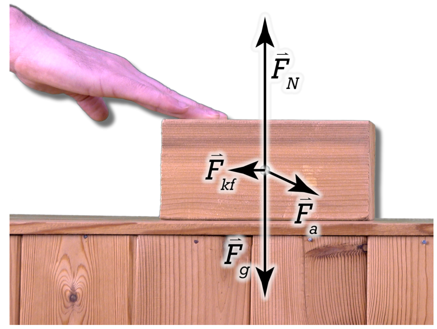
Flipping Physics Lecture Notes:

Example of Energy Transferred Into and Out of a System

http://www.flippingphysics.com/energy-transfer-system-example.html

Example: A 7.50 kg block on a level surface is acted upon by a force applied of 35.0 N at an angle of 25.0° below +x axis. The block starts at rest, the coefficient of kinetic friction between the block and surface is 0.245, and the block is displaced 6.50 m to the right[1]. Find the:

a) Final velocity of the block.
b) Work done by each force acting on the block.
c) Change in internal energy of the block.
d) Change in kinetic energy of the block.
e) And show that the change in energy of the block equals the energy transferred to the block.



$$m = 7.50kg;\ \vec{F}_a = 35.0N @ 25.0^\circ\ below +x\ axis;\ v_i = 0;\ \mu_k = 0.245;\ \Delta\vec{r} = 6.50\hat{i}\ m$$

$$(a) v_f = ?;\ (b) W = ?(\text{each force});\ (c) \Delta E_{internal} = ?;\ (d) \Delta KE = ?;\ (e)\ show\ \Delta E_{system} = \sum T$$

We need to sum the forces in order to determine the acceleration of the block so we can use a uniformly accelerated motion equation to find the final velocity of the block. Let's start by drawing the free body diagram and breaking the force applied into its components in the x and y-directions and use the unit vector form of all the forces.

$$\vec{F}_a = F_{ax}\hat{i} + F_{ay}\hat{j} = \left(F_a\cos\theta\right)\hat{i} + \left(F_a\sin\theta\right)\left(-\hat{j}\right) = (35)\cos(25)\hat{i} - (35)\sin(25)\hat{j} = \left[31.7208\hat{i} - 14.7916\hat{j}\right]N$$

Now, rather than summing the forces in each direction separately, we can sum the forces in all directions simultaneously using one equation:

$$\sum\vec{F} = \vec{F}_a + \vec{F}_g + \vec{F}_N + \vec{F}_{kf} = m\vec{a}$$

$$\sum\vec{F} = \left[31.7208\hat{i} - 14.7916\hat{j}\right] + \left[(mg)\left(-\hat{j}\right)\right] + \left[\left(F_N\right)\hat{j}\right] + \left[\left(\mu_k F_N\right)\left(-\hat{i}\right)\right] = m\left[\left(a_x\right)\hat{i} + (0)\hat{j}\right]$$

And then we can isolate the j unit vector coefficients:

$$\left(\hat{j}\right) \Rightarrow -14.7916 - mg + F_N = 0 \Rightarrow F_N = 14.7916 + mg = 14.7916 + (7.5)(9.81) = 88.3666N$$

And then we can isolate the I unit vector coefficients:

$$\left(\hat{i}\right) \Rightarrow 31.7208 - \mu_k F_N = ma_x \Rightarrow a_x = \frac{31.7208 - \mu_k F_N}{m} = \frac{31.7208 - (0.245)(88.3666)}{7.5} = 1.34279\frac{m}{s^2}$$

Now that we know the acceleration and it is uniform, we can use a uniformly accelerated motion equation to solve for the final velocity of the block:

$$v_f^2 = v_i^2 + 2a\Delta x \Rightarrow v_f = \sqrt{(0)^2 + 2a\Delta r} = \sqrt{(2)(1.34279)(6.5)} = 4.178074 \approx 4.18\frac{m}{s}$$

And we have solved part (a). Yea!

Part (b) is to solve for the work done by each force:

$$W_{F_N} = \vec{F}_N \bullet \Delta\vec{r} = \left(88.3666\hat{j}\right)\bullet\left(6.5\hat{i}\right) = 0$$

$$W_{F_g} = \vec{F}_g \bullet \Delta\vec{r} = \left(-mg\,\hat{j}\right)\bullet\left(6.5\hat{i}\right) = 0$$

$$W_{F_a} = \vec{F}_a \bullet \Delta\vec{r} = \left(31.7208\hat{i} - 14.7916\hat{j}\right)\bullet\left(6.5\hat{i}\right) = (31.7208)(6.5) + (-14.7916)(0) = 206.185 \approx 206J$$

$$W_{F_{kf}} = \vec{F}_{kf} \bullet \Delta\vec{r} = \left(-\mu_k F_N\,\hat{i}\right)\bullet\left(6.5\hat{i}\right) = \left(-\mu_k F_N\right)(6.5) = (-0.245)(88.3666)(6.5) = -140.724 \approx -141J$$

[1] This demonstration is not the correct mass, distance, scale, size, weight, acceleration, conifer, or astral plane. It is intended for visual reference purposes only and should not be confused with reality.

Part (c) is to solve for the change in internal energy of the block:

$$\Delta E_{internal} = -W_{NC} = -W_{F_{kf}} = -\left(-140.724\right) = 140.724 \approx 141J$$

Part (d) is to solve for the change in kinetic energy of the block:
Notice there are currently two ways we could solve this, both are correct:

$$\Delta KE = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = \frac{1}{2}mv_f^{\,2} - 0 = \frac{1}{2}\left(7.5\right)\left(4.178074\right)^2 = 65.4611 \approx 65.5J$$

$$W_{net} = \Delta KE = W_{F_N} + W_{F_g} + W_{F_a} + W_{F_{kf}} = 0 + 0 + 206.185 + \left(-140.724\right) = 65.4611 \approx 65.5J$$

Part (e) is to show that the change in energy of the block equals the energy transferred to the block.

$$\Delta E_{system} = \sum T \Rightarrow \Delta ME + \Delta E_{internal} = W_{F_a} \Rightarrow \Delta KE + \Delta E_{internal} = W_{F_a} \Rightarrow 65.4611 + 140.724 = 206.185 \quad \checkmark$$

And notice the above equation is just a rearrangement of the net work equals change in kinetic energy theorem. The net work equals the sum of all the work done on the block by each force acting on the block. The work done by force normal and force of gravity both equal zero. The work done by the force of kinetic friction equals the negative of the change in internal energy of the system. Rearranging that equation gives you the exact same equation you showed us before using the equation change in energy of the system equals the energy transferred to the system.

$$W_{net} = \Delta KE \Rightarrow W_{F_N} + W_{F_g} + W_{F_a} + W_{F_{kf}} = \Delta KE \Rightarrow W_{F_a} + \left(-\Delta E_{internal}\right) = \Delta KE \Rightarrow \Delta KE + \Delta E_{internal} = W_{F_a}$$