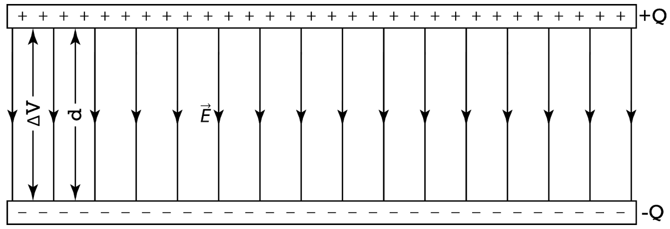
Flipping Physics Lecture Notes:
Capacitors
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-capacitors.html

A capacitor is a way to store electric potential energy in an electric field. The simplest form of a capacitor is a parallel plate capacitor.



Capacitance, C, is defined as the magnitude of the charge stored on one plate divided by the electric potential difference between the two plates: $C \equiv \frac{Q}{\Delta V}$

- Capacitance is always positive.
    - Q, is the charge on the positive plate.
    - ΔV is the positive electric potential difference between the two plates.
- The net charge on a capacitor is zero.
    - $Q_{\text{total}} = +Q + (-Q) = 0$
- $C \equiv \frac{Q}{\Delta V} \Rightarrow \text{Capacitance in } \frac{\text{coulombs}}{\text{volts}} = F\text{, farads}$
    - $\text{charge, } Q \Rightarrow \text{coulombs, } C \;\&\; \text{capacitance, } C \Rightarrow \text{farads, } F$
    - It is not my fault the symbol for capacitance is C and capacitance is charge per electric potential difference and the units for charge are coulombs for which the symbol is C.
- The three-line equal sign, $\equiv$, means "is defined as". This is not a derivation. We made it up. We have simply decided to define the charge on a capacitor divided by the electric potential difference of the capacitor as "capacitance".
- Energy is stored in the electric field of the capacitor.
- The capacitance of a capacitor depends only on the capacitor's physical characteristics. For example, the capacitor's shape and material used to separate the plates of the capacitor.

The basic idea is:

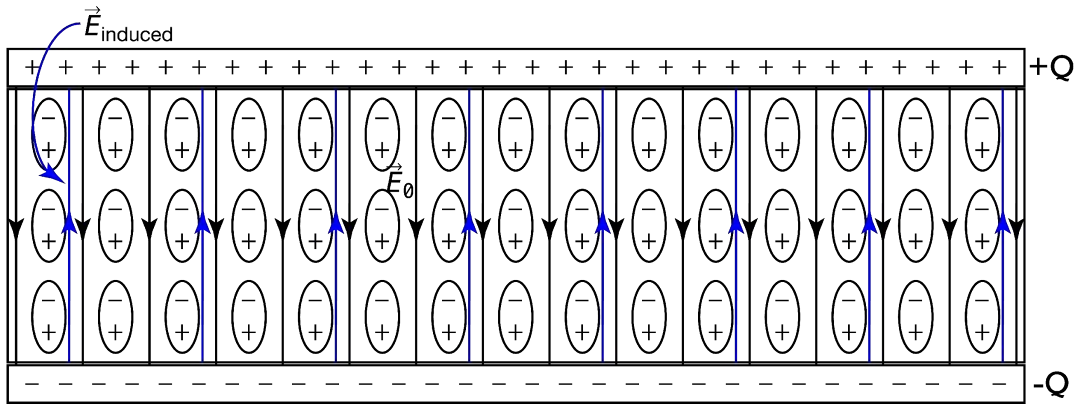
- Start with an uncharged capacitor.
    - No charge on either plate.
    - No electric field between the plates.
- Attach the terminals of a battery to the two plates of the capacitor.
- Charges flow from one plate to the other plate of the capacitor.
- We now have a charged capacitor.
    - Both plates have equal magnitude charge.
    - There is an electric field and an electric potential difference between the plates.
    - Energy is stored in the electric field of the capacitor.

Let's derive the equation for the capacitance of a parallel plate capacitor. We have already derived two equations for two parallel, infinitely large, charged plates with equal magnitude, but opposite sign.

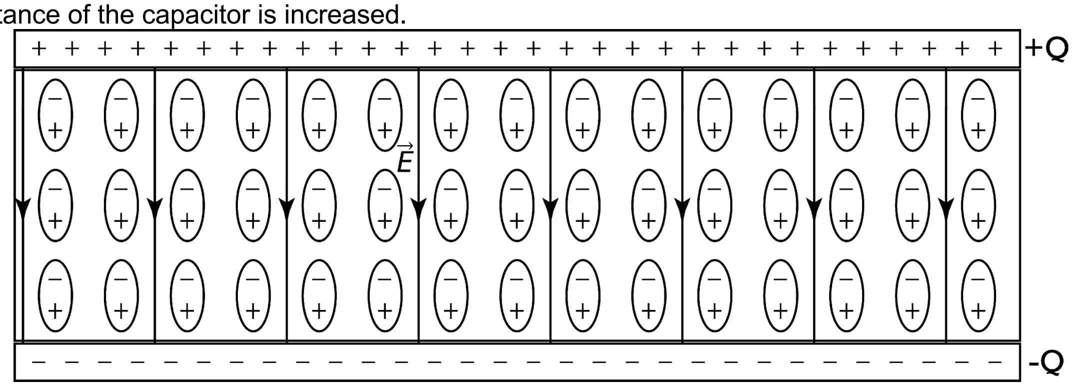
$$E_{\parallel \text{ plates}} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;\&\; \Delta V_{\text{constant E}} = -Ed \Rightarrow \|\Delta V\| = Ed = \left(\frac{Q}{A\varepsilon_0}\right)d$$

$$\& \; C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} \Rightarrow C_{\parallel \text{ plate}} = \frac{\varepsilon_0 A}{d}$$

This assumes there is a vacuum between the two plates. Usually, we place an insulating material between the plates of a capacitor. This is both to help physically separate the two plates and because it increases the capacitance of the capacitor. This insulating material is called a dielectric.



