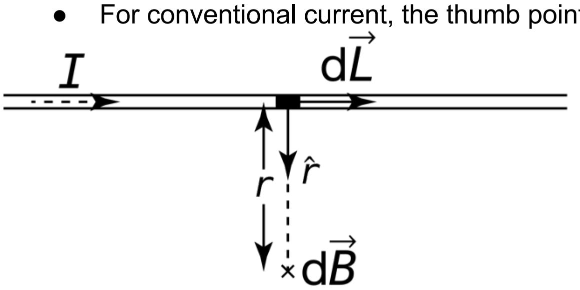
Flipping Physics Lecture Notes:
Biot-Savart and Ampère's Laws
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-biot-savart-ampere.html

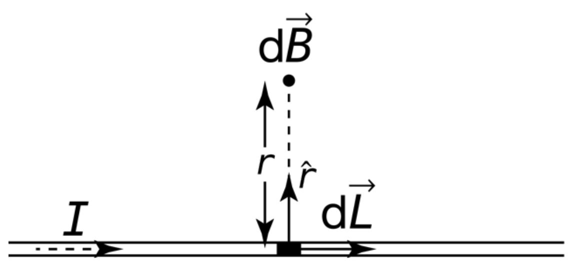
The Biot-Savart law:

- $$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{r}}{r^2}$$
- This is an experimentally determined equation; you cannot derive it.
- Unit vector r is a position vector which points from the location of the infinitesimally small length of the wire, dL, to the location of the infinitesimally small magnetic field, dB.
    - r is the magnitude of the distance between those two points
- Magnetic permeability, μ, is the measurement of the amount of magnetization of a material in response to an external magnetic field. μ_0 is the magnetic permeability of free space:
  $$\mu_0 = 4\pi \times 10^{-7}\frac{T \cdot m}{A}$$
- This equation shows that a current carrying wire creates a magnetic field. In fact, because current is composed of individually moving electric charges, even a single moving electric charge causes a magnetic field.

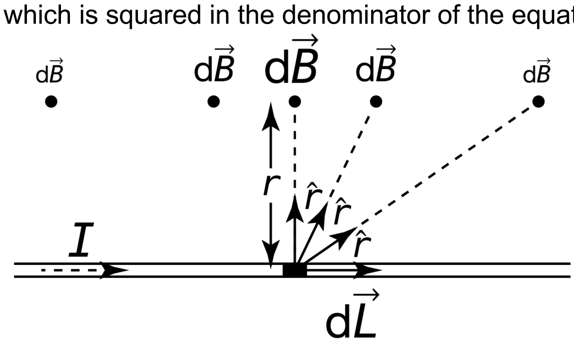
The direction of the magnetic field created by a current carrying wire can be seen using the Biot-Savart law. It is the cross product, so again, we use the right-hand rule!

- Fingers point in the direction of current/wire.
- Fingers curl in the direction of unit vector r.
- For conventional current, the thumb points in the direction of the magnetic field.

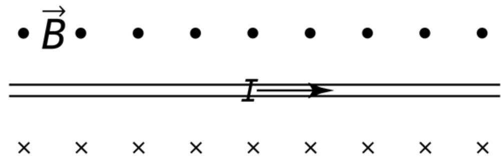


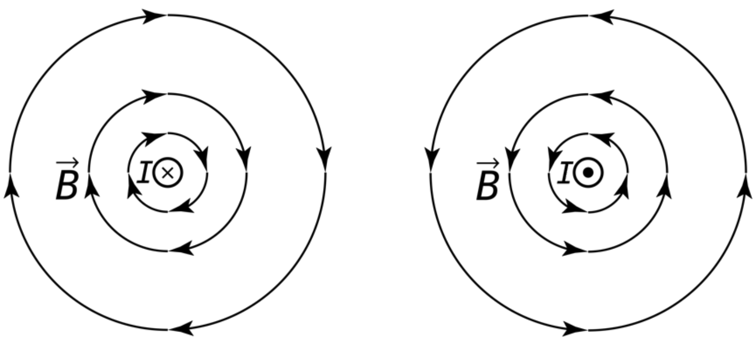


Notice the direction of the magnetic field caused by an infinitesimally small portion of the current carrying wire, dL, is the same along a line parallel to the straight wire, however, the magnitude of the magnetic field decreases as you get farther from a line perpendicular to the straight wire. The direction remains the same because the cross product of dL and unit vector r always gives the same direction. The magnitude decreases as the value of r, which is squared in the denominator of the equation, increases.



