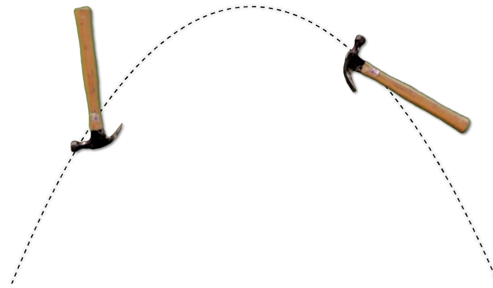
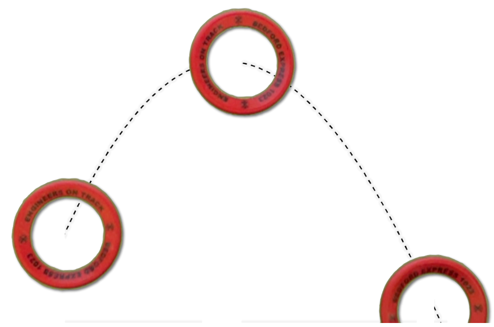
Flipping Physics Lecture Notes:

Introduction to Center of Mass



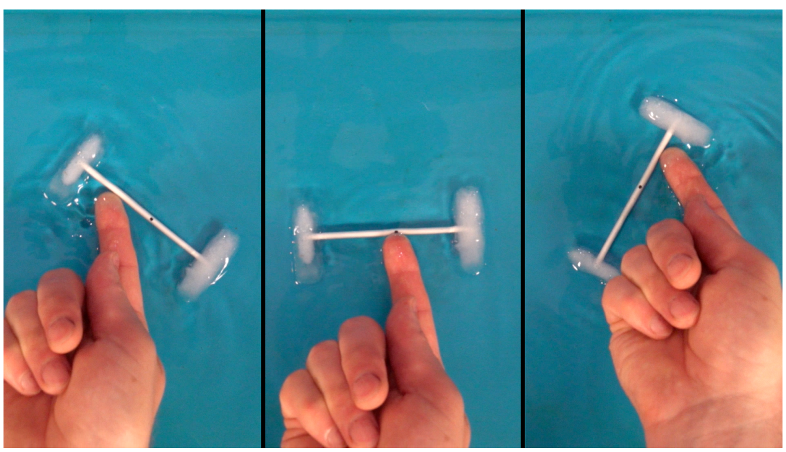
The center of mass of an object is the mass-weighted average position of all of the mass of the object. An object in projectile motion will rotate around its center of mass and its center of mass will follow the same parabolic path we are used to.

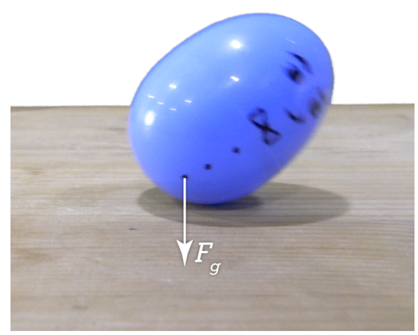
When the acceleration due to gravity is constant, as we consider it to be on the surface of planet Earth, the center of mass and center of gravity of an object are in the same location.



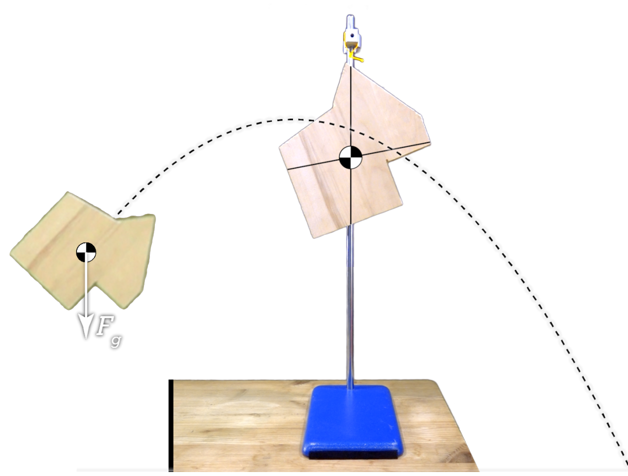
Notice that the center of mass of an object is not a physical location on the object and sometimes is not even located on the object at all. For example, this frisbee ring has its center of mass in the middle, not on the ring itself.

