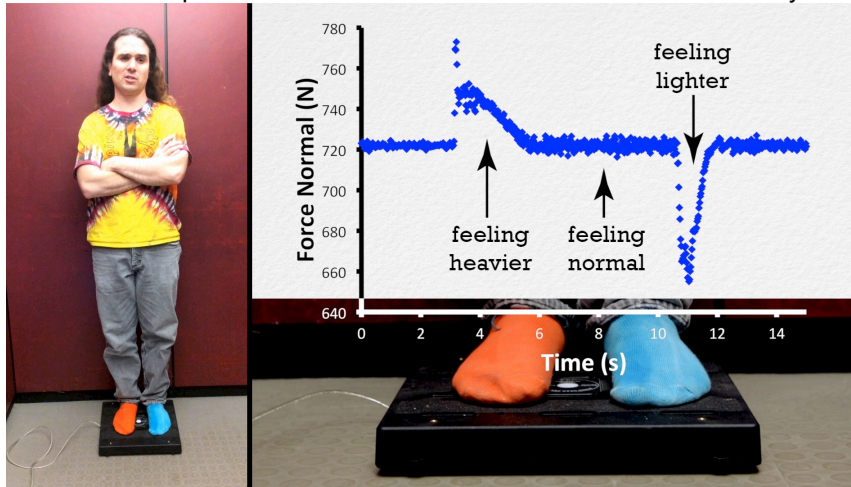


Flipping Physics Lecture Notes:

Do You Feel Your Weight?

No. You do not feel your weight. You feel the force normal acting on you. The normal force acting on you is also called your “apparent weight” because it is what you feel as your weight.

The best example to illustrate this is what a scale measures when you are on an elevator.



- Part 1) The elevator is accelerating upward.
- Part 2) The elevator is moving upward at a constant velocity.
- Part 3) The elevator is accelerating downward.

We can draw a free body diagram to analyze the situation:

$$\sum F_y = F_N - F_g = ma_y \Rightarrow F_N = F_g + ma_y$$

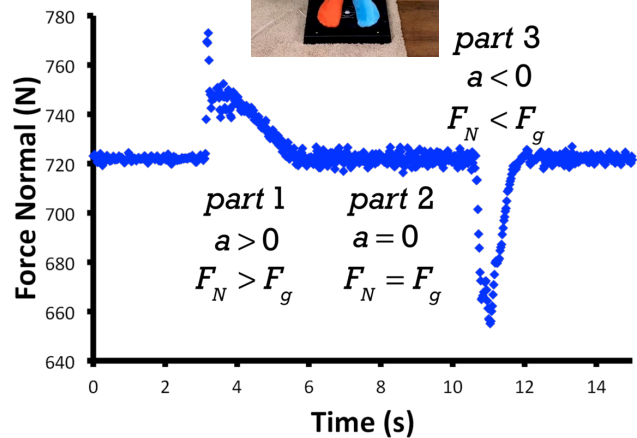
1) $a > 0$ then $ma_y > 0$ and $F_N > F_g$

2) $a = 0$ then $ma_y = 0$ and $F_N = F_g$

3) $a < 0$ then $ma_y < 0$ and $F_N < F_g$

in free fall: $a_y = -g$

$$\Rightarrow F_N = mg + m(-g) = mg - mg = 0 \Rightarrow F_N = 0$$



This is called Apparent Weightlessness, because your force normal is zero and therefore you feel weightless, even though there still is a force of gravity acting on you.



Flipping Physics Lecture Notes:
Letting Go of Your Numbers Dependency
<http://www.flippingphysics.com/numbers-dependency.html>

Many of my students have an inability to move forward in a problem without having numbers to plug in for variables. I call this your “Numbers Dependency”.¹ Today I am going to show an example of what that is, how to work without numbers, and why I think it is important to let go of your numbers dependency. In order to do so, let’s expand on a lesson we already did: Bo in an elevator.² Here is a new problem:



Example: Bo is standing on a scale in an elevator. When the elevator is at rest, the reading on the scale is 722 N. When the elevator is accelerating, the reading on the scale is 745 N, what is Bo’s acceleration?

The following is a typical, numbers dependent solution:

Knowns: $F_{N_1} = 722\text{N}$, $a_{y_1} = 0$, $F_{N_2} = 745\text{N}$, $a_{y_2} = ?$

$$\text{Part 1: } \sum F_y = F_{N_1} - F_g = ma_{y_1} = m(0) = 0 \Rightarrow F_{N_1} = F_g = mg$$

$$\Rightarrow 722 = m(9.81) \Rightarrow m = \frac{722}{9.81} = 73.5984\text{kg}$$

$$\text{Part 2: } \sum F_y = F_{N_2} - F_g = F_{N_2} - mg = ma_{y_2} \Rightarrow 745 - (73.5984)(9.81) = 23 = (73.5984)a_{y_2}$$

$$\Rightarrow a_{y_2} = \frac{23}{73.5984} = 0.312507 \approx 0.313 \frac{\text{m}}{\text{s}^2}$$

This is a typical solution which is not numbers dependent:

Knowns: $F_{N_1} = 722\text{N}$, $a_{y_1} = 0$, $F_{N_2} = 745\text{N}$, $a_{y_2} = ?$

$$\text{Part 1: } \sum F_y = F_{N_1} - F_g = ma_{y_1} = m(0) = 0 \Rightarrow F_{N_1} = F_g = mg \Rightarrow m = \frac{F_{N_1}}{g}$$

$$\text{Part 2: } \sum F_y = F_{N_2} - F_g = F_{N_2} - mg = ma_{y_2} \Rightarrow a_{y_2} = \frac{F_{N_2} - mg}{m} = \frac{F_{N_2} - F_{N_1}}{F_{N_1}/g}$$

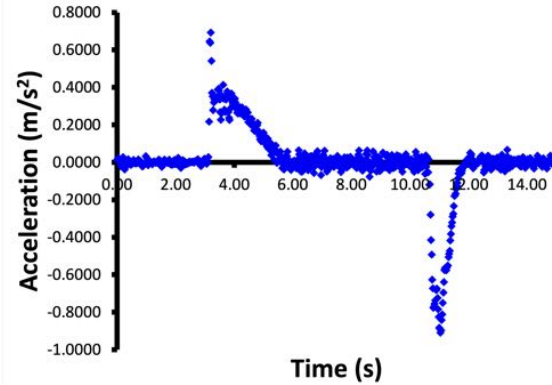
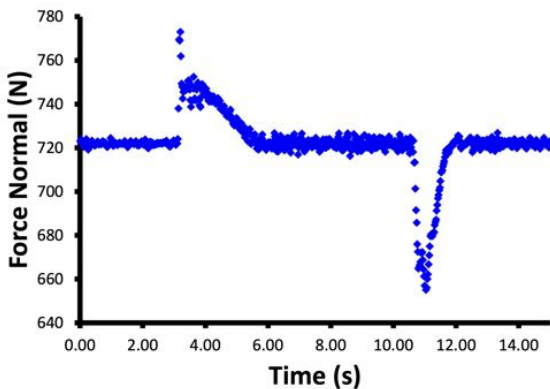
$$\Rightarrow a_{y_2} = g \left(\frac{F_{N_2} - F_{N_1}}{F_{N_1}} \right) = (9.81) \left(\frac{745 - 722}{722} \right) = 0.312507 \approx 0.313 \frac{\text{m}}{\text{s}^2}$$

The advantages of letting go of your numbers dependency are:

- When using primarily numbers there are three times numbers are typed into the calculator. When using primarily variables numbers are typed into the calculator only once. Each time a number is typed into the calculator; it increases the chances of making a mistake.
- Many numbers had to be written and rewritten in the numbers solution. Each time a number has to be written down; it increases the chances of making a mistake.
- The variables solution shows that there is no reason to calculate Bo’s mass.
 - If you need the number for Bo’s mass, the variables solution has an equation you can use to solve for it.
- The variables solution ends with a general equation for acceleration in terms of variables. This means we can now determine Bo’s acceleration given any force normal reading on the scale. This is how labs work. We collect data and perform redundant calculations. For example, now that we have the force normal as a function of time for Bo in the elevator, we can also calculate the acceleration of Bo as a function of time.

