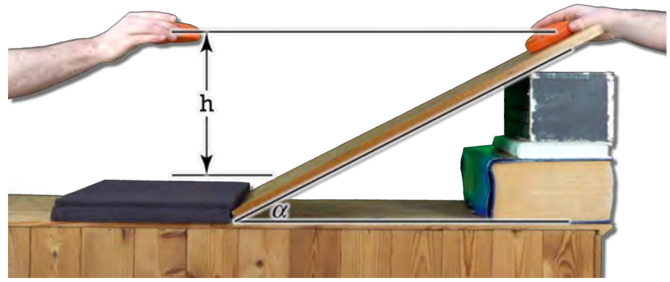
Flipping Physics Lecture Notes:

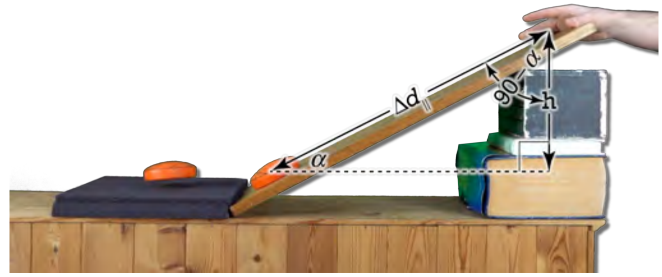
Conservative and Nonconservative Forces
http://www.flippingphysics.com/conservative-nonconservative.html



If an object goes straight down a distance h, the work done by force of gravity on the object equals mgh:

$$W_{F_g} = F_g \Delta r \cos\theta = (mg)(h)\cos(0) = mgh$$

If that object instead slides down an incline of angle alpha through the same vertical distance h, the work done by the force of gravity on the object is still mgh:



$$W_{F_g} = F_g \Delta r \cos\theta = (mg)(\Delta d_{\parallel})\cos(90-\alpha)$$

$$\sin\alpha = \frac{O}{H} = \frac{h}{\Delta d_{\parallel}} \Rightarrow \Delta d_{\parallel} = \frac{h}{\sin\alpha} \;\&\; \sin\alpha = \cos(90-\alpha)$$

$$\Rightarrow W_{F_g} = (mg)\left(\frac{h}{\sin\alpha}\right)(\sin\alpha) = mgh$$

In other words, no matter the angle of the incline, the work done by the force of gravity on the object as the object goes down a distance h is the same. Because the work done by the force of gravity on an object is *independent* of the path taken by the object, the force of gravity is a *conservative force*.

Examples of conservative forces are: gravitational force, spring force, electromagnetic force between two charged particles, and magnetic force between two magnetic poles.

In fact, notice that the work done on the object by the force of gravity equals the negative of the change in gravitational potential energy of the mass:

$$\Delta U_g = U_{gf} - U_{gi} = mgh_f - mgh_i = mg(h_f - h_i) = mg(\Delta h) = mg(-h) = -mgh$$

$$W_{F_g} = mgh = -(-mgh) = -\Delta U_g$$

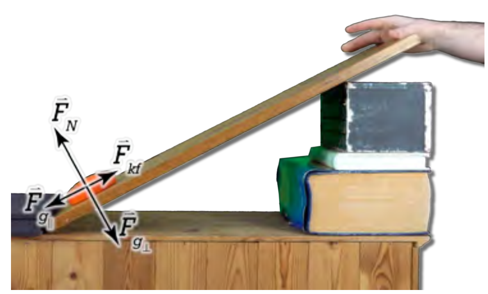
That is because the work done by a conservative force equals the negative of the change in potential energy of the object which is associated with that force.

$$W_{\text{conservative force}} = -\Delta U$$

For example:

- Gravitational potential energy goes with the force of gravity.
- Elastic potential energy goes with spring force.
- Electric potential energy goes with the electromagnetic force.

The work done by the force of friction on an object as the object slides down an incline through a vertical distance h, is *dependent* on the incline angle and therefore the path taken by the object:



$$W_{F_{kf}} = F_{kf}\Delta r\cos\theta = \mu_k F_N \Delta d_{\parallel}\cos(180)$$

$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg\cos\alpha$$

$$\Rightarrow W_{F_{kf}} = \mu_k(mg\cos\alpha)\left(\frac{h}{\sin\alpha}\right)(-1) = -\mu_k mgh\cot\alpha$$

Therefore, because the force of friction is *dependent* on the path the object moves through, the force of friction is a *nonconservative force*.