When you apply a force at or toward the center of mass of the object, the object will not rotate. If you apply a force to the left or right of the center of mass of the object, the object will rotate.





We consider the force of gravity to act on the center of mass of the object. The center of mass of a "Weeble" is very low on the object and when it is tipped over, the center of mass goes up. This is why a Weeble will always right itself.



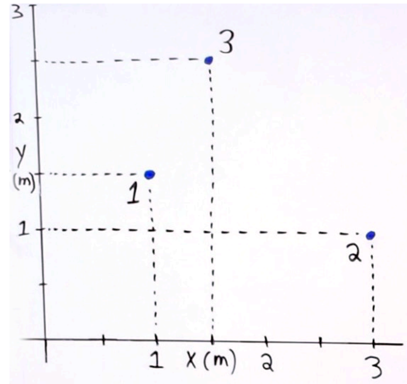
When an object is hung by one point, the force of gravity will pull on the center of mass of the object in an attempt to bring the center of mass to its lowest point. This is why we can hang a flat object to find it's center of mass, because the center of mass will always be below the hanging point.

Flipping Physics Lecture Notes:

Calculating the Center of Mass of a System of Particles

The equation for the position of the center of mass of a system of particles is:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots}$$



Where "m" is the mass of each object and "x" is the distance each object is from a zero reference point. The ellipses (…) mean you add as many expressions as you have objects in the system.

Example: Three point objects are located at various locations on a Cartesian coordinate system. Mass 1, with a mass of 1.1 kg, is located at (1.0,1.5) m. Mass 2, with a mass of 3.4 kg, is located at (3.0,1.0) m. Mass 3, with a mass of 1.3 kg, is located at (1.5,2.5) m. Where is the center of mass of the three-object system?

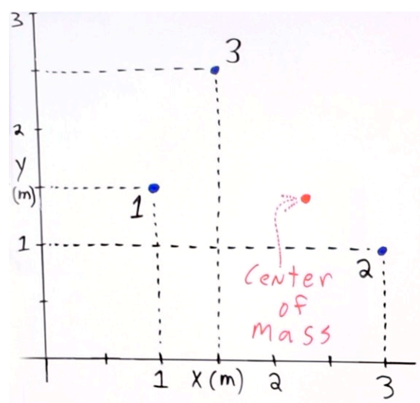
Knowns:

$$m_1 = 1.1kg;\ r_1 = \left(1.0,1.5\right)m;\ m_2 = 3.4kg;\ r_2 = \left(3.0,1.0\right)m;\ m_3 = 1.3kg;\ r_3 = \left(1.5,2.5\right)m;\ r_{cm} = ?$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{\left(1.1\right)\left(1\right) + \left(3.4\right)\left(3\right) + \left(1.3\right)\left(1.5\right)}{1.1 + 3.4 + 1.3} = 2.28448 \approx 2.3\mathrm{m}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{\left(1.1\right)\left(1.5\right) + \left(3.4\right)\left(1\right) + \left(1.3\right)\left(2.5\right)}{1.1 + 3.4 + 1.3} = 1.43103 \approx 1.4\mathrm{m}$$

$$r_{cm} \approx \left(2.3,1.4\right)\mathrm{m}$$

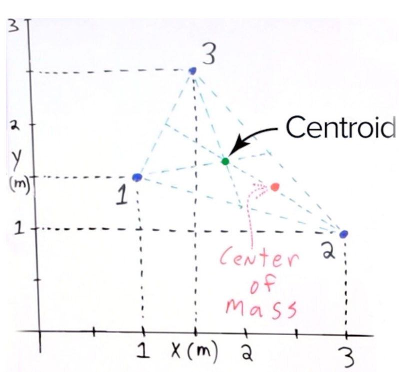


Note: The Center of Mass is different than the Centroid, which is the geometric center, or where the center of mass would be if all of the masses were the same.

$$x_{avg} = \frac{x_1 + x_2 + x_3}{3} = \frac{1 + 3 + 1.5}{3} = 1.8\overline{3} \approx 1.8m$$