¹ “Numbers Dependency” was introduced in “An Introductory Tension Force Problem” <http://www.flippingphysics.com/tension-problem.html>

² “Do You Feel Your Weight? A lesson on Apparent Weight” <https://www.flippingphysics.com/apparent-weight.html>



- From the variable equation for Bo's acceleration in the y-direction we can deduce a lot of information.
 - If force normal 2 is increased, Bo's acceleration is also increased.
 - If the acceleration due to gravity on the planet increases, and the ratio of F_{N2}/F_{N1} remains the same, Bo's acceleration also increases.
 - if $F_{N2} = 0$, then $a_{y2} = g \left(\frac{F_{N2} - F_{N1}}{F_{N1}} \right) = g \left(\frac{0 - F_{N1}}{F_{N1}} \right) = -g$ Bo is in free fall.
- Lastly, without a numbers dependency we can solve problems like this:

Example: Bo is in an elevator and feels like he weighs half his weight. What is his acceleration?

To solve this problem, recall that weight is synonymous with force of gravity and what you feel is not your weight, but rather the force normal which acts on your body. In other words, the force normal in part 2 of our solution equals half of the force of gravity and we know the force of gravity equals the force normal in part 1. That means our solution is:

$$\text{Knowns: } F_{N2} = \frac{F_g}{2} = \frac{F_{N1}}{2}, a_{y1} = 0, a_{y2} = ?$$

$$\text{Part 1: } \sum F_y = F_{N1} - F_g = ma_{y1} = m(0) = 0 \Rightarrow F_{N1} = F_g = mg \Rightarrow m = \frac{F_{N0}}{g}$$

$$\text{Part 2: } \sum F_y = F_{N2} - F_g = F_{N2} - mg = ma_{y2} \Rightarrow a_{y2} = \frac{F_{N2} - mg}{m} = \frac{F_{N2} - F_{N1}}{F_{N1}/g}$$

$$\Rightarrow a_{y2} = g \left(\frac{F_{N2} - F_{N1}}{F_{N1}} \right) = g \left(\frac{F_{N1}/2 - F_{N1}}{F_{N1}} \right) = g \left(\frac{1/2 - 1}{1} \right) = -\frac{g}{2}$$

Bo is currently accelerating downward with the elevator at half the rate of free fall acceleration. As you get further into your physics learning, problems are going to have fewer and fewer numbers in them.

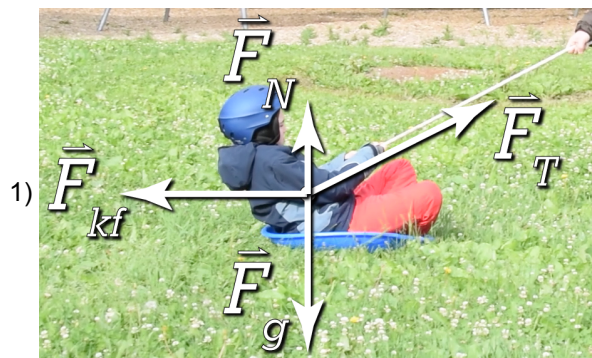
In summary, letting go of your numbers dependency reduces mistakes, allows you to better understand relationships between variables, allows for easier repeated calculations like we do in labs, and the future of your physics learning includes fewer numbers. A numbers dependent solution is like a dead-end one-way street. In order to solve the problem again with different numbers, you need to get out of your car, go back to the beginning of the street, get in a new car, and drive down the street again. However, a solution that is primarily variables is like a turnabout with many, many exits. You just need to pick a path to extend your physics learning, but you do need practice learning how to drive around a turnabout.



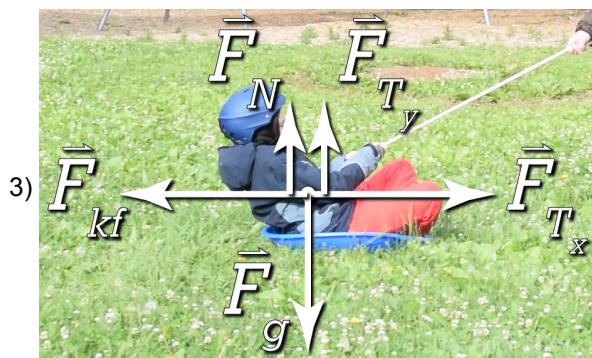
Flipping Physics Lecture Notes:
5 Steps to Solve any Free Body Diagram Problem

- 1) Draw the Free Body Diagram
- 2) Break Forces into Components
- 3) Redraw the Free Body Diagram
- 4) Sum the Forces ($\sum \vec{F} = m\vec{a}$)
- 5) Sum the Forces (in a direction perpendicular to the one in step 4).

For example:



2) $\sin \theta = \frac{O}{H} = \frac{F_{T_y}}{F_T} \Rightarrow F_{T_y} = F_T \sin \theta$ & $\cos \theta = \frac{A}{H} = \frac{F_{T_x}}{F_T} \Rightarrow F_{T_x} = F_T \cos \theta$

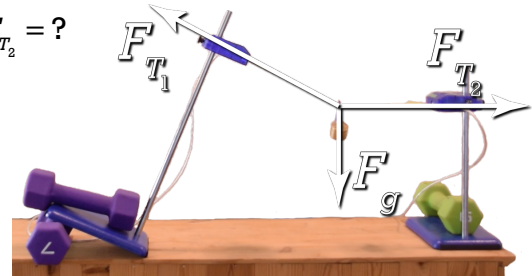
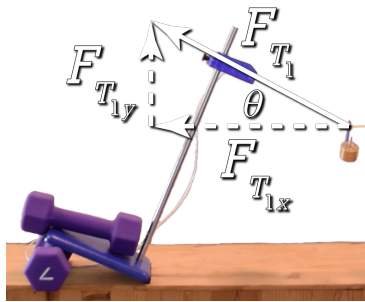


4) $\sum F_y = F_N + F_{T_y} - F_g = ma_y$

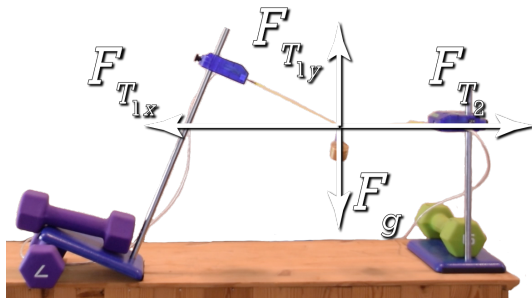
5) $\sum F_x = F_{T_x} - F_{kf} = ma_x$

Flipping Physics Lecture Notes:
An Introductory Tension Force Problem

$$m_{\text{hanging}} = 155.0g \times \frac{1kg}{1000g} = 0.155kg; \theta = 28^\circ; F_{T_1} = ?; F_{T_2} = ?$$



$$\sin \theta = \frac{O}{H} = \frac{F_{T_{1y}}}{F_{T_1}} \Rightarrow F_{T_{1y}} = F_{T_1} \sin \theta \quad \& \quad \cos \theta = \frac{A}{H} = \frac{F_{T_{1x}}}{F_{T_1}} \Rightarrow F_{T_{1x}} = F_{T_1} \cos \theta$$



$$\sum F_y = F_{T_{1y}} - F_g = ma_y = m(0) = 0 \Rightarrow F_{T_{1y}} - F_g = 0 \Rightarrow F_{T_{1y}} = F_g \Rightarrow F_{T_1} \sin \theta = mg$$

$$\Rightarrow F_{T_1} = \frac{mg}{\sin \theta} = \frac{(0.155)(9.81)}{\sin(28)} = 3.238854 \approx \boxed{3.2N}$$

$$\sum F_x = F_{T_2} - F_{T_{1x}} = ma_x = m(0) = 0 \Rightarrow F_{T_2} - F_{T_{1x}} = 0 \Rightarrow F_{T_2} = F_{T_{1x}} = F_{T_1} \cos \theta$$

$$\Rightarrow F_{T_2} = (3.238854) \cos(28) = 2.85974 \approx \boxed{2.9N}$$

Note: The mass hanging is an object in translational equilibrium because the net force acting on it equals zero.