However, now realize that there are, for an infinitely long, straight, current carrying wire, an infinite number of dL's and all of their magnetic fields add up to cause the magnetic field to have a uniform value at a distance r straight out from the wire. And, the magnitude of the magnetic field decreases as r, the distance from the wire, increases.

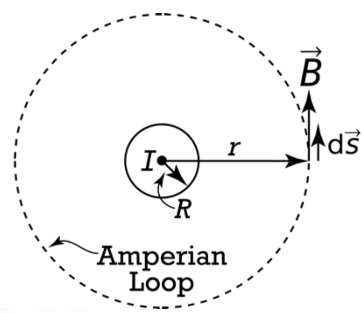




- An alternate right-hand rule exclusively for the magnetic field which surrounds a current carrying wire is:
    - Point thumb in direction of current.
        - Fingers curl in the direction of the magnetic field.
- The Biot-Savart law can also be used to determine the magnitude of the magnetic field a distance r from an infinitely long, straight, current carrying wire. That equation is: $B = \frac{\mu_0 I}{2\pi r}$
    - We now know the magnetic field magnitude is inversely proportional to distance from the wire.

Next up we have Ampère's law, which is the magnetic field equivalent to Gauss' law:

- Gauss' law: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$
    - Closed surface integral and charge inside a Gaussian surface.
- Ampère's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$
    - Closed loop integral and current inside an Amperian loop.



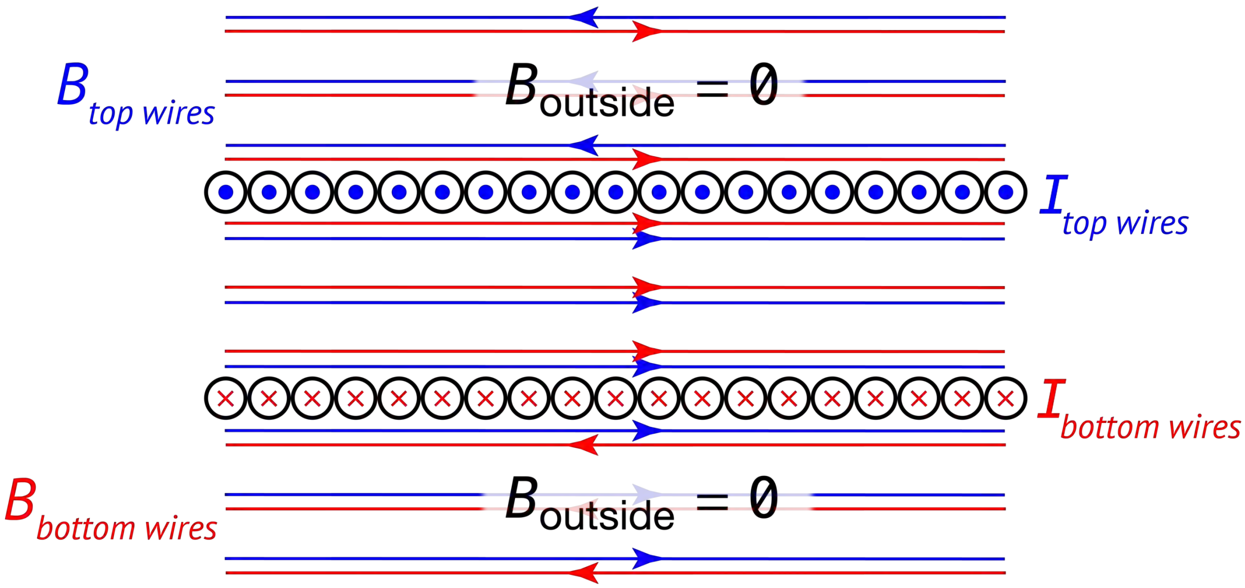
Example: Determine the magnitude of the magnetic field outside an infinitely long, straight, wire with radius R and current I.

Start by drawing an Amperian loop in the shape of a circuit of radius $r \geq R$ which is concentric with the wire. And let's use Ampère's law.

