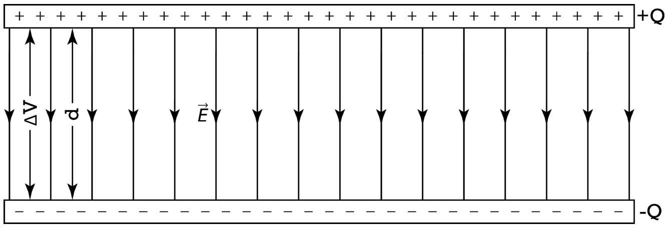
Flipping Physics Lecture Notes:
Parallel Plate Capacitors
http://www.flippingphysics.com/parallel-plate-capacitors.html



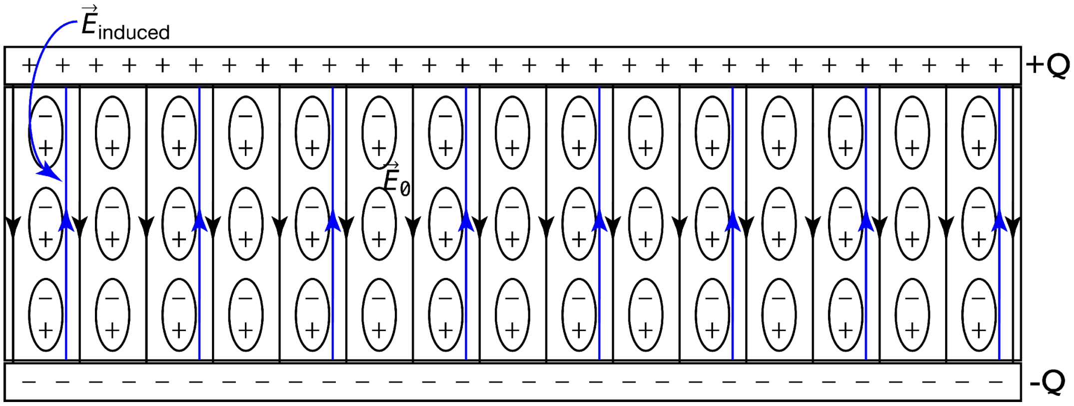
A capacitor is a way to store electric potential energy in an electric field. The simplest form of a capacitor is a parallel plate capacitor.

Let's derive the equation for the capacitance of a parallel plate capacitor. We have already derived two equations for two parallel, infinitely large, charged plates with equal magnitude, but opposite sign.

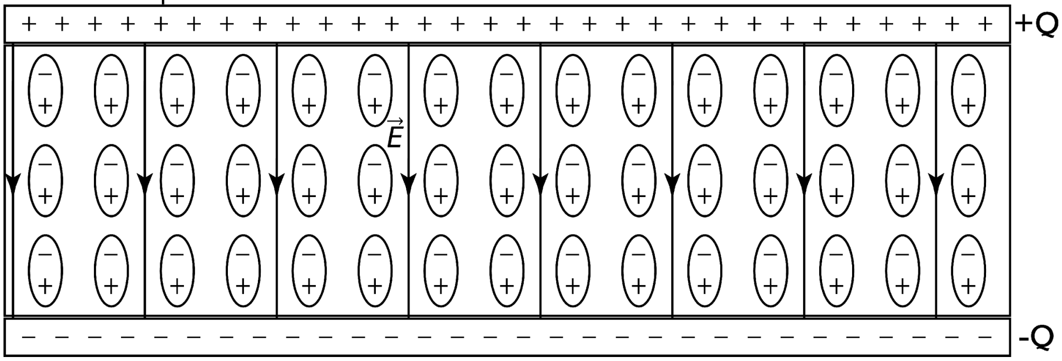
$$E_{\parallel\text{ plates}} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \quad \& \quad \Delta V_{\text{constant E}} = -Ed \Rightarrow \|\Delta V\| = Ed = \left(\frac{Q}{A\varepsilon_0}\right)d$$

$$\& \quad C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} \Rightarrow C_{\parallel\text{ plate}} = \frac{\varepsilon_0 A}{d}$$

This assumes there is a vacuum between the two plates. Usually, we place an insulating material between the plates of a capacitor. This is both to help physically separate the two plates and because it increases the capacitance of the capacitor. This insulating material is called a dielectric.



The charged particles in the dielectric are polarized and induce their own electric field (above in blue) which is opposite the direction of the original electric field of the capacitor E_0 (above in black). The net electric field (below in black) is decreased. Because the electric field is decreased, the electric potential difference across the capacitor is decreased, the charge of the capacitor remains the same, and the capacitance of the capacitor is increased.



$$E = E_0 - E_{\text{induced}} \Rightarrow E \downarrow \;\&\; \|\Delta V\| = Ed$$

$$\Rightarrow \Delta V \downarrow \;\&\; Q \text{ is constant } \;\&\; C = \frac{Q}{\Delta V} \Rightarrow C \uparrow$$

The way we define the effect of a dielectric is with the dielectric constant. The symbol for the dielectric constant is the lowercase Greek letter kappa, κ. It looks basically like a lowercase k. The dielectric constant equals the ratio of the electric permittivity of the dielectric to the electric permittivity of free space.

- $$\kappa = \frac{\varepsilon}{\varepsilon_0}$$
- The dielectric constant has no units.
- Electric permittivity is the measurement of how much a material is polarized when it is placed in an electric field.
  - The easier it is for electrons to change configurations in a material, the larger the dielectric constant of that material.
- The dielectric constant is also sometimes called *relative permittivity*.

We can also determine the relationship between the electric field between the parallel plates of the capacitor with a vacuum and with a dielectric.

$$E_{\text{vacuum}} = \frac{\sigma}{\varepsilon_0} \;\&\; E_{\text{dielectric}} = \frac{\sigma}{\varepsilon} \Rightarrow \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} = \frac{\frac{\sigma}{\varepsilon_0}}{\frac{\sigma}{\varepsilon}} = \frac{\varepsilon}{\varepsilon_0} = \kappa \Rightarrow \kappa = \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} \Rightarrow \kappa = \frac{E_0}{E}$$

And then use that to determine the relationship between the capacitance of the capacitor with a vacuum and the capacitance of the capacitor with a dielectric.

$$C_{\|\text{ plate}} = C_0 = \frac{\varepsilon_0 A}{d} \;\&\; \kappa = \frac{\varepsilon}{\varepsilon_0} \Rightarrow \varepsilon = \kappa\varepsilon_0$$

$$\&\; C_{\text{dielectric}} = C = \frac{\varepsilon A}{d} = \frac{\kappa\varepsilon_0 A}{d} \Rightarrow C_{\text{dielectric}} = \frac{\kappa\varepsilon_0 A}{d} \;\&\; C = \kappa C_0$$

According to the College Board, students are responsible for determining the capacitance only of the following shapes: parallel-plate capacitors, spherical capacitors, and cylindrical capacitors.