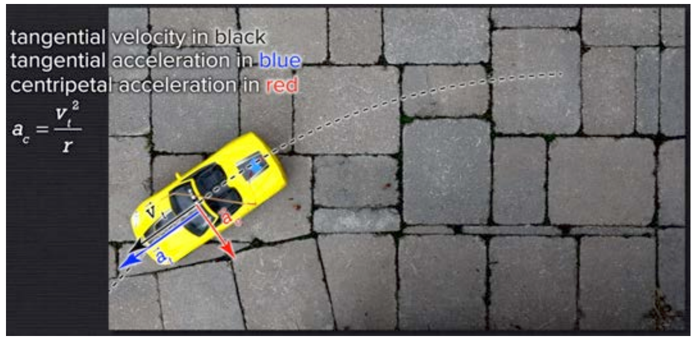
Flipping Physics Lecture Notes:

Non-uniform Circular Motion - Accelerating Car

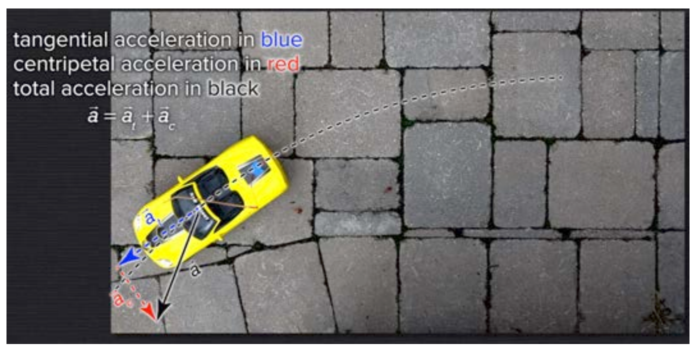
http://www.flippingphysics.com/non-uniform-circular-motion-car.html

We have already discussed the non-uniform circular motion of a ball on the end of a string moving in a vertical circle.[1] The circular motion is non-uniform because the tangential speed of the ball is not constant.



Today we are going to discuss the nonuniform circular motion of a car that is speeding up and turning, which you clearly see in the video[2].

- The initial tangential velocity of the car is zero.
- As the car speeds up the magnitude of the tangential velocity of the car increases, which you can see in the video[3].
- Because the magnitude of the tangential velocity is increasing, the car must be experiencing a tangential acceleration which is in the same direction as the tangential velocity.
    - We are going to assume the tangential acceleration of this car is constant.
- Because the direction of the tangential velocity is changing, there must be a centripetal acceleration which is directed inward.
    - Previously we derived the equation for the centripetal acceleration of an object moving along an arc[4] to be: $a_c = \frac{v_t^2}{r}$
- Because the initial tangential speed of the car is zero, the initial centripetal acceleration of the car is zero.
- As the tangential speed of the car increases, the centripetal acceleration of the car increases.
    - We are going to assume the radius of the arc the car is moving through is constant.
- At this point notice the car has both a tangential acceleration directed tangent to the arc the car is moving through and a centripetal acceleration which is directed inward along the radius of the arc the car is moving through, and those two accelerations are perpendicular to one another. These two accelerations are the components of the acceleration of the car. Any object moving in non-uniform circular motion will have these two components of acceleration.
    - $\vec{a} = \vec{a}_t + \vec{a}_c$



- To find the acceleration of the car at any point, you would need to both find its magnitude and its direction:
    - To find the magnitude of a: $a_t^2 + a_c^2 = a^2 \Rightarrow a = \sqrt{a_t^2 + a_c^2}$
    - To find the direction of a: $\tan\theta = \frac{O}{A} = \frac{a_c}{a_t} \Rightarrow \theta = \tan^{-1}\left(\frac{a_c}{a_t}\right)$