The charged particles in the dielectric are polarized and induce their own electric field (above in blue) which is opposite the direction of the original electric field of the capacitor E_0 (above in black). The net electric field (below in black) is decreased. Because the electric field is decreased, the electric potential difference across the capacitor is decreased, the charge of the capacitor remains the same, and the capacitance of the capacitor is increased.



$$E = E_0 - E_{\text{induced}} \Rightarrow E\downarrow \;\;\&\;\; \|\Delta V\| = Ed$$
$$\Rightarrow \Delta V\downarrow \;\;\&\;\; Q \text{ is constant } \;\&\;\; C = \frac{Q}{\Delta V} \Rightarrow C\uparrow$$

The way we define the effect of a dielectric is with the dielectric constant. The symbol for the dielectric constant is the lowercase Greek letter kappa, κ. It looks basically like a lowercase k. The dielectric constant equals the ratio of the electric permittivity of the dielectric to the electric permittivity of free space.

- $$\kappa = \frac{\varepsilon}{\varepsilon_0}$$
- The dielectric constant has no units.
- Electric permittivity is the measurement of how much a material is polarized when it is placed in an electric field.
    - The easier it is for electrons to change configurations in a material, the larger the dielectric constant of that material.
- The dielectric constant is also sometimes called *relative permittivity*.

We can also determine the relationship between the electric field between the parallel plates of the capacitor with a vacuum and with a dielectric.

$$E_{\text{vacuum}} = \frac{\sigma}{\varepsilon_0} \;\;\&\;\; E_{\text{dielectric}} = \frac{\sigma}{\varepsilon} \;\Rightarrow\; \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} = \frac{\frac{\sigma}{\varepsilon_0}}{\frac{\sigma}{\varepsilon}} = \frac{\varepsilon}{\varepsilon_0} = \kappa \Rightarrow \kappa = \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} \Rightarrow \kappa = \frac{E_0}{E}$$

And then use that to determine the relationship between the capacitance of the capacitor with a vacuum and the capacitance of the capacitor with a dielectric.

$$C_{\parallel \text{ plate}} = C_0 = \frac{\varepsilon_0 A}{d} \ \ \& \ \ \kappa = \frac{\varepsilon}{\varepsilon_0} \Rightarrow \varepsilon = \kappa\varepsilon_0$$

$$\& \ \ C_{\text{dielectric}} = C = \frac{\varepsilon A}{d} = \frac{\kappa\varepsilon_0 A}{d} \Rightarrow C_{\text{dielectric}} = \frac{\kappa\varepsilon_0 A}{d} \ \ \& \ \ C = \kappa C_0$$

According to the College Board, students are responsible for determining the capacitance only of the following shapes: parallel-plate capacitors, spherical capacitors, and cylindrical capacitors.

Next, let’s derive the equation for the energy stored in a capacitor. Starting with an uncharged capacitor, we move one, infinitesimally small charge from one plate to the other plate. Because the electric potential difference between the plates is zero, moving this first charge takes no work. However, moving the next charge does take work because there is now an electric potential difference between the two plates. The work it takes to move a charge equals the change in electric potential energy of the capacitor and it equals the magnitude of the charge which is moved times the electric potential difference the charge is moved through which is the electric potential difference across the capacitor which now has an infinitesimally small electric potential difference across it. We need to identify the infinitesimally small charge as dq and the amount of work it takes to move that charge dW. And take the integral of both sides.

$$Q_i = 0 \ \ \& \ \ W = \Delta U_{\text{elec}} = q\Delta V \Rightarrow \mathrm{d}W = \Delta V \mathrm{d}q$$

$$\& \ \ C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C} \Rightarrow Q = C\Delta V$$

$$W = \int_0^Q \Delta V \,\mathrm{d}q = \int_0^Q \frac{q}{C}\mathrm{d}q = \left[\frac{q^2}{2C}\right]_0^Q = \frac{Q^2}{2C} - \frac{0^2}{2C} \Rightarrow U_C = \frac{Q^2}{2C}$$

$$\Rightarrow U_C = \frac{(C\Delta V)^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(\frac{Q}{\Delta V}\right)\Delta V^2 = \frac{1}{2}Q\Delta V$$

$$\Rightarrow U_C = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}Q\Delta V$$

Note: The energy stored in the capacitor is stored in the electric field of the capacitor and is equal to the amount of work needed to move the charges from one plate to the other.

The capacitor in the disposable camera:

$$C = 120\mu F; \ \Delta V = 330V$$

$$\Rightarrow U_C = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(120 \times 10^{-6}\right)(330)^2 = 6.532 \approx 6.5J$$

$$\Rightarrow U_C = 6.532J \times \left(\frac{1\,eV}{1.6x10^{-19}J}\right) = 4.0825 \times 10^{19} \approx 41 \times 10^{18} eV$$

$$\Rightarrow U_C \approx 41 \times 10^9 \times 10^9 eV \Rightarrow U_C \approx 41 \text{ billion billion } eV$$