Flipping Physics Lecture Notes:
Introduction to Static and Kinetic Friction
by Bobby

Friction:

- A force that tries to prevent two surfaces from sliding relative to one another.
- Is caused by two surfaces rubbing against one another.
- The symbol for the Force of Friction is \vec{F}_f .
- Is a vector and therefore has both magnitude and direction.
- Static:
 - “lacking in movement, action, or change”.
 - surfaces do not slide relative to one another.
- Kinetic:
 - “of, relating to, or resulting from motion”.
 - surfaces do slide relative to one another.
- Is not dependent on surface area.
 - As surface area increases, pressure decreases and Force of Friction remains the same.

$$P = \frac{F}{A}$$

- Direction:
 - \parallel to surfaces
 - Opposes sliding/motion
 - Independent of \vec{F}_a

Comment from mr.p:

Newton's Third Law states: $\vec{F}_{12} = -\vec{F}_{21}$: for every force object 1 exerts on object 2, there is an equal but opposite force object 2 exerts on object 1.

In Bobby's example, the force of friction he talks about is the force of friction the ramp exerts on the block, which is up the incline. There is also a force of friction the block exerts on the ramp, which is down the incline. These two different forces of friction form a Newton's Third Law Force Pair. They are equal in magnitude, opposite in direction, and act on two different objects.



Flipping Physics Lecture Notes:
Introduction to the Coefficient of Friction

$\vec{F}_f = \mu \vec{F}_N$ The force of friction equals the coefficient of friction times the force normal.

The Coefficient of Friction:

- The symbol is the Greek lowercase letter mu, which is μ .
- $\vec{F}_f = \mu \vec{F}_N \Rightarrow \mu = \frac{\vec{F}_f}{\vec{F}_N} \Rightarrow \frac{N}{N} = 1$
 - μ is dimensionless, it has no units.
- Depends on the materials the two contacting surfaces are made of.

Material 1	Material 2	Coefficient of Friction	
		Static	Kinetic
Aluminum	Steel (mild)	0.61	0.47
Glass	Glass	0.9-1.0	0.4
Glass	Nickel	0.78	0.56
Oak	Oak (parallel grain)	0.62	0.48
Oak	Oak (cross grain)	0.54	0.32
Steel (mild)	Steel (mild)	0.74	0.57
Steel (hard)	Steel (hard)	0.78	0.42

- From The Engineers Handbook*: “Extreme care is needed in using friction coefficients and additional independent references should be used. For any specific application the ideal method of determining the coefficient of friction is by trials.”
 - Coefficients of friction are experimentally determined.
 - The only way to know for sure is to perform the experiment.
- $\mu_s > \mu_k$
 - It is harder to get an object moving than it is to keep it moving.
- A typical range of values for μ is 0 – 2.
 - Highest published value I have seen is 4*, which is a bit extreme because it is for drag racing tires on dry concrete.

* www.engineershandbook.com/Tables/frictioncoefficients.htm

* hpwizard.com/tire-friction-coefficient.html



Flipping Physics Lecture Notes: Understanding the Force of Friction Equation

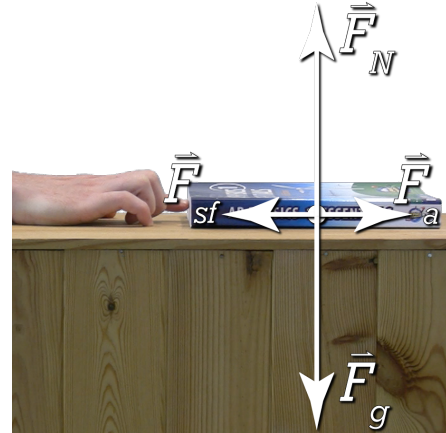
$\vec{F}_f = \mu \vec{F}_N$ The force of friction equals the coefficient of friction times the force normal.

Kinetic Friction: $\vec{F}_{kf} = \mu_k \vec{F}_N$

Static Friction: $\vec{F}_{sf} \leq \mu_s \vec{F}_N$ & $\vec{F}_{sf_{max}} = \mu_s \vec{F}_N$

Static friction example Free Body Diagram:

- The force of static friction is to the left because if there were no friction, the book would slide to the right, so the force of static friction is preventing the book from sliding to the right.
- $\sum F_x = F_a - F_{sf} = ma_x = m(0) = 0 \Rightarrow F_a = F_{sf}$
- The force of static friction increase or decreases in an attempt to prevent the object from moving and, in this case, once the magnitude of the force applied exceeds the maximum magnitude of the force of static friction, the book starts to move and the friction switches to kinetic friction.



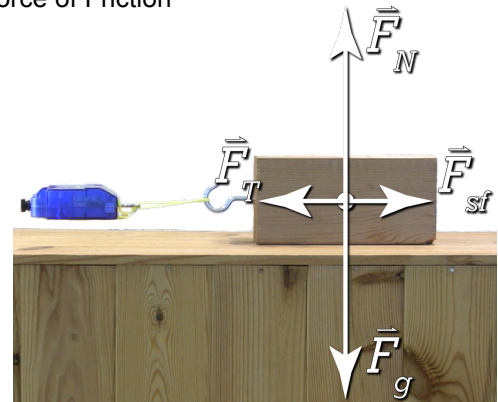


Flipping Physics Lecture Notes: Experimentally Graphing the Force of Friction

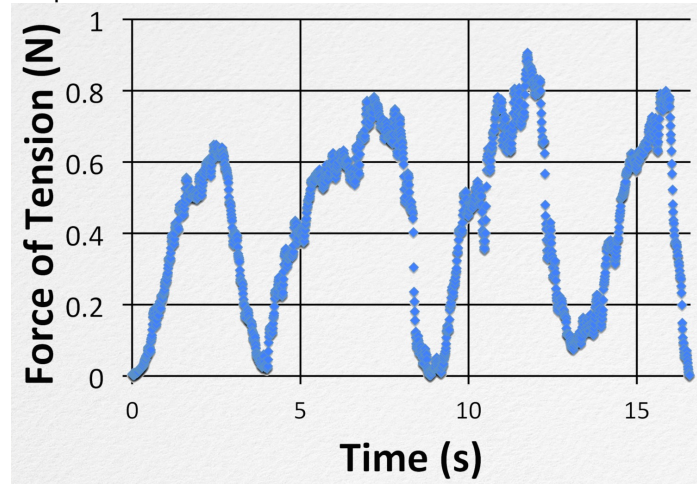
First we draw the Free Body Diagram.
Then we sum the forces in the x-direction.