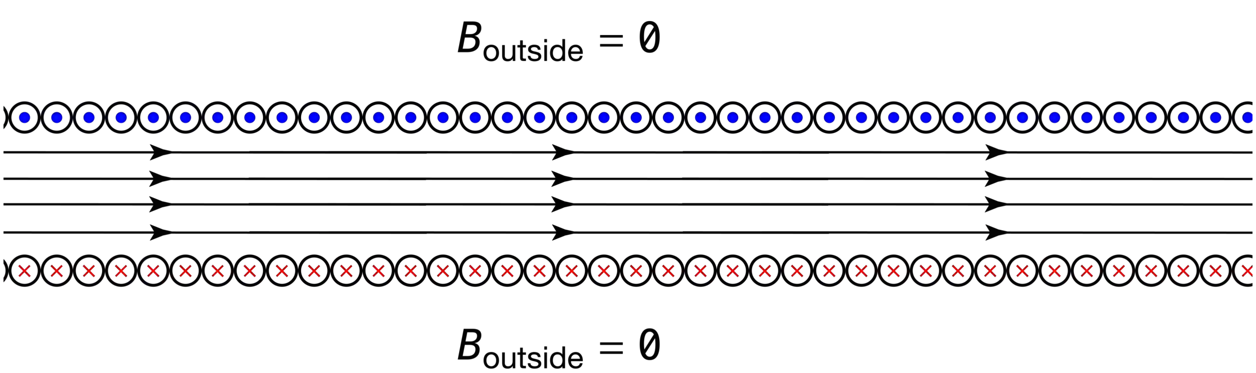
$$\Rightarrow \oint B ds \cos\theta = \oint B ds \cos 0^\circ = B \oint ds = B(2\pi r) = \mu_0 I_{in}$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

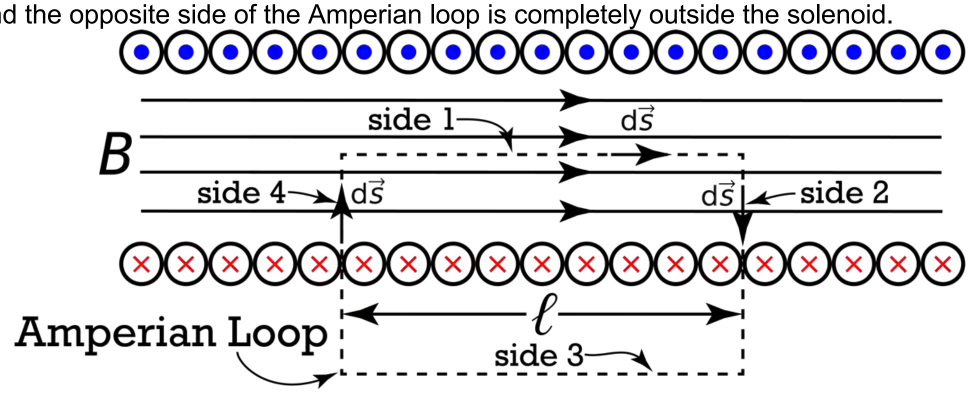
Next let's look at a solenoid. A very common tool for creating a uniform magnetic field. A typical solenoid is a single, very long, current carrying, insulated wire wrapped to form a hollow cylinder. An ideal solenoid has a length which is much, much larger than its diameter. The cross section of a solenoid looks like this.



Outside the solenoid, the magnetic field caused by the current in the top wires completely cancels out the magnetic field caused by the bottom wires. In other words, an ideal solenoid has zero magnetic field outside the cylinder of the solenoid. (ideal solenoid below)



Now let's derive the equation for the magnetic field inside an ideal solenoid. In order to do so, we begin with Ampère's law and draw an Amperian loop. Just like Gaussian surfaces, we want to pick Amperian loops to have sides which are at integer multiples of 90° relative to the magnetic field, and such that the magnetic field is uniform on the sides of the Amperian loop. For an ideal solenoid, we pick an Amperian loop shape of a rectangle with one side inside the solenoid and parallel to the magnetic field inside the solenoid and the opposite side of the Amperian loop is completely outside the solenoid.



And now we can begin using Ampère's law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \Rightarrow \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s} = \mu_0 I_{in}$$

For side 3, the magnetic field is zero outside the solenoid, so that integral equals zero. For sides 2 and 4, the magnetic field and ds are 90° to one another and the cosine of 90° is zero, so both of those integrals equal zero. That means, the only integral which remains is the integral for side 1.

$$\Rightarrow \int_1 \vec{B}\cdot d\vec{s} = B\int_1 ds\cos 0° = B\ell = \mu_0 N I \;\;\&\;\; I_{in} = NI$$

$$\Rightarrow B = \frac{\mu_0 N I}{\ell} \;\;\&\;\; n = \frac{N}{\ell} \Rightarrow B_{solenoid} = \mu_0 n I$$

Where "n" is the turn density of the solenoid.