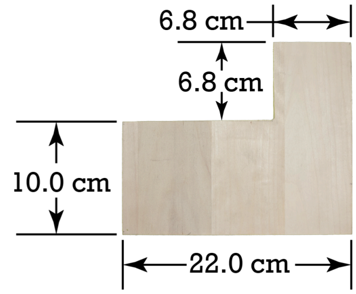
$$y_{avg} = \frac{y_1 + y_2 + y_3}{3} = \frac{1.5 + 1 + 2.5}{3} = 1.\overline{6} \approx 1.7m$$

$$r_{centroid} \approx \left(1.8,1.7\right)m$$



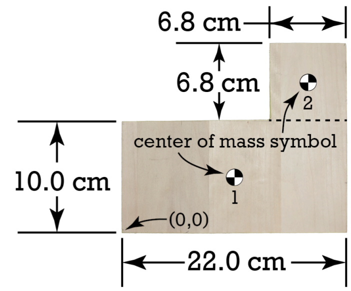
Flipping Physics Lecture Notes:

Center of Mass of an Irregular Object



Where is the center of mass of an "L" shaped, constant density, constant thickness block with the dimensions shown in the illustration?

Set the zero, zero location or origin at the lower leftmost corner of the block. Split the block into symmetrical shapes with known centers of mass locations.



Piece 1 is 22.0 by 10.0 cm and piece 2 is 6.8 by 6.8 cm. Both pieces have, due to symmetry and constant density, a center of mass at their geometric center:

$$r_1 = (11,5)cm \; \& \; r_2 = \left(22 - \frac{6.8}{2}, 10 + \frac{6.8}{2}\right) = (18.6, 13.4)cm$$

We can use the equation for center of mass of a system of particles to determine the center of mass of the "L" shaped block: $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

The issue here is we do not know the masses of the two pieces. However, we know both pieces have the same density. Therefore, we can set those two densities equal to one another:♥

$$\rho_1 = \rho_2 \Rightarrow \frac{m_1}{\forall_1} = \frac{m_2}{\forall_2} \Rightarrow \frac{m_1}{A_1 \times t} = \frac{m_2}{A_2 \times t} \Rightarrow \frac{m_1}{A_1} = \frac{m_2}{A_2} \Rightarrow m_1 = \frac{A_1}{A_2} m_2$$

This gives us a relationship between mass 1 and mass 2, which we can substitute back into the equation for center of mass of a system of particles.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{\frac{A_1}{A_2} m_2 x_1 + m_2 x_2}{\frac{A_1}{A_2} m_2 + m_2} = \frac{\frac{A_1}{A_2} x_1 + x_2}{\frac{A_1}{A_2} + 1} = \left(\frac{A_2}{A_2}\right) \frac{\frac{A_1}{A_2} x_1 + x_2}{\frac{A_1}{A_2} + 1} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

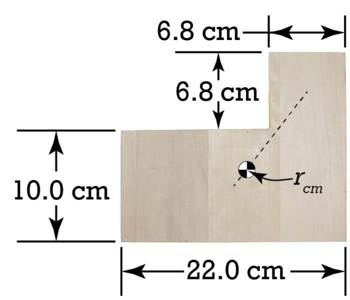
Notice in this step we have multiplied by $\frac{m_2}{m_2}$ and $\frac{m_2}{m_2} = 1$.

It turns out, if all the pieces of the object have the same density and thickness, we can substitute in the area of each piece for the mass of each piece. This can be very helpful to remember!

Now we can substitute in equations for area and numbers:

$$x_{cm} = \frac{L_1 w_1 x_1 + L_2 w_2 x_2}{L_1 w_1 + L_2 w_2} = \frac{(22)(10)(11) + (6.8)^2 (18.6)}{(22)(10) + (6.8)^2} = 12.31995 \approx 12cm$$

$$y_{cm} = \frac{L_1 w_1 y_1 + L_2 w_2 y_2}{L_1 w_1 + L_2 w_2} = \frac{(22)(10)(5) + (6.8)^2 (13.4)}{(22)(10) + (6.8)^2} = 6.45889 \approx 6.5cm$$



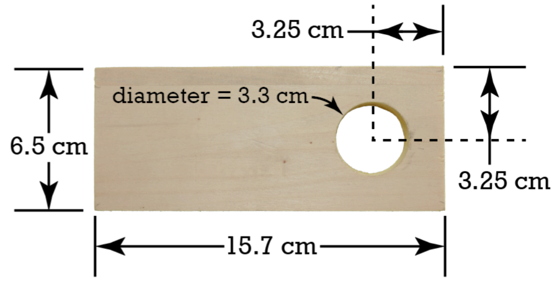
$$\boxed{r_{cm} \approx (12, 6.5)cm}$$

This is logical because the center of mass of the "L" shaped block should be somewhere between the centers of mass of the two pieces, but much closer to the center of piece 1.

