Flipping Physics Lecture Notes:
Electric Potential
http://www.flippingphysics.com/apcem-electric-potential.html

The electrostatic force is a conservative force, therefore:

$$F_x = -\frac{dU}{dx} \Rightarrow F_e = -\frac{dU_e}{dr} \Rightarrow dU_e = -\vec{F}_e \cdot d\vec{r} \Rightarrow \int dU_e = -\int \vec{F}_e \cdot d\vec{r}$$

$$\Rightarrow \Delta U_e = -\int \vec{F}_e \cdot d\vec{r} \;\; \& \;\; \vec{E} = \frac{\vec{F}_e}{q} \Rightarrow \vec{F}_e = q\vec{E} \Rightarrow \Delta U_e = -\int_A^B q\vec{E} \cdot d\vec{r}$$

$$\Rightarrow \Delta U_e = -q\int_A^B \vec{E} \cdot d\vec{r} = -q\int_A^B E\cos\theta\, dr$$

We have determined the change in electric potential energy experienced by a charged particle which has moved from point A to point B in an electric field. Notice, because the electrostatic force is a conservative force, this change in electric potential energy does not depend on the path taken from point A to point B.

To make a comparison to gravitational potential energy, if we lift an object vertically upward in a constant downward gravitational field, it will experience a positive change in gravitational potential energy.

The same is true for a positively charged object, if we move a positively charged object vertically upward in a constant downward electric field, it will experience a positive change in electric potential energy.

Notice the negative and the dot product in the equation. As we move a charge in a direction opposite the direction of the field, the direction of the displacement of the charge and the direction of the field are opposite to one another, therefore, the angle between those two directions is 180°, the cosine of 180° is negative one, which makes the change in potential energy of a positive charge positive.

$$\Rightarrow \Delta U_e = -q\int_A^B E\cos(180^\circ)\, dr = -q\int_A^B E(-1)\, dr = q\int_A^B E\, dr$$

Next, we need to define **electric potential**. Just like we define the electric field in terms of the force experienced by a small, positive test charge, we define the electric potential in terms of the energy experienced by a small, positive test charge.

- $$\vec{E} = \frac{\vec{F}_e}{q} \Rightarrow V = \frac{U_e}{q} \text{ in volts, } V = \frac{J}{C}$$
- The symbol for electric potential is V. Yes, I know. The symbol for electric potential, V, is the same as the symbol for the units for electric potential, volts, V. It's not my fault.
- Electric potential is a **scalar** attribute of a **vector** electric field which does not depend on any electric charges which could be placed in that field.
    - The fact that electric potential is a scalar can be very helpful in this class.
    - This scalar can be either positive or negative for any given location.
    - Just like gravitational potential energy, we need to either assign a location where it equals zero, or follow a convention for assigning the zero potential location.
- Most often we work with **electric potential difference** not just electric potential. Electric potential difference is the difference in the electric potential between two points:[1]

[1] I know. I know. … Duh! … But it had to be said.

$$\Delta V = V_f - V_i = V_B - V_A \;\;\&\;\; V = \frac{U_e}{q}$$

$$\Rightarrow \Delta V = \frac{\Delta U_e}{q} = \frac{-q\int_A^B \vec{E}\cdot d\vec{r}}{q} = -\int_A^B \vec{E}\cdot d\vec{r}$$

- We will often set the initial electric potential, or electric potential at point A, equal to zero.
- Realize we can rearrange every integral to form a derivative:

$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{r} \Rightarrow dV = -\vec{E}\cdot d\vec{r} \Rightarrow E_r = -\frac{dV}{dr}$$

Now that we have volts, the units for electric potential, it is important to realize the units for the electric field can be given in terms of volts as well.

$$\vec{E} = \frac{\vec{F}_e}{q} \text{ in } \frac{N}{C} = \left(\frac{N}{C}\right)\left(\frac{m}{m}\right) = \left(\frac{J}{C}\right)\left(\frac{1}{m}\right) = \frac{V}{m} \Rightarrow \frac{N}{C} = \frac{V}{m}$$