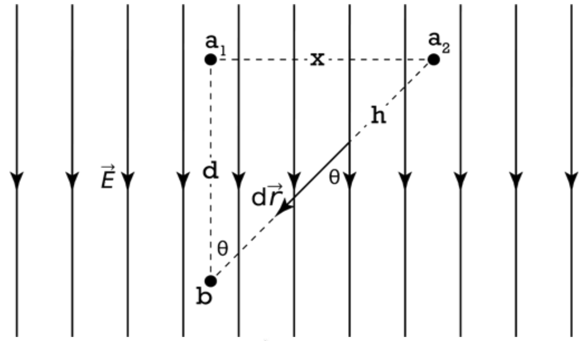
Flipping Physics Lecture Notes:
Equipotential Surfaces
http://www.flippingphysics.com/equipotential.html

Let's look at determining the electric potential difference when moving at an angle relative to a uniform electric field. We already know the electric potential difference when moving from point a_1 to b:

$$\Delta V_{a_1 \to b} = -Ed$$

Let's determine the electric potential difference when moving from point a_2 to b:



$$\Delta V_{a_2 \to b} = -\int_{a_2}^{b} E \cdot dr = -\int_{a_2}^{b} E \cos\theta \, dr = -\int_{a_2}^{b} E\left(\frac{d}{h}\right) dr = -\left(\frac{Ed}{h}\right)\int_{a_2}^{b} dr$$

$$\Rightarrow \Delta V_{a_2 \to b} = -E\left(\frac{d}{h}\right)(h) = -Ed \Rightarrow \Delta V_{a_1 \to b} = \Delta V_{a_2 \to b}$$

The electric potential difference is the same for both of these because points a_1 and a_2 have the same electric potential.

$$\Delta V_{a_2 \to a_1} = -\int_{a_2}^{a_1} E \cdot dr = -E\int_{a_2}^{a_1} \cos\theta \, dr = -E\int_{a_2}^{b} \cos(90^\circ) \, dr = 0$$