♥ I always add a horizontal slash through my $\forall$ for volume. This is to differentiate it from v for velocity.

Flipping Physics Lecture Notes:

Center of Mass of an Object with a Hole



Where is the center of mass of a rectangular, constant density, constant thickness block that has a hole in it? Dimensions are shown in the illustration.

Set the zero, zero location or origin at the lower leftmost corner of the block. Split the block into symmetrical shapes with known centers of mass locations.

Piece 1 is a whole rectangular block, which is 15.7 by 6.5 cm.
- This theoretical piece does not have a hole in it.

Piece 2 is a hole with negative mass, which has a diameter of 3.3 cm.
- This theoretical piece is the hole that we need to remove from the rectangle by subtracting it.

Realize because both pieces have the same y-position center of mass, then the center of mass of the block will have the same center of mass: 3.25 centimeters up from zero.

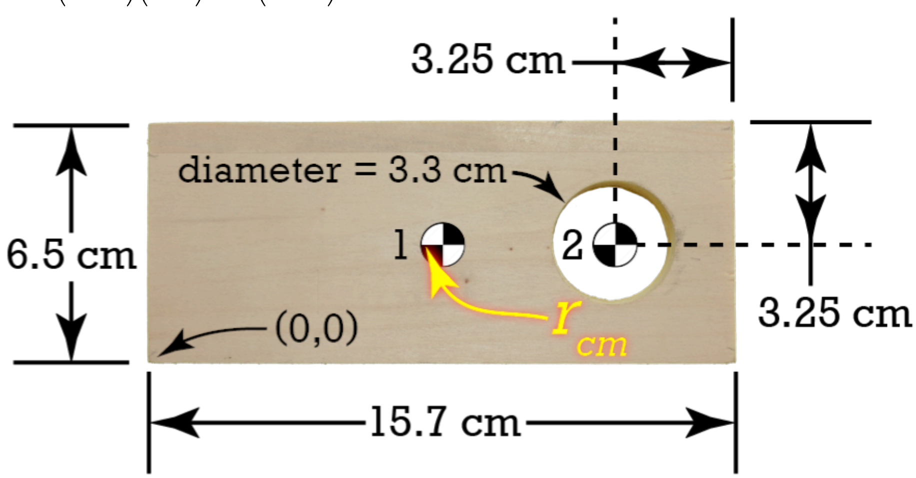
Both pieces have, due to symmetry and constant density, a center of mass at their geometric center:

$$x_1 = \frac{15.7}{2} = 7.85cm \ \& \ x_2 = 15.7 - 3.25 = 12.45cm \qquad \& \qquad r_2 = \frac{d_2}{2} = \frac{3.3}{2} = 1.65cm$$

We can use the equation for center of mass of a system of particles to determine the center of mass of the block with a hole in it: $\Rightarrow x_{cm} = \frac{m_1 x_1 + (-m_2) x_2}{m_1 + (-m_2)} = \frac{A_1 x_1 + (-A_2) x_2}{A_1 + -(A_2)} = \frac{L_1 w_1 x_1 - \pi (r_2)^2 x_2}{L_1 w_1 - \pi (r_2)^2}$

Remember the mass of the hole is negative, which is why we are subtracting mass 2 in the equation. Also, in our previous lesson we derived why we can replace area with mass in this equation. Please see that video for that derivation. http://www.flippingphysics.com/center-of-mass-hole.html

$$\Rightarrow x_{cm} = \frac{(15.7)(6.5)(7.85) - \pi (1.65)^2 (12.45)}{(15.7)(6.5) - \pi (1.65)^2} = 7.4292 \Rightarrow \boxed{x_{cm} \approx 7.4cm \ \& \ y_{cm} \approx 3.2cm}$$

