Flipping Physics Lecture Notes:

Nonuniform Circular Motion - Ball in a Vertical Circle

http://www.flippingphysics.com/non-uniform-circular-motion-ball.html

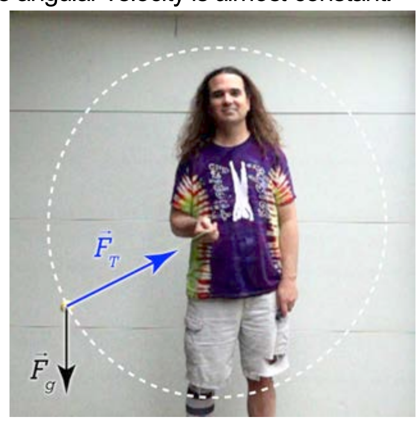
We have already discussed Uniform Circular Motion or the motion of an object moving with a constant angular velocity. Like the small, yellow chip on top of the record, because its angular velocity is constant, the angular acceleration of the chip is zero, the tangential acceleration of the chip is zero, and its motion is often called Uniform Circular Motion.

$$\vec{\omega} = \text{constant} \Rightarrow \vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} = \frac{0}{\Delta t} = 0 \ \& \ a_t = r\alpha = r(0) = 0$$

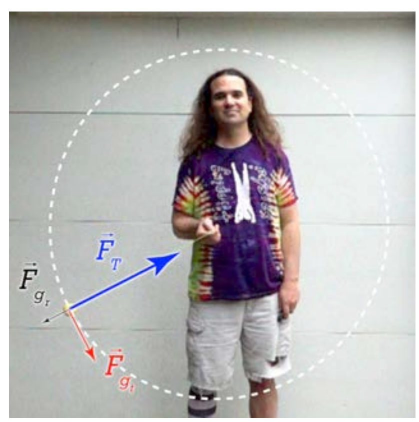
When the angular velocity of an object is not constant, the object has an angular acceleration, the object also has a tangential acceleration, and the motion of the object is called Non-uniform Circular Motion because the angular velocity of the object is not constant.

$$\vec{\omega} \neq \text{constant} \Rightarrow \vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} \neq 0 \ \& \ a_t = r\alpha \Rightarrow a_t \neq 0$$

A good example of Non-uniform Circular Motion is a ball on the end of a string moving in a vertical circle. At first glance it might appear that the angular velocity of this ball is moving at a constant angular velocity, however, when you look at the forces acting on the ball, you will see that the angular velocity of the ball cannot be constant, and this must be Non-uniform Circular Motion. However, from the video, you can see the angular velocity is *almost* constant.



The force of gravity always acts straight down on the ball and the force of tension always acts inward, toward the center of the circle. Now, because the ball is moving in a circle, there must always be a centripetal force acting on the ball. In other words, there must always be a net force acting inward on the ball. Because the force of gravity changes direction relative to the "in-direction" as the ball moves around the vertical circle, the force of tension needs to increase and decrease to maintain that net inward force. That is why the force of tension has a maximum magnitude when the ball is at the bottom of its path where the force of gravity is pointed directly out away from the center of the vertical circle opposite the direction of the force of tension. And the force of tension has a minimum magnitude when the ball is at the top of its path where the force of gravity is pointed directly in toward the center of the vertical circle which is the same direction as the force of tension.



In order to understand how these forces affect the motion of the ball, we need to break forces into components. Because the ball is moving in a circle, we need to break forces into components that are in the "in-direction" and tangent to the "in-direction". Let's call these directions:

- "in-direction" = radial direction
    - Radial direction corresponds to the radius of the circle.
- Tangent to the radial direction = tangential direction

We do not need to find any components of the force of tension because the force of tension is always directed radially inward. The force of gravity we need to break into the tangential force of gravity component and the radial force of gravity component. There are several things which happens to these force of gravity components:

For the radial component of the force of gravity:

- From the top, as the ball moves down, the inward radial component of the force of gravity decreases in magnitude until it reaches zero when the string is horizontal.
- Then the radial component of the force of gravity increases in magnitude and is directed radially outward.

