Flipping Physics Lecture Notes:

Ballistic Pendulum

http://www.flippingphysics.com/ballistic-pendulum.html

Example: A dart moving horizontally at a constant speed embeds itself in a hanging, stationary wood block. The wood block and dart rise to a maximum vertical height. Solve for the speed of the dart before the collision in terms of the mass of the dart, the mass of the wood block, the maximum vertical height of the dart and wood block, and known constants. This is called a Ballistic Pendulum.

Moment #1: Right before the collision.
Moment #2: Right after the collision.
Moment #3: Dart and wood block at maximum height.

$$\text{knowns: } m_d;\ m_w; v_{w1}=0;\ h_3;\ v_{d1}=?$$

From 1 → 2: Collision between dart and wood block. Conservation of Linear Momentum in the x-direction.

$$\sum \vec{F}_x = \frac{d\vec{p}}{dt} = 0 \Rightarrow \sum \vec{p}_1 = \sum \vec{p}_2 \Rightarrow m_d\vec{v}_{d1} + m_w\vec{v}_{w1} = m_d\vec{v}_{d2} + m_w\vec{v}_{w2} \Rightarrow m_d\vec{v}_{d1} = \left(m_d + m_w\right)\vec{v}_2 \Rightarrow \vec{v}_{d1} = \left(\frac{m_d + m_w}{m_d}\right)\vec{v}_2$$

Note: Because the dart and wood block are stuck together after the collision, they have the same velocity.

$$\vec{v}_{d2} = \vec{v}_{w2} = \vec{v}_2$$

We need to solve for the velocity of the dart and wood block at point 2. Let's put this equation in our equation holster!

From 2 → 3: After the collision. Conservation of Mechanical Energy. Horizontal zero line at the center of mass of the dart and wood block right after the collision.

$$ME_2 = ME_3 \Rightarrow \frac{1}{2}m_t v_2^{\ 2} = m_t g h_3 \Rightarrow \frac{1}{2}v_2^{\ 2} = g h_3 \Rightarrow v_2 = \sqrt{2gh_3}$$

Which we can substitute back into the holstered equation.

$$\Rightarrow v_{d1} = \left(\frac{m_d + m_w}{m_d}\right)\sqrt{2gh_3}$$

Does this equation make sense?
If we keep everything else constant …
According to the equation, if we increase the velocity of the dart at 1, height 3 increases.

$$\text{unchanged: } m_d;\ m_w;\ g \text{ and } v_{d1}\uparrow \Rightarrow h_3\uparrow$$

$$v_{d1}\uparrow \Rightarrow p_1\uparrow \Rightarrow p_2\uparrow \Rightarrow v_2\uparrow \Rightarrow KE_2\uparrow \Rightarrow PE_{g3}\uparrow \Rightarrow h_3\uparrow$$

if the velocity of the dart at 1 is again increased, however, we want the height at 3 to be the same as before, and if the only variable we change is the mass of the wood block, how would we have to change the mass of the wood block to make that happen?

$$\text{unchanged: } m_d;\ h_3;\ g \text{ and } v_{d1}\uparrow \Rightarrow m_w\uparrow$$

$$v_{d1}\uparrow \Rightarrow p_1\uparrow \Rightarrow p_2\uparrow \ \&\ h_3 = \text{unchanged} \Rightarrow v_2 = \text{unchanged} \Rightarrow m_w\uparrow \Rightarrow p_2\uparrow$$

if the velocity of the dart at 1 is again increased, however, we want the height at 3 to be the same as before, … if the only variable we change is the mass of the dart, how would we have to change the mass of the dart to make that happen?

$$\text{unchanged: } m_w;\ h_3;\ g \text{ and } v_{d1}\uparrow \Rightarrow m_d\downarrow$$

$$v_{d1}\uparrow \Rightarrow p_1\uparrow \ \&\ \text{need } p_1 = \text{unchanged} \Rightarrow m_d\downarrow$$

To confirm the equation, let's take some measurements: m_d = 0.0051 kg; m_w = 0.1329 kg; h_3 = 0.056m

$$\Rightarrow v_{d1} = \left(\frac{m_d + m_w}{m_d}\right)\sqrt{2gh_3} = \left(\frac{0.0051 + 0.1329}{0.0051}\right)\sqrt{(2)(9.81)(0.056)} = 28.363 \approx 28\frac{m}{s}$$

In other words, our calculations predict the speed of the dart before the collision to be roughly 28 m/s. And, the speed of the dart is something we can measure!

$$\Delta x_{d1} = 0.297m;\ \Delta t_{d1} = 10\text{frames}\left(\frac{1\text{sec}}{960\text{frames}}\right) = 0.01041\bar{6}\,\text{sec}$$

$$v_{d1} = \frac{\Delta x_{d1}}{\Delta t_{d1}} = \frac{0.297m}{0.01041\bar{6}\,\text{sec}} = 28.512\frac{m}{s}$$

And we can determine our relative error:

$$E_r = \frac{O - A}{A} \times 100 = \left(\frac{28.512 - 28.363}{28.363}\right) \times 100 = 0.525325 \approx 0.53\%$$

The Physics Works!!