

Flipping Physics Lecture Notes:
Finding the Force on a Ball from a Dent http://www.flippingphysics.com/force-ball-dent.html

A 67 N ball is dropped from a height of 79.8 cm above a bag of sand. If the ball makes a 9.0 mm deep dent in the sand, what is the average force the sand applies on the ball during the collision?

$$
F_{g}=67 N ; h_{i}=79.8 \mathrm{~cm}\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)=0.798 \mathrm{~m} ; \text { dent }=9.0 \mathrm{~mm}\left(\frac{1 \mathrm{~m}}{1000 \mathrm{~mm}}\right)=0.009 \mathrm{~m} ; F_{\mathrm{a}}=?(\text { average })
$$

We could use free fall to determine the final velocity of the ball right before it strikes the sand, which is the same as the initial velocity of the collision. (Alternatively, we could use conservation of mechanical energy to find this velocity.) Then we could use uniformly accelerated motion equations to determine the acceleration of the ball during the collision because we know the initial velocity during the collision, the final velocity of the ball during the collision is zero, and we know the displacement during the collision is 9.0 mm down. Then we could draw a free body diagram and sum the forces in the $y$-direction to determine the average force on the ball during the collision. Or ....

We could remember the Work Energy Theorem.
(What I like to call the Net Work equals Change in Kinetic Energy Theorem.)
Setting the initial point at the top where the ball is dropped and the final point at the bottom where the ball momentarily stops before moving back upward, we get:
$W_{\text {net }}=\Delta K E=K E_{f}-K E_{i}=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}=0-0=0$

$W_{\text {net }}=W_{F_{\mathrm{a}}}+W_{F_{g}}=0 \Rightarrow W_{F_{\mathrm{a}}}=-W_{F_{g}} \Rightarrow F_{\mathrm{a}} \Delta r_{F_{\mathrm{a}}} \cos \theta_{F_{\mathrm{a}}}=-F_{g} \Delta r_{F_{g}} \cos \theta_{F_{g}}$
$\Rightarrow F_{a}(0.009) \cos (180)=-(67)(0.798+0.009) \cos (0) \Rightarrow F_{a}=6007 . \overline{6} \approx 6.0 \times 10^{3} \mathrm{~N}$
$\Rightarrow F_{a}=6007 . \overline{6} N\left(\frac{1 l b}{4.448 N}\right)=1350.64 \approx 1400 \mathrm{lb}$
I would argue that remembering the Net Work equals Change in Kinetic Energy Theorem makes the solution much simpler.

But why is the average force applied by the sand on the ball about 90 times greater than the force of gravity?
$\frac{F_{a}}{F_{g}}=\frac{6007 . \overline{6}}{67}=89 . \overline{6} \approx 9.0 \times 10^{1}$
Because the force applied acts on the ball for a distance which is about 90 times smaller than the distance during which the force of gravity acts on the ball.

$$
\begin{aligned}
& F_{a} \Delta r_{F_{a}} \cos \theta_{F_{a}}=-F_{g} \Delta r_{F_{g}} \cos \theta_{F_{g}} \Rightarrow F_{\mathrm{a}} \Delta r_{F_{a}} \cos (180)=-F_{g} \Delta r_{F_{g}} \cos (0) \\
& \Rightarrow-F_{a} \Delta r_{F_{a}}=-F_{g} \Delta r_{F_{g}} \Rightarrow \frac{F_{a}}{F_{g}}=\frac{\Delta r_{F_{g}}}{\Delta r_{F_{a}}}=\frac{0.798+0.009}{0.009}=89 . \overline{6} \approx 9.0 \times 10^{1}
\end{aligned}
$$