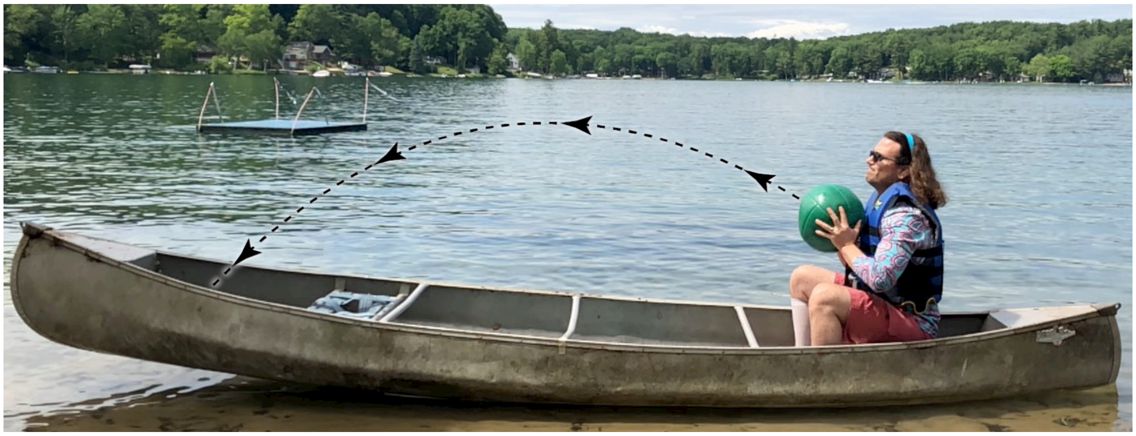


This is logical because the center of mass of the block with a missing piece should be closer to the center of mass of piece 1 rather than the center of mass of the hole.

Flipping Physics Lecture Notes:

Throwing a Ball in a Boat - Demonstrating Center of Mass

When I throw the massive ball to the left such that it lands in the other end of the canoe, what will happen to the positions of the objects?



In the absence of an external force, a system of particles at rest will remain at rest. In other words, if the net external force on a system is zero, the center of mass of a system will remain in the same location. When the ball moves to the left, the center of mass of the system moves to the left *relative to the boat*. However, because the center of mass of the system does not change relative to planet Earth, *everything else in the system moves to the right.* In other words, if the ball goes to the left, the canoe and I must move to the right.

Before we do any calculations, let's ask one more question. When I throw the ball to the left such that it does not land in the canoe, what will happen?

In this case the ball is not actually a part of the system because the ball does not land in the boat. In other words, the net external force on the system does not equal zero; there is a net *rightward* external force on the system and the canoe and I will continue to move to the right at a constant velocity after releasing the ball.

Now back to the ball landing in the canoe. We can calculate how far the canoe and I move. In order to do so, we need to calculate the initial and final positions of the center of mass of the system and then take the difference between them.

Mass knowns: $m_b = 6.5kg; m_p = 72kg; m_c = 33kg$

Note: b = ball, c = canoe, p = mr.p

Position knowns: $x_{bi} = 108cm; x_{pi} = 133cm = x_{pf}; x_{ci} = 0 = x_{cf}; x_{bf} = -157cm$

Note: All positions are relative to the center of the canoe and the position final of the ball is negative because the final position of the ball is to the left of the zero reference point.

$$x_i = \frac{m_b x_{bi} + m_p x_{pi} + m_c x_{ci}}{m_b + m_p + m_c} = \frac{(6.5)(108) + (72)(133) + (33)(0)}{6.5 + 72 + 33} = 92.179\text{cm}$$

$$x_f = \frac{m_b x_{bf} + m_p x_{pf} + m_c x_{cf}}{m_b + m_p + m_c} = \frac{(6.5)(-157) + (72)(133) + (33)(0)}{6.5 + 72 + 35} = 76.731\text{cm}$$

$$\Delta x_{cm} = x_f - x_i = 76.731 - 92.179 = -15.448 \approx -15\text{cm}$$

In other words, the center of mass of the system moves 15 centimeters to the left *relative to the system*. In order for the center of mass of the system to stay in the same location relative to the rest of the planet, the zero reference point of the system (the center of the canoe) needs to move 15 centers to the right *relative to the planet*. The measurement is confirmed in the video and again, THE PHYSICS WORKS!!