Points a_1 and a_2 are on an equipotential surface. An equipotential surface (or line):
- Has the same electric potential at every point on the surface (or line)
- Is always perpendicular to the electric field
  - Therefore, the electric field has no component along the equipotential line
- Equipotential lines are sometimes called isolines
- And it takes zero work to move a charged object along an equipotential surface
  - $W = q\Delta V \Rightarrow W_{\text{equipotential surface}} = q(0) = 0$
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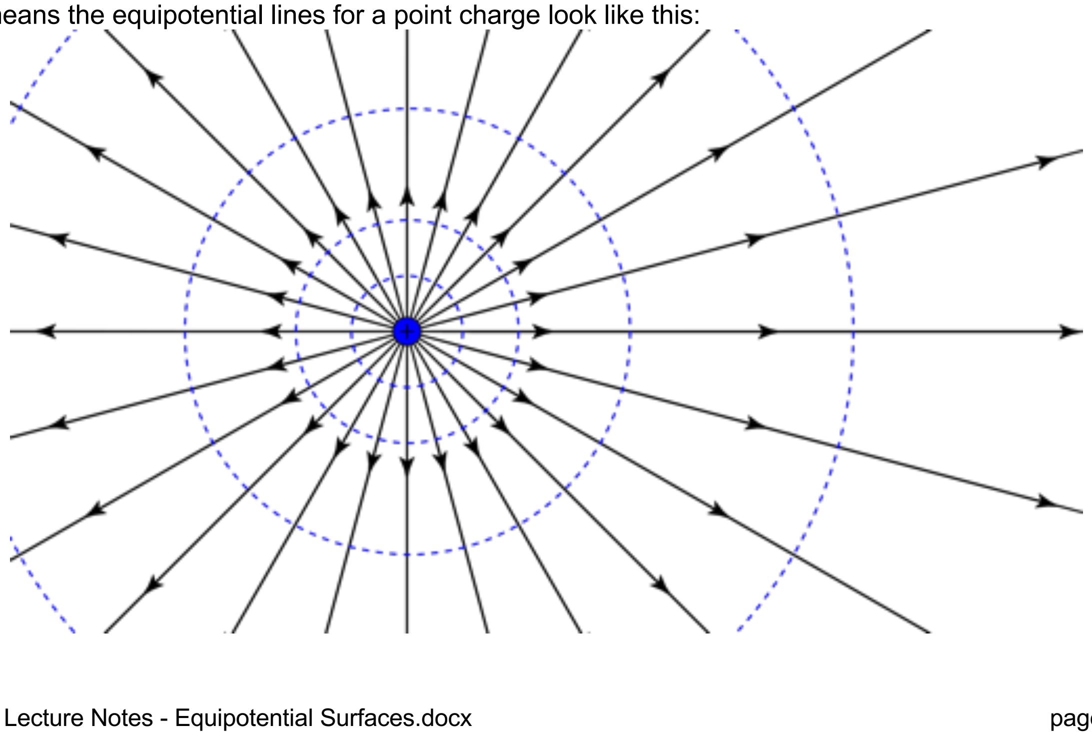
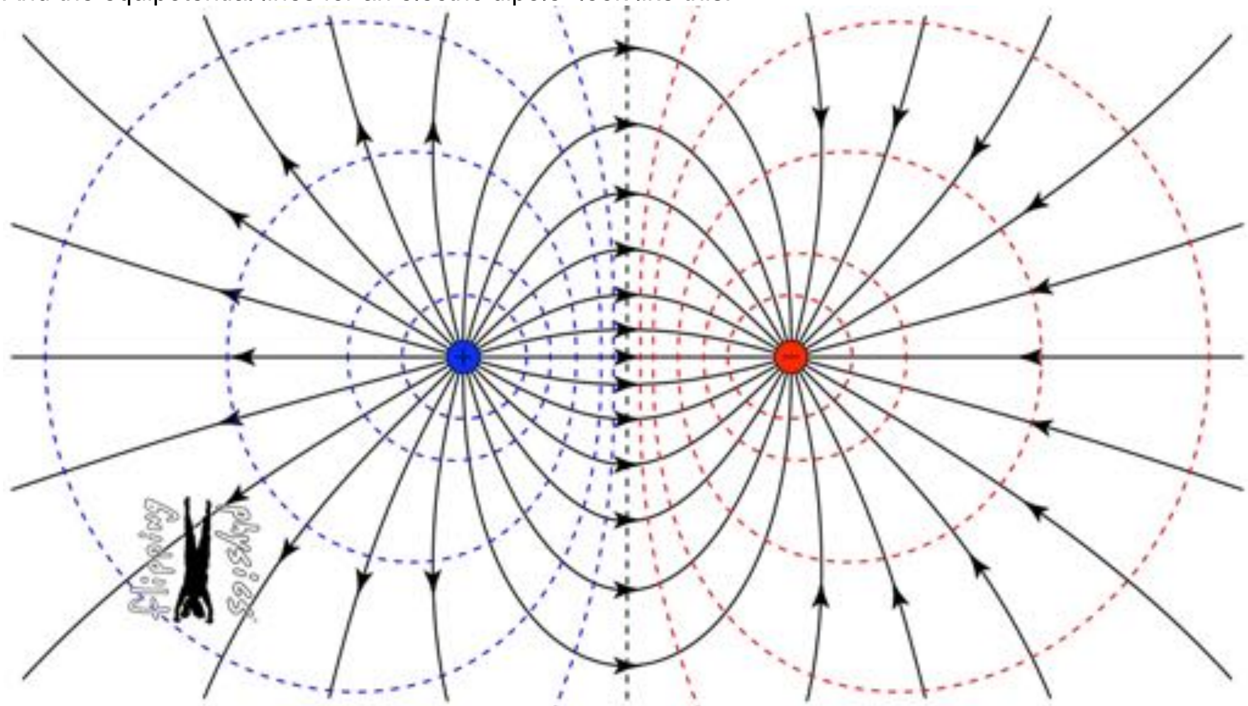
This means the equipotential lines for a point charge look like this:



And the equipotential lines for an electric dipole[1] look like this:



The equation for the electric potential which surrounds and is caused by a point charge is:

$$V_{\text{point charge}} = \frac{kq}{r}$$

This equation assigns our location of zero electric potential to be infinitely far away.

We can use the relationship between electric potential and electric potential energy to determine the electric potential energy which surrounds and is caused by a point charge:

$$V = \frac{U_{\text{elec}}}{q} \Rightarrow U_{\text{elec}} = qV \Rightarrow U_{\text{2 point charges}} = q_1\left(\frac{kq_2}{r}\right)$$

$$\Rightarrow U_{\text{2 point charges}} = \frac{kq_1 q_2}{r}$$

[1] This is a simple example of an electric dipole which is a pair of electric charges of equal magnitude, but opposite sign separated by some typically small distance.