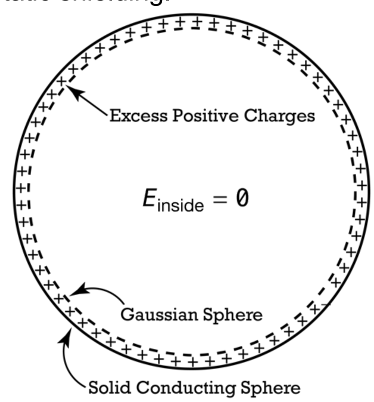
Flipping Physics Lecture Notes:
Conductors in Electrostatic Equilibrium
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-conductors-electrostatic-equilibrium.html

Conductors are materials where the electrons are free to move rather easily, however, when they are in electrostatic[1] equilibrium, this means the charges are stationary in the object. There are four things you need to remember about conductors in electrostatic equilibrium:

1) The electric field inside a conductor in electrostatic equilibrium equals zero. $E_{\text{inside}} = 0$
   a. If the electric field inside were not equal to zero, charges would have a net electrostatic force acting on them and they would accelerate, therefore the conductor would not be in electrostatic equilibrium.
      i. $E_{\text{inside}} \neq 0 \Rightarrow F_e = qE \neq 0 \Rightarrow$ not in electrostatic equilibrium
   b. Notice that this means that anything inside a conductor in electrostatic equilibrium is shielded from all external electric fields. This is called electrostatic shielding.
2) All excess charges are located on the surface (or surfaces) of the conductor.
   a. Solid conducting sphere example:
      i. Draw a Gaussian surface as a concentric sphere with a radius slightly smaller than the radius of the sphere.
      ii. Using Gauss' law, because there is no electric field inside the conductor in electrostatic equilibrium, we know the left-hand side of the equation equals zero.
      iii. Therefore, there must be zero net charge inside the Gaussian sphere and all the excess charges must be outside the Gaussian sphere.
      iv. Therefore, all the excess charges are on the surface of the conductor.



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \ \& \ E_{\text{inside}} = 0 \Rightarrow 0 = \frac{q_{\text{enclosed}}}{\varepsilon_0} \Rightarrow q_{\text{enclosed}} = 0$$

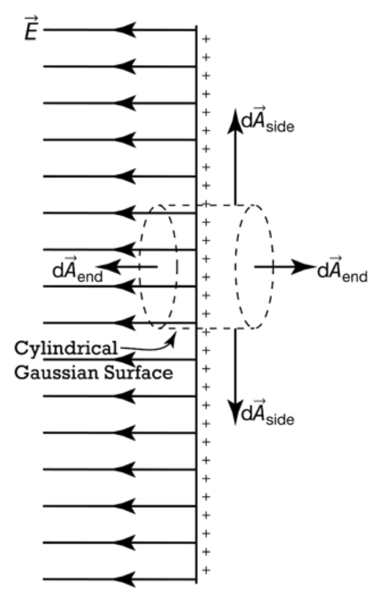
3) The electric field just outside the surface of a conductor in electrostatic equilibrium is:

$$E_{\substack{\text{just}\\\text{outside}}} = \frac{\sigma_{\text{local}}}{\varepsilon_0} \ \& \perp \text{ to surface}$$

   a. If the electric field had a component parallel to the surface of the conductor, the charges would move, and the conductor would no longer be in electrostatic equilibrium. Therefore, the electric field at the surface of a conductor in electrostatic equilibrium must be perpendicular to the surface.
      i. Because equipotential surfaces are always perpendicular to the electric field, the surface of a conductor in electrostatic equilibrium must be an equipotential surface.

