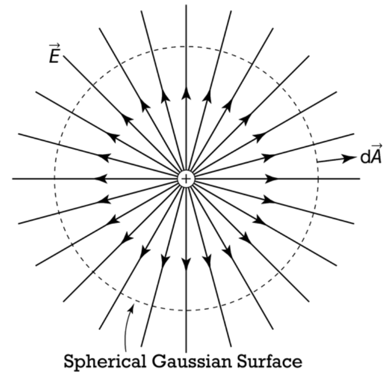
Flipping Physics Lecture Notes:
Gauss's Law - Point Charge Electric Flux
http://www.flippingphysics.com/gauss-law.html



Let's determine the electric flux passing through a sphere which is concentric to and surrounds a positive point charge.

Notice we cannot use $\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$ because the electric field is not uniform. We need to use the integral equation for electric flux:

$$\Phi_E = \vec{E} \cdot \vec{A} \Rightarrow d\Phi_E = \vec{E} \cdot d\vec{A} \Rightarrow \int d\Phi_E = \int \vec{E} \cdot d\vec{A} \Rightarrow \Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E dA \cos\theta = \int E dA \cos(0) = \int E dA = E \int dA = EA$$

$$\vec{F}_{21} = k\frac{(q_1)(q_2)}{r^2}\hat{r}_{21} \ \& \ \vec{E} = \frac{\vec{F}_e}{q} \Rightarrow E_{+\text{ point charge}} = \frac{\frac{kqQ}{r^2}}{q} = \frac{kQ}{r^2} \ \& \ A_{\text{sphere}} = 4\pi r^2$$

$$\Rightarrow \Phi_E = \left(\frac{kQ}{r^2}\right)\left(4\pi r^2\right) = 4\pi kQ = 4\pi\left(\frac{1}{4\pi\varepsilon_0}\right)Q = \frac{Q}{\varepsilon_0}$$

In other words, the electric flux through a closed Gaussian surface is:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

This is Gauss' law!

Gauss' law relates electric flux through a Gaussian surface to the charge enclosed by the Gaussian surface:

- A Gaussian surface is a three-dimensional closed surface
- While the Gaussian surfaces we usually work with are imaginary, the Gaussian surface could actually be a real, physical surface
- Typically, we choose the shapes of our Gaussian surfaces such that the electric field generated by the enclosed charge is either perpendicular or parallel to the various sides of the Gaussian surface. This greatly simplifies the surface integral because all the angles are multiples of 90° and the cosine of those angles have a value of -1, 0, or 1.
- As long as the amount of charge enclosed in a Gaussian surface is constant, the total electric flux through the Gaussian surface does not depend on the size of the Gaussian surface.
- Gauss' law is the first of Maxwell's equations which are a collection of equations which fully describe electromagnetism.

Notice then that, if the net charge inside a closed Gaussian surface is zero, then the net electric flux through the Gaussian surface is zero.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

This is why the net electric flux through the closed rectangular box in the example in our previous lesson was zero.