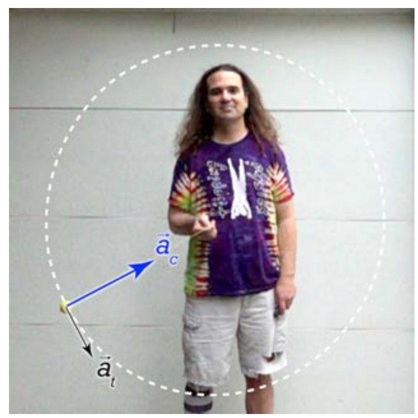
- The radial component of the force of gravity reaches its maximum magnitude at the bottom of the arc and then decreases in magnitude until it reaches zero when the string is horizontal.
- Then the radial component of the force of gravity increases in magnitude, is directed radially inward, and reaches its maximum magnitude at the top of the arc.

For the tangential component of the force of gravity:

- At the top, there is no tangential component of the force of gravity.
- As the ball moves downward the tangential component of the force of gravity is in the direction of motion of the ball and increases in magnitude until the string is horizontal.
- Then the tangential component of the force of gravity decreases in magnitude until the ball reaches the bottom of the arc where there is no tangential component of the force of gravity.
- As the ball moves upward the tangential component of the force of gravity is opposite the direction of motion of the ball and increases in magnitude until the string is horizontal.
- Then the tangential component of the force of gravity decreases in magnitude until the ball reaches the top of the arc.

Because net force equals mass times acceleration, we can now look at the acceleration in the tangential direction and the acceleration in the radial direction (which is directed inward and called centripetal acceleration). Let's start with the tangential acceleration, because the only force acting in the tangential direction is the tangential component of the force of gravity, the tangential acceleration will be proportional to the tangential component of the force of gravity.

$$\sum \vec{F}_t = \vec{F}_{g_t} = m\vec{a}_t \Rightarrow F_{g_t} \propto a_t$$



- At the top, there is no tangential acceleration.
- As the ball moves downward the tangential acceleration is in the direction of motion of the ball and increases in magnitude until the string is horizontal. In other words, the tangential speed of the ball is increasing.
- Then the tangential acceleration decreases in magnitude until the ball reaches the bottom of the arc where there is no tangential acceleration. In other words, the tangential speed of the ball is increasing all the way from the top to the bottom. And the tangential speed of the ball is at a maximum at the bottom of the circle.
- As the ball moves upward the tangential acceleration is opposite the direction of motion of the ball and increases in magnitude until the string is horizontal.
- Then the tangential acceleration decreases in magnitude until the ball reaches the top of the arc. In other words, the tangential speed of the ball is decreasing all the way from the bottom to the top. And the tangential speed of the ball is at a minimum at the top of the circle.
- Because the tangential acceleration of the ball is not zero, the tangential speed of the ball is not constant, and this must be Non-uniform Circular Motion.

Because centripetal acceleration equals tangential speed squared over radius, as the tangential speed of the ball increases, the centripetal acceleration of the ball increases, and as the tangential speed of the ball decreases, the centripetal acceleration of the ball decreases. In other words:

- From the top to the bottom, the centripetal acceleration of the ball increases. And the centripetal acceleration of the ball has its maximum magnitude at the bottom of the circle.
- From the bottom to the top, the centripetal acceleration of the ball decreases. And the centripetal acceleration of the ball has its minimum magnitude at the top of the circle.
- At the top, the centripetal acceleration of the ball is nonzero because the ball does have a tangential speed.

Because the ball has its maximum tangential speed at the bottom of the circle and its minimum tangential speed at the top of the circle, we would expect the time for the ball to travel through the bottom semicircle to be less than the time for the ball to travel through the top semicircle. And that is exactly what we see in the video.