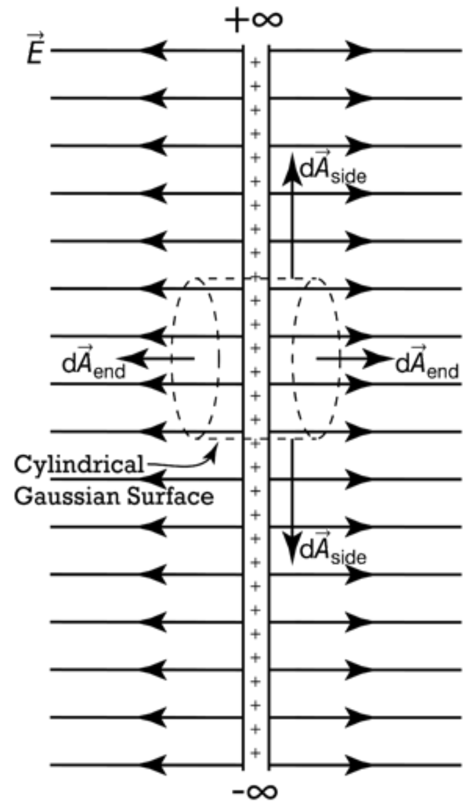
Flipping Physics Lecture Notes:
Gauss's Law - Charged Plane Electric Field
http://www.flippingphysics.com/gauss-law-plane.html

Determine the electric field which surrounds an infinitely large, thin plane of positive charges with uniform surface charge density, σ:

First off, we know the electric field will be directed normal to and away from the infinite plane of positive charges. This is because the plane is infinitely large; therefore, every component of the electric field, dE, which is parallel to the plane of charges and is caused by infinitesimally small, charged pieces of the plane, dq, will cancel out leaving only electric field components of dE which are perpendicular to the plane and directed away from the plane.

We pick a Gaussian surface such that it is a cylinder with ends parallel to the plane of charges and a side parallel to the electric field and use Gauss' law. The two ends of the Gaussian cylinder are equidistant from the charged plane.



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\Phi_E = \int_{\text{side}} \vec{E} \cdot d\vec{A}_{side} + \int_{\substack{\text{left}\\\text{end}}} \vec{E} \cdot d\vec{A}_{end} + \int_{\substack{\text{right}\\\text{end}}} \vec{E} \cdot d\vec{A}_{end} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = \int_{\text{side}} EdA\cos\theta_{side} + \int_{\substack{\text{left}\\\text{end}}} EdA\cos\theta_{end} + \int_{\substack{\text{right}\\\text{end}}} EdA\cos\theta_{end} = \frac{q_{in}}{\varepsilon_0}$$

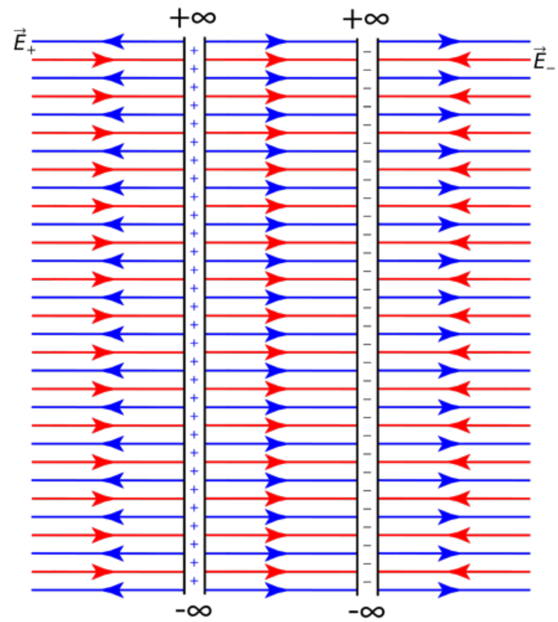
$$\Rightarrow \Phi_E = \int_{\text{side}} EdA\cos(90°) + \int_{\substack{\text{left}\\\text{end}}} EdA\cos(0°) + \int_{\substack{\text{right}\\\text{end}}} EdA\cos(0°) = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = E\int_{\substack{\text{left}\\\text{end}}} dA + E\int_{\substack{\text{right}\\\text{end}}} dA = E(2A_{end}) = \frac{q_{in}}{\varepsilon_0}$$

$$\& \ \sigma = \frac{Q}{A} = \frac{q_{in}}{A_{end}} \Rightarrow q_{in} = \sigma A_{end} \Rightarrow E(2A_{end}) = \frac{\sigma A_{end}}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Notice this electric field is uniform and is independent of the distance from the infinite plane of charges.

And notice what happens if we have two infinite parallel planes of charges, one with positive charge and one with negative charge:



The electric field outside the planes of charges cancels out to give zero electric field outside the planes of charges:

$$E_{\text{outside}} = 0$$

And between the two planes of charges, the electric fields add together:

$$E_{\text{between}} = 2E_{\text{one plate}} = 2\left(\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

And we have begun our journey towards determining the capacitance of a parallel plate capacitor…

Notice that a positively charged particle moving through a constant electric field will experience an electrostatic force in the direction of the electric field. This force will be constant and equal to qE. In other words, the motion of a charged particle through a constant electric field will have similar characteristics to a mass moving through the constant gravitational field near the surface of a planet. This is very similar to projectile motion.