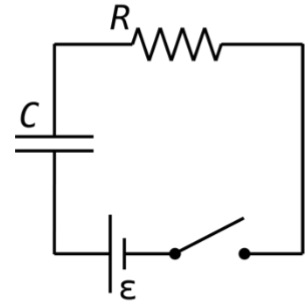
Flipping Physics Lecture Notes:
RC Circuit Time Constant
http://www.flippingphysics.com/rc-circuit-time-constat.html



We have already determined the equations for charge and current as functions of time while charging a capacitor in an RC circuit.

Now we get to talk about the time constant!
In the equations for charge and current as functions of time, there appears this expression: $e^{-\frac{t}{RC}}$

The time constant equals whatever appears in the denominator of that fraction. In other words, for an RC circuit, the time constant equals resistance times capacitance. The symbol for the time constant is the lowercase Greek letter tau, τ: $\tau = RC$

Before we discuss further what the times constant is, let's determine its units:

$$\tau = RC \Rightarrow \Omega F = \left(\frac{V}{A}\right)\left(\frac{C}{V}\right) = \frac{C}{A} = \frac{C}{\frac{C}{s}} = \frac{1}{\frac{1}{s}} = s$$

$$R = \frac{\Delta V}{I} \Rightarrow \Omega = \frac{V}{A} \;\&\; C = \frac{Q}{\Delta V} \Rightarrow F = \frac{C}{V} \;\&\; I = \frac{dq}{dt} \Rightarrow A = \frac{C}{s}$$

The units for the time constant are seconds; it is the *time* constant.

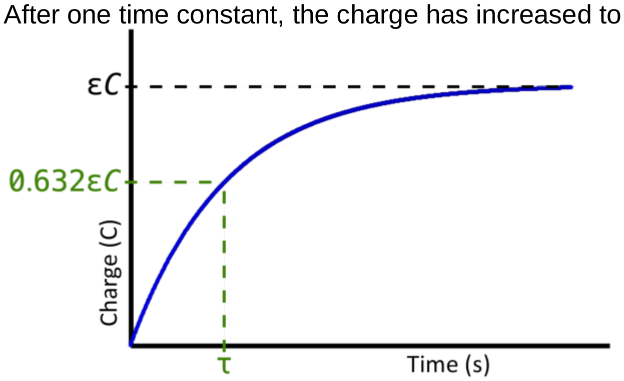
Let's replace RC with the time constant in our charge equation:

$$q(t) = q_{max}\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow q(t) = q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$$

And determine the charge on the capacitor after one time constant:

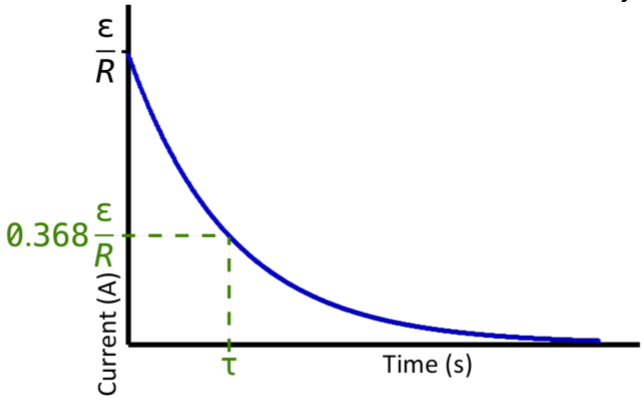
$$q(\tau) = q_{max}\left(1 - e^{-\frac{\tau}{\tau}}\right) = q_{max}\left(1 - e^{-1}\right) = q_{max}(1 - 0.368) \Rightarrow q(\tau) = 0.632 q_{max}$$

After one time constant, the charge has increased to 63.2% of its maximum value.



$$i(t) = i_{max} e^{-\frac{t}{\tau}} \Rightarrow i(\tau) = i_{max} e^{-\frac{\tau}{\tau}} = i_{max} e^{-1} \Rightarrow i(\tau) = 0.368 i_{max}$$

After one time constant, the current has decreased by 63.2% from its maximum value.



The time constant is the time it takes for a change of 63.2%. If you want to know more about the time constant, I talk about it in more detail in my video *Time Constant and the Drag Force*: https://www.flippingphysics.com/drag-force-time-constant.html

There are similar equations for discharging a capacitor through a resistor which we are not going to derive today.

Please realize the following two calculus equations are on the AP Equation Sheet:

$$\int \frac{dx}{x+a} = \ln|x+a| \quad \& \quad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$