

When an anthropomorphic¹ charge has no choice but to go through two circuit elements, those two circuit elements are in *series*. For example, a charge which goes through resistor 1 has no choice but to also go through resistor 2. There is no other path for the anthropomorphic charge to choose.

The currents through the three circuit elements must all be equal:

$$I_t = I_1 = I_2$$

The “t” in the subscript refers to the current at the terminals of the battery which is the current delivered by the battery to the circuit.

The electric potential difference across the battery equals the summation of the electric potential difference across the two resistors:

$$\Delta V_{\text{bottom wire} \rightarrow \text{top wire}} = \epsilon = \Delta V_1 + \Delta V_2$$

(If you’d prefer to look at this in terms of the electric potential difference around the loop in the circuit:)

$$\Delta V_{\text{loop}} = V_f - V_i = V_a - V_a = 0 = \epsilon - \Delta V_1 - \Delta V_2 \Rightarrow \epsilon = \Delta V_1 + \Delta V_2$$

We know Ohm’s law: $\Delta V = IR$; therefore, ...

$$\Rightarrow \epsilon = I_t R_{\text{eq}} = I_1 R_1 + I_2 R_2$$

$$\Rightarrow R_{\text{eq}} = R_1 + R_2$$

The “eq” in the subscript means equivalent. In other words, R_{eq} is one resistor with the equivalent resistance of the two resistors.

Therefore, the equation for the equivalent resistance of n resistors in series is:

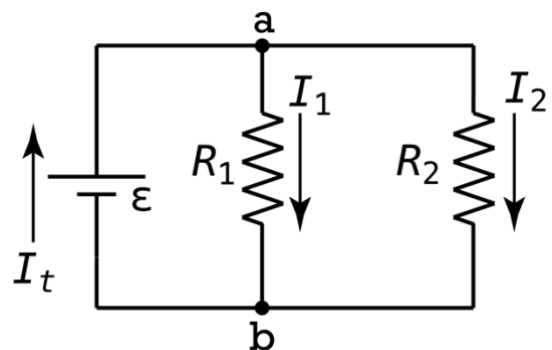
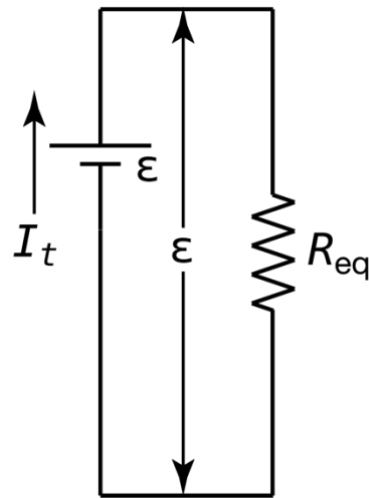
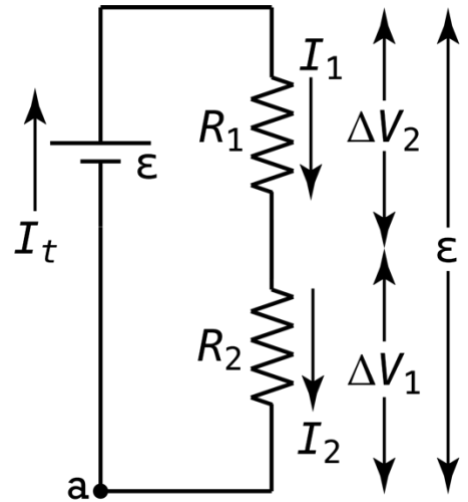
$$R_{\text{eq series}} = \sum_n R_n = R_1 + R_2 + \dots$$

When an anthropomorphic charge has the choice between two circuit elements and then the paths through those two circuit elements reconverge without going through another circuit element, the two circuit elements are in *parallel*.

When circuit elements are in parallel, their electric potential differences are equal:

$$\epsilon = \Delta V_1 = \Delta V_2$$

Note the junctions at points a and b. Due to conservation of charge, the net current going into a junction equals the net current coming out of a junction. For junction a:



¹ *Anthropomorphism*: Giving human characteristics or behaviors to non-human objects.

$$I_{\text{in}} = I_{\text{out}} \Rightarrow I_t = I_1 + I_2$$

We can then use Ohm's law:

$$\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow \frac{\epsilon}{R_{\text{eq}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

And we get the equivalent resistance for the two resistors in parallel:

$$\Rightarrow R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$$

And the equivalent resistance for n resistors in parallel:

$$\Rightarrow R_{\text{eq parallel}} = \left(\sum_n \frac{1}{R_n} \right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots \right)^{-1}$$

When we add a resistor in series, the equivalent resistance increases.

When we add a resistor in parallel, the equivalent resistance decreases.