$$\sum F_x = F_{sf} - F_T = ma_x = m(0) = 0 \Rightarrow F_{sf} = F_T$$

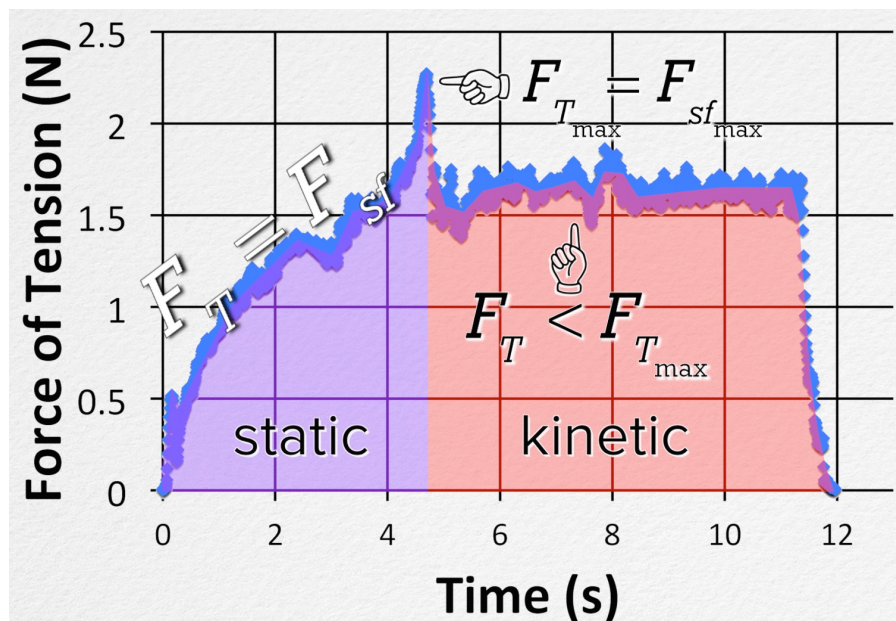
Which shows that, as long as the block isn't accelerating, the Force of Static Friction and the Force of Tension are equal in magnitude.



Graph when the block doesn't move.



Now I pull until the block moves. The first part is all static friction, when the block is stationary and the force of tension equals the force of static friction. This means the maximum force of static friction is also the force of tension. Then the block starts to move and the friction switches to kinetic. After the block starts to move, the force of tension decreases, because the coefficient of static friction is greater than the coefficient of kinetic friction: $\mu_s > \mu_k$. This graph shows it is harder to put an object in motion than it is to keep something moving.





Flipping Physics Lecture Notes:
Does the Book Move?
An Introductory Friction Problem

Example Problem: You apply a horizontal force of 2.0 Newtons to a book with a mass of 0.674 kg. The values for the coefficients of friction between the book and the incline are $\mu_s = 0.27$ and $\mu_k = 0.24$.

(a) Does the book move? (b) What is the acceleration of the book?

$$F_a = 2.0\text{N (horizontal)}; m = 0.674\text{kg}; \mu_s = 0.27; \mu_k = 0.24;$$

(a) Does the book move?

(b) $a = ?$

$$\sum F_y = F_N - F_g = ma_y = m(0) = 0 \Rightarrow F_N = F_g = mg$$

$$\sum F_x = F_a - F_f = ma_x \Rightarrow F_a - F_{sf_{\max}} = ma_x \Rightarrow F_a - \mu_s F_N = ma_x \Rightarrow F_a - \mu_s mg = ma_x$$

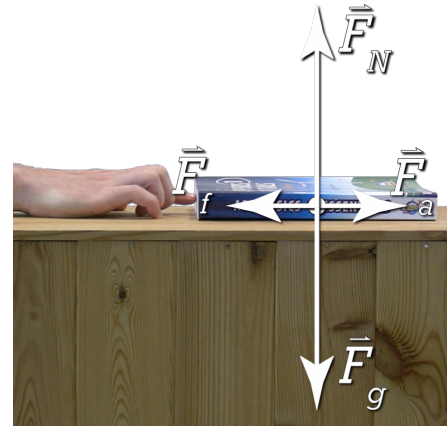
$$\Rightarrow 2 - (0.27)(0.674)(9.81) = (0.674)a_x \Rightarrow 2 - 1.7852 = 0.674a_x$$

(a) Because the net force in the x-direction is positive, the book will move to the right.

(b) Now that the book is moving, the friction is no longer static, it is kinetic.

$$\sum F_x = F_a - F_f = ma_x \Rightarrow F_a - F_{kf} = F_a - \mu_k F_N = F_a - \mu_k mg = ma_x \Rightarrow a_x = \frac{F_a - \mu_k mg}{m}$$

$$\Rightarrow a_x = \frac{2 - (0.24)(0.674)(9.81)}{0.674} = 0.61296 \approx \boxed{0.61 \frac{m}{s^2}}$$

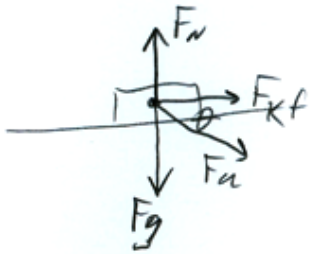


Thomas-Palmer Physics In-Class Lecture Notes:

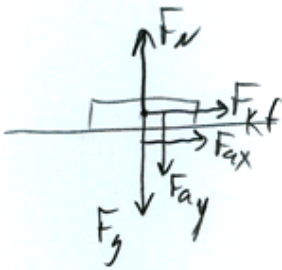
A Friction Review Problem - The Original Billy Bobby and Bo
Thank You, Mr. Thomas-Palmer, for these notes. (Oh wait, that's me.)

CH 04.4: BASIN OF BELLIGERANT
BOSSA NOVA BANQUETS
Friction (no Incline) Review Problem

$v_i = -3.0 \text{ m/s}$ $m = 52.4 \text{ g} \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 0.0524 \text{ kg}$ $\mu_k = 0.17$
 $\Delta x = 75 \text{ cm} \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) = 0.75 \text{ m}$ $F_a = 0.20 \text{ N} @ 15^\circ \text{ below Horiz.}$



$\sin \theta = \frac{op}{h} = \frac{F_{ay}}{F_a} \Rightarrow F_{ay} = F_a \sin \theta$
 $F_{ay} = 0.2 \sin 15 = 0.051764 \text{ N}$
 $\cos \theta = \frac{adj}{h} = \frac{F_{ax}}{F_a} \Rightarrow F_{ax} = F_a \cos \theta = 0.2 \cos 15 = 0.19319 \text{ N}$



$\sum F_y = F_n - F_g - F_{ay} = m a_y = m(0) = 0$
 $F_n - F_g - F_{ay} = 0 \Rightarrow F_n = F_g + F_{ay}$
 $F_n = m g + F_{ay} = (0.0524)(9.8) + (0.051764)$
 $F_n = 0.565284 \text{ N}$

$\sum F_x = F_{k} + F_{ax} = m a_x \Rightarrow a_x = \mu_k \frac{F_n + F_{ax}}{m}$

$a_x = \frac{(0.17)(0.565284) + 0.19319}{0.0524} = 5.5208 \text{ m/s}^2$

UAM $v_{fx}^2 = v_{ix}^2 + 2 a_x \Delta x \Rightarrow v_{fx}^2 - v_{ix}^2 = 2 a_x \Delta x$

$\Delta x = \frac{v_{fx}^2 - v_{ix}^2}{2 a_x} = \frac{0^2 - (-3)^2}{(2)(5.5208)} = -0.81510 \text{ m} \approx -0.92 \text{ m}$

it slides too far, AHHH!



Flipping Physics Lecture Notes:

Everybody Brought Mass to the Party!

You have to be able to identify when mass cancels out of an equation.

Example #1:

$$mg \cos \theta - \mu_k mg \sin \theta = ma \Rightarrow \frac{\cancel{m}g \cos \theta}{\cancel{m}} - \frac{\mu_k \cancel{m}g \sin \theta}{\cancel{m}} = \frac{\cancel{m}a}{\cancel{m}} \Rightarrow g \cos \theta - \mu_k g \sin \theta = a$$

The "equation" is the party and the individuals are delineated by a subtraction, addition or equal sign. "Everybody" brought mass to the party! " $mg \cos \theta$ " brought mass. " $\mu_k mg \sin \theta$ " brought mass. " ma " brought mass. Therefore we can be equitable and take mass from everybody.