All conservative forces have the following two equivalent properties:

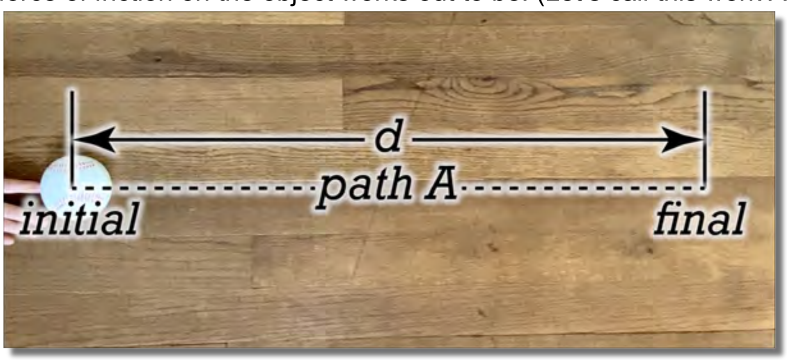
1) The work done by a conservative force on an object moving between any two points is independent of the path taken by the object.
2) The work done by a conservative force on an object moving through any closed path equals zero. (A closed path is a path where the initial and final points are the same location.)

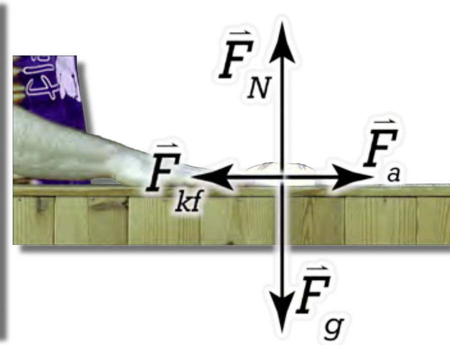
If a force does not have these two equivalent properties, it is a nonconservative force.

Clearly, we have shown the first property to be true, let's look at work done on an object as it moves through a closed path in order to investigate the second property. We have already shown that the work done on an object by the force of gravity equals mass times acceleration due to gravity times height, regardless of the path taken. For a closed path, the initial and final heights will be the same and the work done on the object by the force of gravity will equal zero. That shows that the force of gravity has the second equivalent property necessary to be a conservative force.

$$W_{F_g} = mgh = mg(0) = 0\left(\text{closed path}\right)$$

Now let's look at *nonconservative* forces in more detail. If we have an object on a level surface and we displace that object a straight-line distance d, from an initial point to a final point, the work done by the force of friction on the object works out to be: (Let's call this work A and path A.)

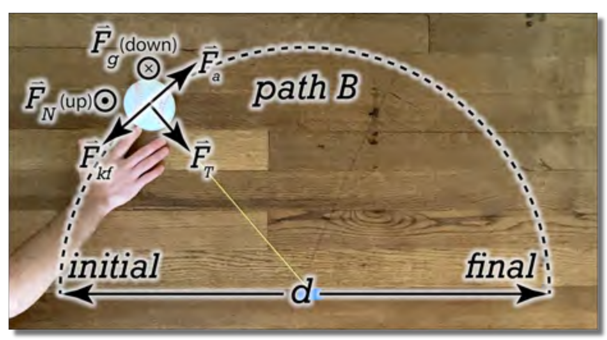




$$W_A = F_{kf}\Delta r_A \cos\theta = \left(\mu_k F_N\right)\left(d\right)\cos\left(180\right) = -\mu_k mgd$$

$$\sum F_y = F_N - F_g = ma_y = m(0) = 0 \Rightarrow F_N = F_g = mg$$

Now instead let's tie a string to the object and move it from the same initial point to the same final point, only this time the path of the object will form half a circle. The work done by the force of friction on the object now is: (Let's call it work B and path B.)



$$W_B = F_{kf}\Delta r_B \cos\theta = \left(\mu_k F_N\right)\left(\frac{C}{2}\right)\cos\left(180\right) = -\mu_k\left(mg\right)\left(\frac{\pi d}{2}\right) = -\frac{\pi}{2}\mu_k mgd \neq -\mu_k mgd = W_A$$

$$C = \pi D = \pi d \Rightarrow \frac{C}{2} = \frac{\pi d}{2}$$

In other words, the work done by the force of friction on the object is different depending on the path taken by the object. That makes the force of friction a nonconservative force because it does not have the first of the two equivalent properties.

The difference here is that more energy has been converted to thermal energy when going along path B than when going along path A. The work that went into the system via the force applied moving the object was converted into thermal energy in the object via work done by the force of friction.

Now, we should know, because the two conservative force properties are equivalent, that the force of friction will also not have the second, equivalent property, the work over a closed path for the force of friction will not equal zero, however, let's just confirm that. Let's determine the work done by the force of friction acting on an object moving through one full circle of radius d over 2. In other words, from the original initial point all the way through one circumference back to that same point. The work done by the force of friction on the object works out to be: (Let's call it work O and path O)

$$W_O = F_{kf}\Delta r_O \cos\theta = \left(\mu_k F_N\right)\left(C\right)\cos\left(180\right) = -\mu_k\left(mg\right)\left(\pi d\right) = -\pi\mu_k mgd \neq 0$$

Which, clearly, does not equal zero. So, the work done by the force of friction on an object over a closed path does not equal zero and the force of friction is a nonconservative force. Again, work done by the force of friction converts kinetic energy into thermal energy and the object is warmer after going through the circle than it was before.