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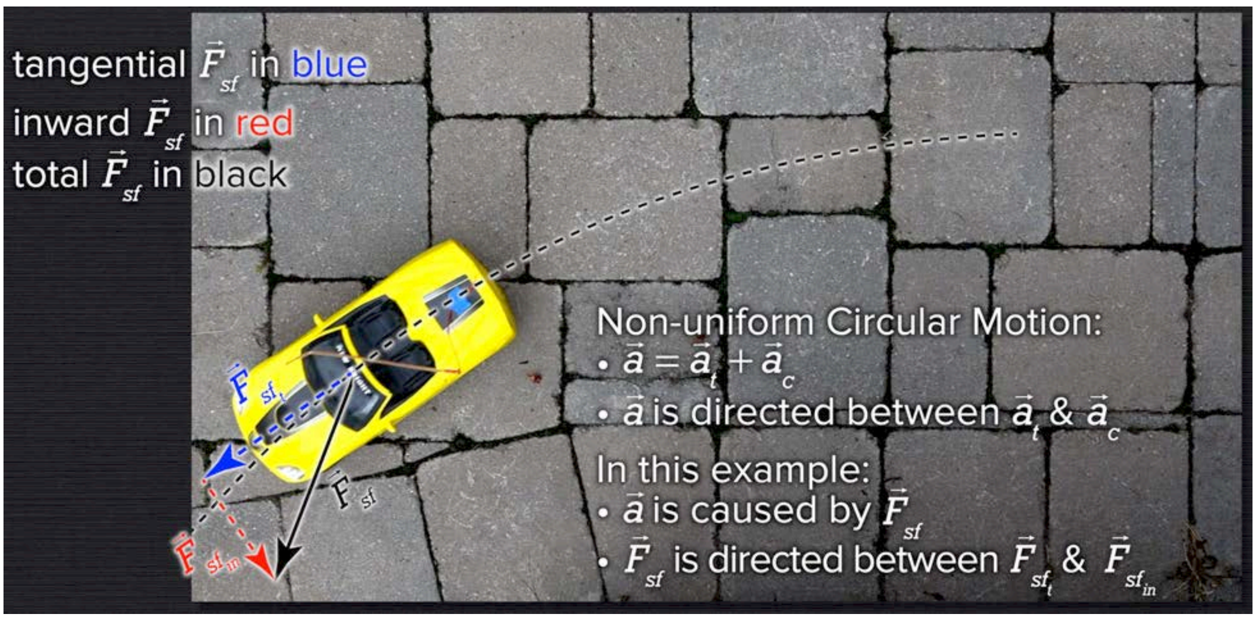
[1] http://www.flippingphysics.com/non-uniform-circular-motion-ball.html
[2] I mean really, you have to watch the video, right? The link is at the very top of this page, eh!
[3] Okay, I am going to stop writing “as you can see in the video” each time, but really, people, watch video.
[4] http://www.flippingphysics.com/centripetal-acceleration-derivation.html

- The only force acting on the car in the horizontal plane the car is moving in is the force of static friction, therefore, according to Newton's Second Law, the forces which are causing the tangential and centripetal accelerations are the components of the Force of Static Friction:
  - $$\sum \vec{F}_{in} = \vec{F}_{sf_{in}} = m\vec{a}_c \;\&\; \sum \vec{F}_t = \vec{F}_{sf_t} = m\vec{a}_t$$
    - Yes, there are actually 4 forces of static friction acting on the car, one for each tire. We are simplifying the example and modeling it as one force of static friction.
    - Yes, there is a force of gravity acting down on the center of mass of the car and 4 normal forces acting up on the car (one for each tire), however, we are only concerning ourselves with the forces which act in the horizontal plane the car is moving in.
- The components of the Force of Static Friction are proportional to the components of the acceleration.
- The components of the force of static friction add up to the force of static friction.
- The force of static friction starts tangent to the circle where the tangential speed and centripetal acceleration are zero, and then increases in magnitude and rotates its direction such that it slowly acts more and more in an inward direction.



The main takeaways from this lesson are:

- Non-uniform Circular Motion has both a tangential and centripetal component to its linear acceleration, therefore, the linear acceleration of the object acts in a direction between the tangential and inward directions.
- In this example, that linear acceleration is caused by the force of static friction which acts in the same direction as the acceleration of the car.