Example #2:

$$0 = \mu_k mg \sin \theta + mgh \Rightarrow \frac{0}{m} = \frac{\mu_k \cancel{m}g \sin \theta}{\cancel{m}} + \frac{\cancel{m}gh}{\cancel{m}} \Rightarrow 0 = \mu_k g \sin \theta + gh$$

Zero is nobody and because $\frac{0}{m} = 0$, we can take mass from nobody. Everybody brought mass to the party!

Example #3:

$$mgh + \frac{1}{2}mv^2 = 4kx^2 \Rightarrow \frac{\cancel{m}gh}{\cancel{m}} + \frac{\frac{1}{2}\cancel{m}v^2}{\cancel{m}} = \frac{\frac{1}{2}kx^2}{m} \Rightarrow gh + \frac{v^2}{2} = \frac{kx^2}{2m}$$

$\frac{1}{2}kx^2$ didn't bring mass to the party, so we can't take mass from everybody.

Everybody did *not* bring mass to the party. ☹

Example #4:

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + mgL(1 - \cos \theta) \Rightarrow \frac{\frac{1}{2}\cancel{m}v_i^2}{\cancel{m}} = \frac{\frac{1}{2}\cancel{m}v_f^2}{\cancel{m}} + \frac{\cancel{m}gL(1 - \cos \theta)}{\cancel{m}} \Rightarrow \frac{1}{2}v_i^2 = \frac{1}{2}v_f^2 + gL(1 - \cos \theta)$$

Everybody brought mass to the party! ☺



Flipping Physics Lecture Notes:

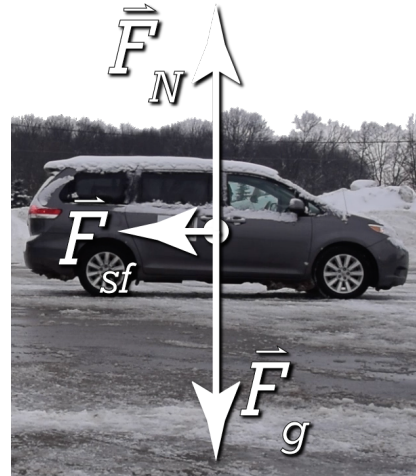
Determining the Static Coefficient of Friction between Tires and Snow

Example: A car with anti-lock brakes driving on snow has an initial velocity of 8.9 m/s and slows to a stop in 3.12 seconds. Determine the coefficient of friction between the tires and the snow.

Knowns: $v_i = 8.9 \frac{m}{s}$; $\Delta t = 3.12 s$; $v_f = 0$; $\mu_s = ?$

$$\begin{aligned} \sum F_y &= F_N - F_g = ma_y = m(0) = 0 \Rightarrow F_N = F_g = mg \\ \sum F_x &= -F_{sf} = ma_x \Rightarrow -\mu_s F_N = ma_x \Rightarrow -\mu_s mg = ma_x \\ \Rightarrow -\mu_s g &= a_x \Rightarrow \mu_s = -\frac{a_x}{g} \end{aligned}$$

All we need is the acceleration in the x direction and we can use our Uniformly Accelerated Motion equations to find a_x .



$$v_f = v_i + a\Delta t \Rightarrow v_f - v_i = a\Delta t \Rightarrow a = \frac{v_f - v_i}{\Delta t} = \frac{0 - (8.9)}{3.12} = -2.85256 \frac{m}{s^2}$$

And we can now solve for the coefficient of friction.

$$\mu_s = -\frac{a_x}{g} = -\frac{-2.85256}{9.81} = 0.290781 \approx 0.29$$



I actually did 9 trials and the average of all nine trials gave me: $\mu_{s_{average}} = 0.299972 \approx 0.30$

$$\text{FYI: } v_i = 8.9 \frac{m}{s} \times \frac{3600s}{1hr} \times \frac{1mi}{1609m} = 19.913 \approx 20 \frac{mi}{hr} \quad (\text{okay, fine. With 2 sig figs: } v_i \approx 2.0 \times 10^1 \frac{mi}{hr})$$

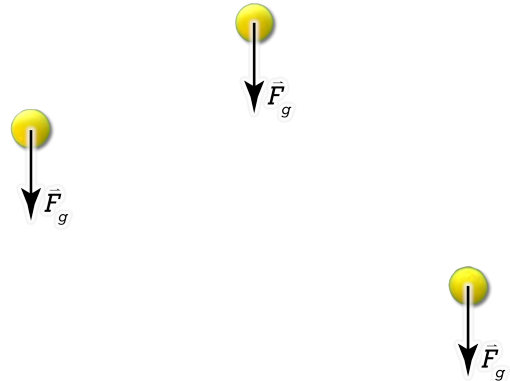


Flipping Physics Lecture Notes:

g is Positive.

<http://www.flippingphysics.com/+g.html>

The acceleration due to gravity, g , is **positive**. Here on Earth, it has a value of roughly 9.81 meters per second squared. And, again, g is positive. However, I understand that many students question that. And I think this is why they typically do. For the first month or so of physics class, every time we use the acceleration due to gravity, it is as a part of the acceleration experienced by an object in free fall. We have shown many examples of that. We have thrown a medicine ball up in the air¹. We have dropped dictionaries². From a moving car, we have dropped a ball and had it land in a bucket³. And we have launched a ball from a nerd-a-pult and landed the ball in a basket⁴. Every time we did that the acceleration of the object in the y -direction was equal to negative 9.81 meters per second squared. So, again, I understand, because the acceleration of the object was always negative 9.81 meters per second squared in each of these problems, students tend to infer that the acceleration due to gravity is also negative 9.81 meters per second squared. It is not. g is **positive** 9.81 meters per second squared.



Now that we have learned Newton's Second Law, we can better understand why $g = +9.81 \text{ m/s}^2$ near the surface of planet Earth. Let's begin by looking at the simple example of a ball in projectile motion. The free body diagram of the forces acting on any object in projectile motion has only one force in it, the force of gravity acting straight down. When we sum the forces in the y -direction on the ball, we get this:

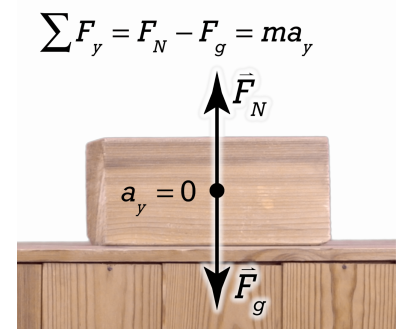
$$\sum F_y = -F_g = ma_y \Rightarrow -mg = ma_y \Rightarrow a_y = -g \text{ \& } g_{\text{Earth}} = +9.81 \frac{\text{m}}{\text{s}^2} \Rightarrow a_y = -9.81 \frac{\text{m}}{\text{s}^2}$$

In other words, the acceleration of an object in projectile motion near the surface of planet Earth equals negative 9.81 m/s^2 . That free fall acceleration is caused by the force of gravity. The reason free fall acceleration is negative, is because the force of gravity is down or toward the center of the planet.

Notice, the acceleration due to gravity, g , is not the acceleration of the object, but rather, it is the value by which you multiply the mass of an object to get the force of gravity the Earth exerts on an object.