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r} = -\int_A^B E\cos\theta\, dr = -\int_A^B E\cos(90^\circ)\, dr = 0$$

   b. If we zoom way in on the surface of the conductor in electrostatic equilibrium, we can draw a Gaussian cylinder



[1] Electrostatics is the study of electromagnetic phenomena that occur when there are no moving charges.

with its cylindrical axis normal to the surface of the conductor.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = \int_{\text{side}} E dA \cos\theta_{\text{side}} + \int_{\substack{\text{left}\\\text{end}}} E dA \cos\theta_{\text{end}} + \int_{\substack{\text{right}\\\text{end}}} E dA \cos\theta_{\text{end}} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$\Rightarrow \int_{\text{side}} E dA \cos(90°) + \int_{\substack{\text{left}\\\text{end}}} E dA \cos(0°) + \int_{\substack{\text{right}\\\text{end}}} (0)\, dA \cos\theta_{\text{end}} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

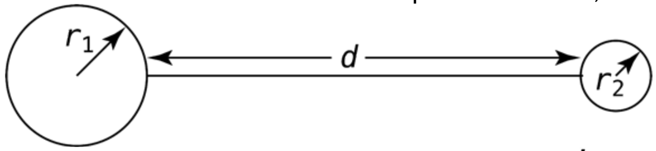
$$\& \ \sigma = \frac{Q}{A} \Rightarrow \sigma_{\text{local}} = \frac{q_{\text{in}}}{A_{\text{end}}} \Rightarrow q_{\text{in}} = \sigma_{\text{local}} A_{\text{end}}$$

$$\Rightarrow E \int_{\substack{\text{left}\\\text{end}}} dA = E A_{\text{end}} = \frac{\sigma_{\text{local}} A_{\text{end}}}{\varepsilon_0} \Rightarrow E = \frac{\sigma_{\text{local}}}{\varepsilon_0}$$

4) For an irregular shape, the local surface charge density is at its maximum where the radius of curvature is at its minimum. In other words, the largest number of excess charges per area will be where the radius of curvature is the smallest.

$$\sigma_{\text{local}} = \text{maximum @ } r_{\text{curvature}} = \text{minimum}$$

   a. To prove this, we have two conducting spheres connected by a long conducting wire with the whole system in electrostatic equilibrium.
      i. This system is a conductor in electrostatic equilibrium. In other words, when two conductors are brought into contact with one another, the charges redistribute such that both conductors are at the same electric potential. Please realize this happens so quickly that the time for this to occur is considered to be negligible.
   b. The radius of sphere 2 is smaller than the radius of sphere 1, and the distance, d, between the two spheres is much, much larger than either radius.



$$r_2 < r_1 \ \& \ d >> r_1 \ \& \ V_1 = V_2 \Rightarrow \frac{kq_1}{r_1} = \frac{kq_2}{r_2} \Rightarrow \frac{q_1}{r_1} = \frac{q_2}{r_2}$$

$$\Rightarrow q_1 = \left(\frac{r_1}{r_2}\right) q_2 \ \& \ \frac{r_1}{r_2} > 1 \Rightarrow q_1 > q_2$$

$$E_1 = \frac{kq_1}{(r_1)^2} \ \& \ E_2 = \frac{kq_2}{(r_2)^2} \Rightarrow \frac{E_1}{E_2} = \frac{\frac{kq_1}{(r_1)^2}}{\frac{kq_2}{(r_2)^2}} = \left(\frac{kq_1}{(r_1)^2}\right)\left(\frac{(r_2)^2}{kq_2}\right) = \frac{q_1 (r_2)^2}{q_2 (r_1)^2}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{\left(\left(\frac{r_1}{r_2}\right) q_2\right)(r_2)^2}{q_2 (r_1)^2} = \frac{r_2}{r_1} \Rightarrow E_2 = \left(\frac{r_1}{r_2}\right) E_1 \ \& \ \frac{r_1}{r_2} > 1$$

$$\Rightarrow E_2 > E_1 \ \& \ E = \frac{\sigma_{\text{local}}}{\varepsilon_0} \Rightarrow \sigma_2 > \sigma_1 \Rightarrow \text{if } r_2 < r_1 \text{ then } \sigma_2 > \sigma_1$$