Flipping Physics Lecture Notes:

Impulse Derivation and Demonstration
http://www.flippingphysics.com/impulse-derivation-demo.html

Newton's Second Law in terms of momentum is: $\sum \vec{F} = \frac{d\vec{p}}{dt}$

We can take the integral of both sides and solve for change in momentum:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \sum \vec{F}\,dt = d\vec{p} \Rightarrow \int_{t_i}^{t_f} \sum \vec{F}\,dt = \int_{p_i}^{p_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

And we have solved for impulse: $\Rightarrow \text{Impulse} = \vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \sum \vec{F}\,dt$

And remember the Impulse Approximation states that the force of impact is so much larger than all the other forces acting on the object that we can consider the force of impact to equal the net force:

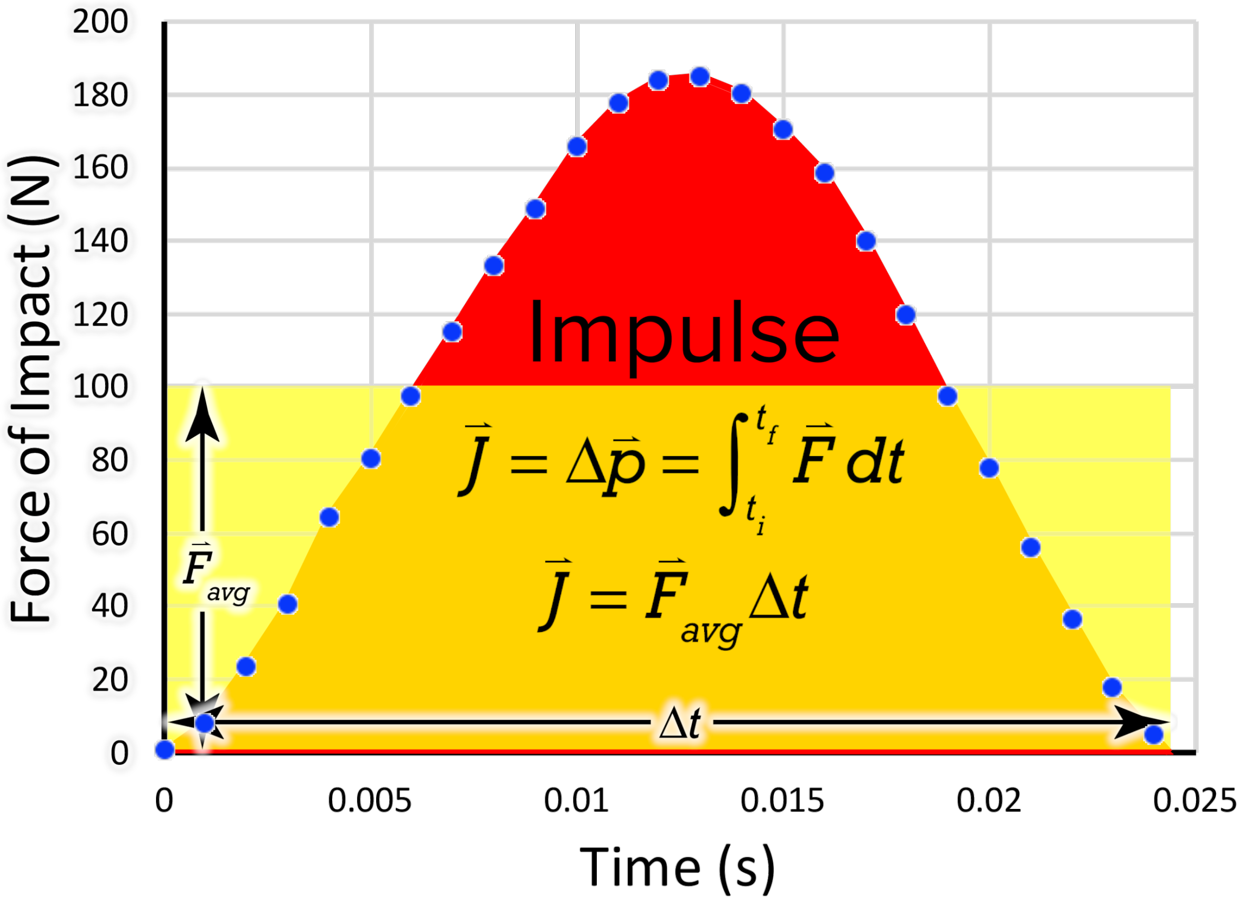
$$\text{Impulse Approximation: } F_{impact} >> F_{all\ other} \Rightarrow \sum \vec{F} \approx \vec{F}_{impact}$$

That means we now impulse in terms of the force of impact: $\Rightarrow \text{Impulse} = \vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}\,dt$

Things to know about impulse:

- Impulse is a vector.
- The symbol for impulse is capital J, though I will sometimes write out Impulse instead.
  - The symbol is **NOT** capital I, that is for rotational inertia (also called moment of inertia).
- Impulse is equal to 3 equivalent expressions:
  - Change in momentum: $\vec{J} = \Delta\vec{p}$
  - The integral of the force of impact with respect to time: $\vec{J} = \int_{t_i}^{t_f} \vec{F}\,dt$
  - The area "under" a force vs. time function: $\vec{J} = \text{area "under" force vs. time function}$
    - This is the definition of an integral.
- The units for Impulse are newtons times seconds: $\vec{J} = \int_{t_i}^{t_f} \vec{F}\,dt \Rightarrow N \bullet s = \left(\frac{kg \bullet m}{s^2}\right)s = \frac{kg \bullet m}{s}$
  - Which are the same as the units for momentum. However, we typically use N•s for impulse and kg•m/s for momentum to help differentiate between the two.
- Please do not confuse impulse with work: $W = \int_{x_i}^{x_f} F_x\,dx \text{ does } NOT \text{ equal } \vec{J} = \int_{t_i}^{t_f} \vec{F}\,dt$

A typical graph of force as a function of time during a collision looks like this data for the rubber playground ball I dropped above a force platform:



The area “under” the curve is the impulse during the collision.
The same area can be determined using the average force and change in time. In other words, *another* equation for impulse is: $\vec{J} = \vec{F}_{avg}\Delta t$

So yeah, there are a lot of equations for impulse:

$$\text{Impulse} = \vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}\,dt = \text{area "under" force vs. time function} = \vec{F}_{avg}\Delta t \neq W = \int_{x_i}^{x_f} F_x\,dx$$