The acceleration of an object and acceleration due to gravity, g , are two different things: $a \neq g$

An object at rest on a surface will still have the force of gravity acting on it. You still multiply mass times the acceleration due to gravity to get the magnitude of the force of gravity acting on the object, however, in this case, the acceleration of the object is zero. Again, the acceleration of the object and the acceleration due to gravity are two different things. And the acceleration due to gravity, g , near the surface of planet Earth is **positive** 9.81 meters per second squared.



$$\sum F_y = F_N - F_g = ma_y$$

¹ <https://www.flippingphysics.com/introduction-to-free-fall.html>

² <https://www.flippingphysics.com/dropping-dictionaries.html>

³ <https://www.flippingphysics.com/projectile-motion-problem-part-1-of-2.html>

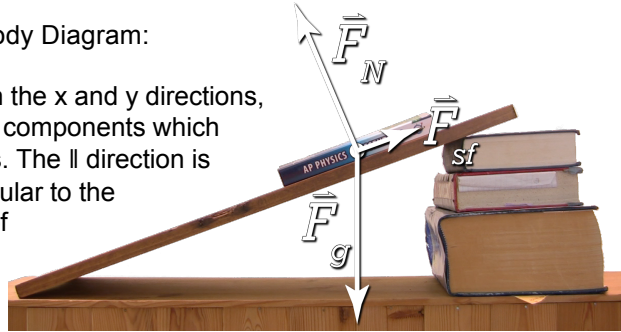
⁴ <https://www.flippingphysics.com/nerd-a-pult.html>

Flipping Physics Lecture Notes:

Breaking the Force of Gravity into its Components on an Incline

A book at rest on an incline has the following Free Body Diagram:

Previously we resolved the vectors into components in the x and y directions, however, on an incline we usually resolve vectors into components which are in the \parallel (parallel) and \perp (perpendicular) directions. The \parallel direction is parallel to the incline and the \perp direction is perpendicular to the incline. This means we only need to break the Force of Gravity into components and not the Force Normal or the Force of Static Friction.



Before we determine the equations for the Force of Gravity components, we need to discuss the angles.

θ_1 is called the incline angle. θ_2 is the angle between the force of gravity perpendicular and the force of gravity. θ_3 is the angle between the incline and the force of gravity. Because the incline and the force of gravity parallel are parallel to one another, θ_3 is also the angle between the force of gravity parallel and the force of gravity.

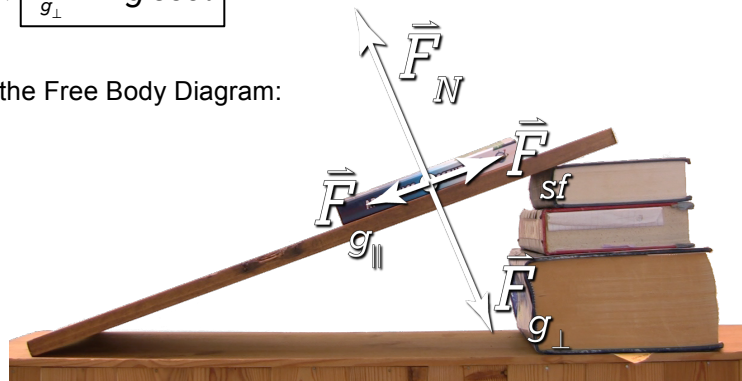
$$180^\circ = \theta_1 + 90^\circ + \theta_3 \Rightarrow 90^\circ = \theta_1 + \theta_3 \Rightarrow \theta_1 = 90^\circ - \theta_3 \quad \& \quad 90^\circ = \theta_2 + \theta_3 \Rightarrow \theta_2 = 90^\circ - \theta_3 = \theta_1$$

So the incline angle and the angle we use to determine the Force of Gravity components are the same.

$$\sin \theta = \frac{O}{H} = \frac{F_{g\parallel}}{F_g} \Rightarrow F_{g\parallel} = F_g \sin \theta \Rightarrow \boxed{F_{g\parallel} = mg \sin \theta}$$

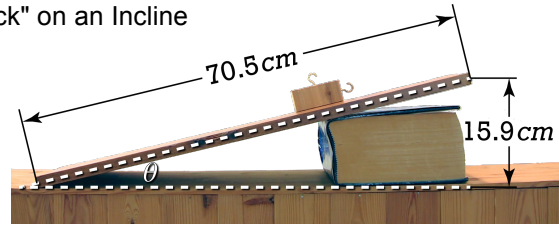
$$\cos \theta = \frac{A}{H} = \frac{F_{g\perp}}{F_g} \Rightarrow F_{g\perp} = F_g \cos \theta \Rightarrow \boxed{F_{g\perp} = mg \cos \theta}$$

And then, of course, we should redraw the Free Body Diagram:



First we need the incline angle:

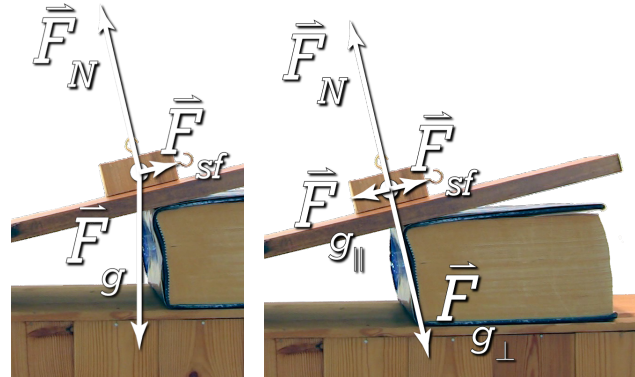
$$\sin \theta = \frac{O}{H} = \frac{15.9 \text{ cm}}{70.5 \text{ cm}} \Rightarrow \theta = \sin^{-1} \left(\frac{15.9}{70.5} \right) = 13.0342^\circ$$



The mass of the block:

$$m_{\text{block}} = 121 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 0.121 \text{ kg}$$

- Draw The Free Body Diagram.
- Break the Force of Gravity into its parallel and perpendicular components.
- Redraw the Free Body Diagram.



Newton's Second Law in both directions:

$$\sum F_{\parallel} = F_{sf} - F_{g_{\parallel}} = ma_{\parallel} = m(0) = 0 \Rightarrow F_{sf} = F_{g_{\parallel}} = mg \sin \theta$$

$$\Rightarrow F_{sf} = F_{g_{\parallel}} = (0.121)(9.81) \sin(13.0342) = 0.267709 \approx 0.268 \text{ N}$$

$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg \cos \theta$$

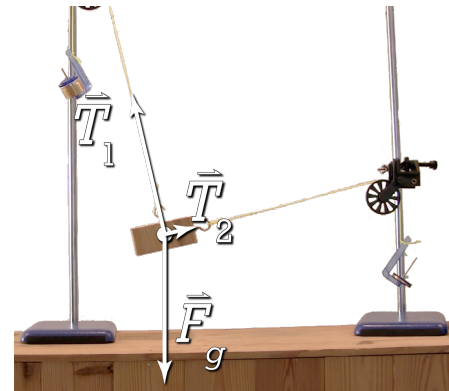
$$\Rightarrow F_N = F_{g_{\perp}} = (0.121)(9.81) \cos(13.0342) = 1.15643 \approx 1.16 \text{ N}$$

The "Magic Trick" math:

$$m_{\parallel} = m \sin \theta = (121) \sin(13.0342) = 27.2894 \approx 27.3 \text{ g}$$

$$m_{\perp} = m \cos \theta = (121) \cos(13.0342) = 117.883 \approx 118 \text{ g}$$

The "Floating Block" replaces the Force Normal with Force of Tension 1 and the Force of Static Friction with Force of Tension 2.



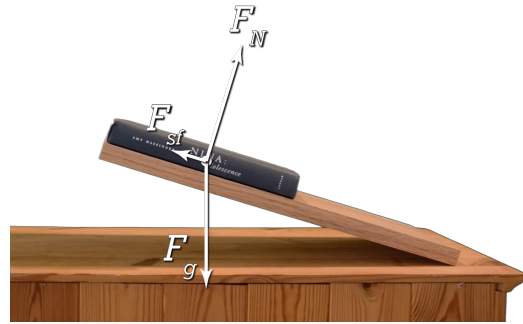
Flipping Physics Lecture Notes:

An Introductory Static Friction on an Incline Problem

Example: A book is resting on a board. One end of the board is slowly raised. The book starts to slide when the incline angle is 15° . What is the coefficient of static friction between the book and the incline?

