Flipping Physics Lecture Notes:

System of Particles Translational Motion
http://www.flippingphysics.com/system-particles-motion.html

Previously we determined the x-position of the center of mass of a system of particles to be:

$x_{cm} = \dfrac{\sum_i m_i x_i}{m_{total}}$ Where i goes from 1 to the total number of particles in the system.

We can use the same equation to describe the 3-dimensional position of an object:

$$x_{cm} = \frac{\sum_i m_i x_i}{m_{total}};\ y_{cm} = \frac{\sum_i m_i y_i}{m_{total}};\ z_{cm} = \frac{\sum_i m_i z_i}{m_{total}}$$

However, we do not have to use 3 separate equations to identify the 3-dimensional position of the center of mass of a system of particles. We can use the r position vector and unit vectors instead.

Recall that the position of a single particle can be defined as: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Therefore, the center of mass of a system of particles can be defined as:

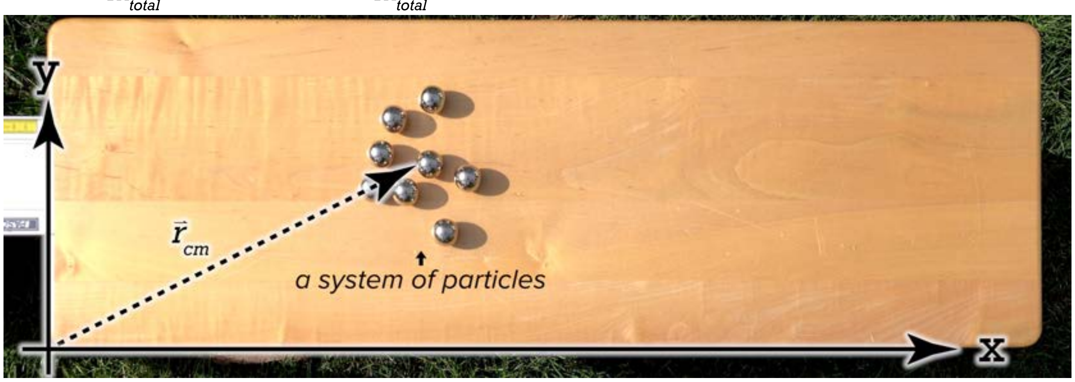
$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k} = \frac{\sum_i m_i x_i \hat{i}}{m_{total}} + \frac{\sum_i m_i y_i \hat{j}}{m_{total}} + \frac{\sum_i m_i z_i \hat{k}}{m_{total}} = \frac{\sum_i m_i x_i \hat{i} + \sum_i m_i y_i \hat{j} + \sum_i m_i z_i \hat{k}}{m_{total}}$$

In other words, when we define the position of the ith particle using the r position vector:

$$\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$$

We can define the position of the center of mass of a system of particles to be:

$$\Rightarrow \vec{r}_{cm} = \frac{\sum_i m_i\left(x_i\hat{i} + y_i\hat{j} + z_i\hat{k}\right)}{m_{total}} \Rightarrow \vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{m_{total}}$$



To be clear, because the r position vector describes the location of a single particle in three-dimensional space, the r center of mass position vector describes the location of a system of particles in three-dimensional space.

Therefore, we can define the velocity of the center of mass of a system of particles as:

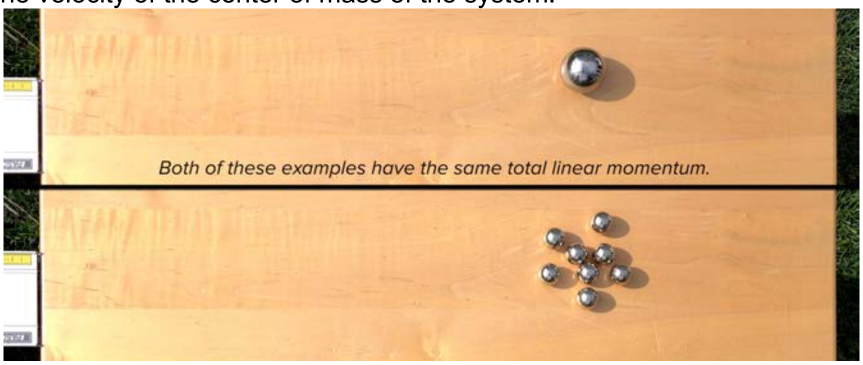
$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum_i m_i \vec{r}_i}{m_{total}}\right) = \frac{1}{m_{total}}\sum_i\left(m_i\frac{d\vec{r}_i}{dt}\right) \Rightarrow \vec{v}_{cm} = \frac{\sum_i m_i \vec{v}_i}{m_{total}}$$

Note: the mass of each particle is constant, and therefore the total mass of the system is also constant.

This means we can also define the momentum of a system of particles as:

$$m_{total}\vec{v}_{cm} = m_{total}\left(\frac{\sum_i m_i\vec{v}_i}{m_{total}}\right) = \sum_i m_i\vec{v}_i = \sum_i \vec{p}_i = \vec{p}_{total}$$

The total momentum of the system of particles equals the total mass of the system times the velocity of the center of mass of the system. That means, total linear momentum of the system of particles is the same as the linear momentum of a single particle with a mass equal to the total mass of the system which is moving at by the velocity of the center of mass of the system.



We can also define the acceleration of the center of mass of a system of particles as:

$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum_i m_i\vec{v}_i}{m_{total}}\right) = \frac{1}{m_{total}}\sum_i\left(m_i\frac{d\vec{v}_i}{dt}\right) \Rightarrow \vec{a}_{cm} = \frac{\sum_i m_i\vec{a}_i}{m_{total}}$$

And if we multiply that equation by the total mass of the system:

$$m_{total}\vec{a}_{cm} = m_{total}\left(\frac{1}{m_{total}}\sum_i m_i\vec{a}_i\right) = \sum_i m_i\vec{a}_i = \sum_i \vec{F}_i$$

Now, the forces on acting any particle in the system will include both internal and external forces. In other words, internal forces will occur when particles within the system collide with one another, however, those forces will form a Newton's third law force pair and will therefore cancel one another out. That means the sum of the internal forces acting on the system will be zero and summing the internal and external forces acting on the system will equal only the external forces acting on the system.

$$\sum\vec{F}_{internal} = 0 \Rightarrow \sum\vec{F}_{internal} + \sum\vec{F}_{external} = \sum\vec{F}_{external} = m_{total}\vec{a}_{cm}$$

Therefore, when the net external force acting on a system equals zero, the change in velocity of the system equals zero, and the linear momentum of the system will be constant.

$$\sum\vec{F}_{external} = m_{total}\vec{a}_{cm} = 0 \Rightarrow \vec{a}_{cm} = 0 \Rightarrow \frac{d\vec{v}_{cm}}{dt} = 0 \Rightarrow \vec{v}_{cm} = \text{constant} \Rightarrow m_{total}\vec{v}_{cm} = \vec{p}_{total} = \text{constant}$$

Again, the linear momentum of the system of particles will remain constant only when the net external force acting on the system of particles equals zero.

And yes, it does take 8 spheres of half the radius of the larger sphere to have the same mass as a single sphere, assuming the same density.

$$r_L = 2r_S \;\&\; \rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow \frac{V_L}{V_S} = \frac{\frac{4}{3}\pi r_L^{\,3}}{\frac{4}{3}\pi r_S^{\,3}} = \frac{r_L^{\,3}}{r_S^{\,3}} = \frac{\left(2r_S\right)^3}{r_S^{\,3}} = \frac{8r_S^{\,3}}{r_S^{\,3}} = 8 \Rightarrow V_L = 8V_S$$