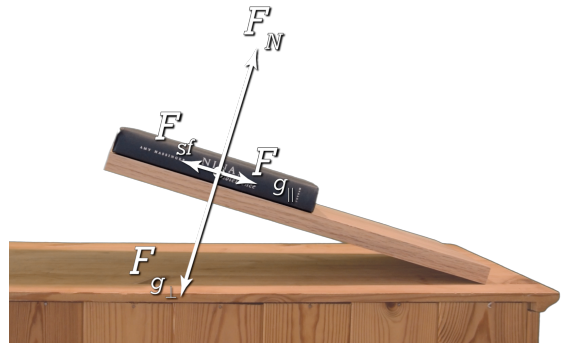
Knowns: $\theta = 15^\circ$; $\mu_s = ?$

Draw the Free Body Diagram.



Break the Force of Gravity into its components.

Redraw the Free Body Diagram.



$$\sum F_{\parallel} = F_{g_{\parallel}} - F_{sf} = ma_{\parallel} = m(0) = 0 \Rightarrow F_{g_{\parallel}} = F_{sf_{\max}} \Rightarrow mg \sin \theta = \mu_s F_N$$

Note: The acceleration in the parallel direction is zero because this is *static* friction. In other words, the book is not sliding relative to the incline yet.

We need the Force Normal.

$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg \cos \theta$$

$$\& \ mg \sin \theta = \mu_s F_N \Rightarrow mg \sin \theta = \mu_s mg \cos \theta \Rightarrow \sin \theta = \mu_s \cos \theta \Rightarrow \mu_s = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\mu_s = \tan(15) = 0.267949 \approx \boxed{0.27}$$



Flipping Physics Lecture Notes:

Calculating the Uncertainty of the Coefficient of Friction

Example: A book is resting on a board. One end of the board is slowly raised. The book starts to slide when the incline angle is 15° . What is the coefficient of static friction between the book and the incline?

As shown at <http://www.flippingphysics.com/static-friction-incline.html> the answer is:

$$\mu_s = \tan \theta = \tan(15) = 0.267949 \approx 0.27$$

The reality is that the measurements were as such:

Incline Angle (in degrees)	Tangent of Incline Angle and μ_s (6 significant digits)
13	0.230867
13	0.230867
14	0.248328
15	0.267949
15	0.267949
15	0.267949
16	0.286745
17	0.305731
17	0.305731
17	0.305731

The average value is $\mu_{s \text{ average}} = 0.271855 \approx 0.27$

The standard deviation is $0.027527 \approx 0.03$
 (Method #2 from this WikiHow shows how to calculate standard deviation.
<http://www.wikihow.com/Calculate-Uncertainty>)

Therefore the coefficient of static friction is 0.27 with an uncertainty of plus or minus 0.03: $\mu_s \approx 0.27 \pm 0.03$

Note: I have illustrated the uncertainty with plus or minus 1 standard deviation, this means roughly 68%¹ of the measurements should fall within the range 0.24 – 0.30. If you would prefer, you can use 2 standard deviations instead. This would mean 95% of the measurements should fall within the range. Or, you could even use 3 standard deviations. This would mean 99.7% of the measurements should fall within the range. FYI: The symbol for standard deviation is typically a lowercase, Greek letter sigma, σ .

Average Value	Number of Standard Deviations (# of σ)	Uncertainty	Average Value with Uncertainty	Value Range	Predicted % of Measurements within the Range
0.27	1	$\sigma = 0.027527 \approx 0.03$	0.27 ± 0.03	0.24 – 0.30	68%
0.27	2	$2\sigma = (2)(0.027527) = 0.055054 \approx 0.06$	0.27 ± 0.06	0.21 – 0.33	95%
0.27	3	$3\sigma = (3)(0.027527) = 0.082582 \approx 0.08$	0.27 ± 0.08	0.19 – 0.35	99.7%

Also note: This way of calculating uncertainty is very useful and will be used more often as you get further into science and engineering. However, we will not be using it very often in our algebra based physics class.

¹ <http://www.statisticshowto.com/68-95-99-7-rule/>



Flipping Physics Lecture Notes:

An Introductory Kinetic Friction on an Incline Problem

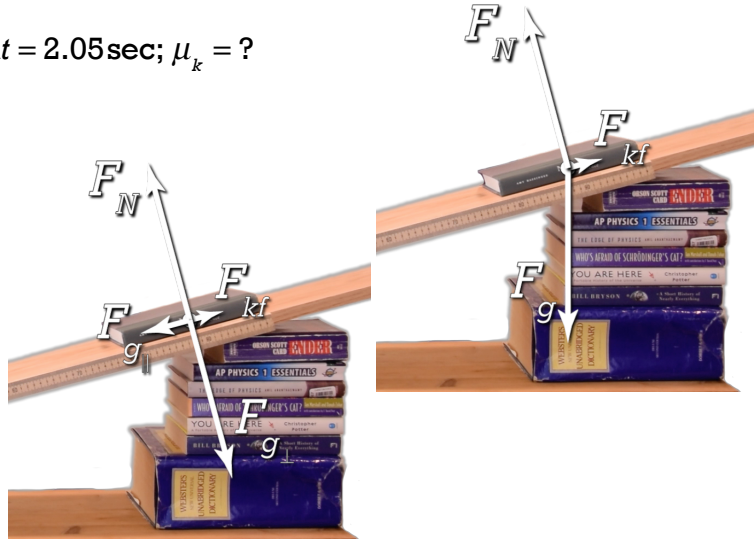
Example: You place a book on a 14° incline and then let go of the book. If the book takes 2.05 seconds to travel 0.78 meters, what is the coefficient of kinetic friction between the book and the incline?

Knowns: $\theta = 14^\circ$; $v_i = 0$; $\Delta d_{\parallel} = 0.78\text{m}$; $\Delta t = 2.05\text{sec}$; $\mu_k = ?$

Draw the Free Body Diagram.

Break Force of Gravity into its components.

Redraw the Free Body Diagram.



$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg \cos \theta$$

$$\sum F_{\parallel} = F_{k_f} - F_{g_{\parallel}} = ma_{\parallel} \Rightarrow \mu_k F_N - mg \sin \theta = ma_{\parallel} \Rightarrow \mu_k (mg \cos \theta) - mg \sin \theta = ma_{\parallel}$$

$$\Rightarrow \mu_k g \cos \theta - g \sin \theta = a_{\parallel} \Rightarrow \mu_k g \cos \theta = g \sin \theta + a_{\parallel} \Rightarrow \mu_k = \frac{g \sin \theta + a_{\parallel}}{g \cos \theta}$$

Now we need the acceleration in the parallel direction. Use a Uniformly Accelerated Motion equation.

$$\Delta d_{\parallel} = v_i \Delta t + \frac{1}{2} a_{\parallel} \Delta t^2 = (0) \Delta t + \frac{1}{2} a_{\parallel} \Delta t^2 = \frac{1}{2} a_{\parallel} \Delta t^2 \Rightarrow 2 \Delta d_{\parallel} = a_{\parallel} \Delta t^2 \Rightarrow a_{\parallel} = \frac{2 \Delta d_{\parallel}}{\Delta t^2}$$

$$\Rightarrow 2 \Delta d_{\parallel} = a_{\parallel} \Delta t^2 \Rightarrow a_{\parallel} = \frac{2 \Delta d_{\parallel}}{\Delta t^2} = \frac{(2)(0.78)}{(2.05)^2} = 0.371208 \frac{\text{m}}{\text{s}^2}$$

$$\mu_k = \frac{(9.81) \sin 14 + (0.371208)}{(9.81) \cos 14} = 0.288326 \approx 0.29$$

Note: This cannot be correct because previously we determined the coefficient of static friction between this book and this incline to be 0.27 and $\mu_k < \mu_s$. The mistake we made was that the displacement in the parallel direction is negative, which makes the acceleration negative, which changes our answer.

$$\Rightarrow a_{\parallel} = \frac{(2)(-0.78)}{(2.05)^2} = -0.371208 \frac{\text{m}}{\text{s}^2} \quad \& \quad \mu_k = \frac{(9.81) \sin 14 + (-0.371208)}{(9.81) \cos 14} = 0.21030 \approx \boxed{0.21}$$



Flipping Physics Lecture Notes:

An incline, 2 free body diagrams, and a pulley.
What could be more fun?

<http://www.flippingphysics.com/incline-masses-pulley.html>

A 55 g mass is attached to a light string, which is placed over a frictionless, massless pulley, and attached to a 199 g block which is on a board inclined at 39.3° as shown. Assuming the block starts at rest and the μ_k between the incline and block is 0.38, how long will it take the block to move 13 cm?

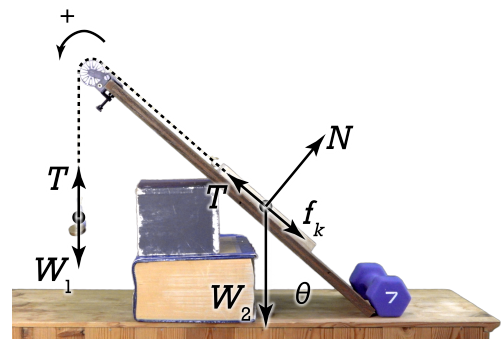
Identify the 55 g mass as mass 1 and the 199 g mass as mass 2.

$$m_1 = 55g; m_2 = 199g; \mu_k = 0.38; \theta = 39.3^\circ; \Delta d_{\parallel} = 13cm; \Delta t = ?$$

Now we draw the free body diagrams:

On mass 1 there is a weight or force of gravity acting down, let's call it weight 1, W_1 . There is also a force of tension acting upward, T .

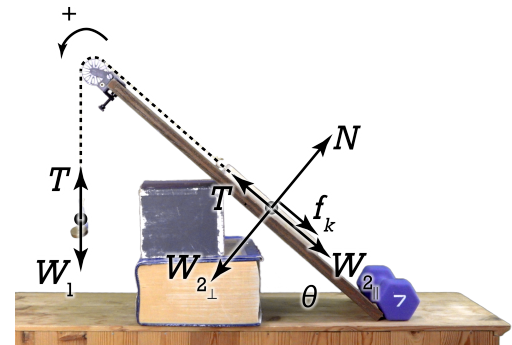
On mass 2 there is a weight or force of gravity acting down, let's call it weight 2, W_2 . There is a force normal, N , acting up and perpendicular to the incline. There is also a force of tension acting up the incline, T .



Because the pulley is massless and frictionless, those two forces of tension are equal in magnitude. If the pulley had mass, the two forces of tension would not be equal in magnitude. If the pulley were not frictionless, the two forces of tension would not be equal in magnitude. Do not worry, pulleys will have mass and friction in later problems. ☺

The force of kinetic friction, f_k , is parallel to the incline and opposes motion, therefore, it is either up or down the incline. We need to predict which direction the masses will move in order to determine the direction of the force of kinetic friction. Let's predict mass 1 will move down and mass 2 will move up the incline. Therefore, the force of kinetic friction is opposite the direction mass 2 will move and is down the incline.

Break the Weight 2 or Force of Gravity 2 into its components. And add the direction we think the objects will move.



Now we can begin using Newton's Second Law and sum the forces. Whenever we sum the forces, we need to identify:

- The object (or objects) we are summing the forces on.
- The direction we are summing the forces in.

The goal with summing the forces is to solve for the acceleration of the two masses, so we can then use a uniformly accelerated motion equation to solve for the change in time.

$$\sum_{\substack{\text{+ direction} \\ \text{on both}}} F = W_1 - T + T - f_k - W_{2\parallel} = m_t a_{\parallel} \Rightarrow m_1 g - \mu_k N - m_2 g \sin \theta = (m_1 + m_2) a_{\parallel}$$

At this point notice there are two unknown variables: force normal and acceleration parallel. This equation goes in our equation holster and we move on to another net force equation.

$$\sum_{\substack{\perp \text{ direction} \\ \text{on mass 2}}} F = N - W_{2\perp} = m_2 a_{\perp} = m_2 (0) = 0 \Rightarrow N = W_{2\perp} = m_2 g \cos \theta$$

We now have an equation for the force normal which we can substitute into our equation holster equation.

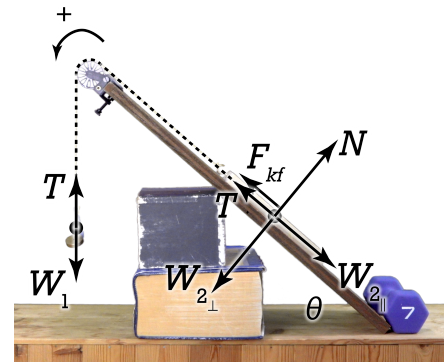
$$\Rightarrow m_1 g - \mu_k (m_2 g \cos \theta) - m_2 g \sin \theta = (m_1 + m_2) a_{\parallel} \Rightarrow a_{\parallel} = \frac{m_1 g - \mu_k m_2 g \cos \theta - m_2 g \sin \theta}{m_1 + m_2}$$

And now we can substitute in numbers, remembering to convert all of the masses to kilograms. Actually, technically we do not have to convert the masses to kilograms because the units for mass cancel out of the equation, however, it is still a good habit to get into. Another smiley face → ☺

$$\Rightarrow a_{\parallel} = \frac{(0.055)(9.81) - (0.38)(0.199)(9.81)\cos(39.3) - (0.199)(9.81)\sin(39.3)}{0.055 + 0.199} = -5.00390 \frac{m}{s^2}$$

And we get a negative for our acceleration...

This means we predicted the incorrect direction for the masses to move. We now need to go back, change the direction of the force of kinetic friction, and redo the problem. Weeeeeeee! Please realize we cannot simply “make the acceleration positive” because we have the incorrect direction for the force of kinetic friction. Here, have two smiley faces! ☺☺



$$\sum_{\substack{+ \text{ direction} \\ \text{on both}}} F = W_1 - T + T + f_k - W_{2\parallel} = m_1 a_{\parallel} \Rightarrow m_1 g + \mu_k N - m_2 g \sin \theta = (m_1 + m_2) a_{\parallel}$$

$$\Rightarrow m_1 g + \mu_k (m_2 g \cos \theta) - m_2 g \sin \theta = (m_1 + m_2) a_{\parallel} \Rightarrow a_{\parallel} = \frac{m_1 g + \mu_k m_2 g \cos \theta - m_2 g \sin \theta}{m_1 + m_2}$$

$$\Rightarrow a_{\parallel} = \frac{(0.055)(9.81) + (0.38)(0.199)(9.81)\cos(39.3) - (0.199)(9.81)\sin(39.3)}{0.055 + 0.199} = -0.483741 \frac{m}{s^2}$$

And now we can use a uniformly accelerated motion equation to solve for the change in time:

$$\Delta d_{\parallel} = v_i \Delta t + \frac{1}{2} a \Delta t^2 \Rightarrow 2 \Delta d_{\parallel} = a \Delta t^2 \Rightarrow \Delta t = \sqrt{\frac{2 \Delta d_{\parallel}}{a}} = \sqrt{\frac{(2)(-0.13)}{-0.483741}} = 0.733129 \approx 0.73 \text{ sec}$$