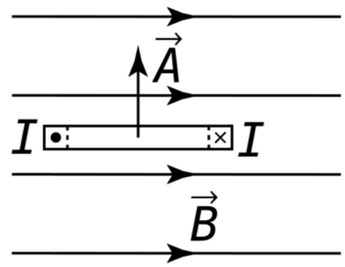
Flipping Physics Lecture Notes:
Electromagnetic Induction
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-electromagnetic-induction.html

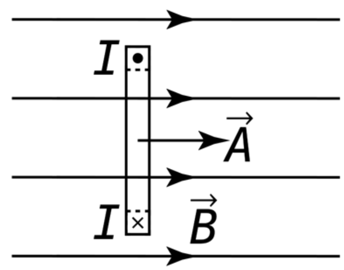
Before we learn about electromagnetic induction, we need to learn about magnetic flux. Before we do that, let's review electric flux:

- Electric flux is the measure of the number of electric field lines which pass through a surface.
- When the electric field is uniform, and the surface is a two-dimensional plane:
  $\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$
- The general equation for electric flux: $\Phi_E = \int \vec{E} \cdot d\vec{A}$

Magnetic flux:



- Magnetic flux is the measure of the number of magnetic field lines which pass through a surface.
- When the magnetic field is uniform, and the surface is a two-dimensional plane: $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$
- The general equation for magnetic flux:
  $$\Phi_B = \int \vec{B} \cdot d\vec{A} \Rightarrow T \cdot m^2 = \text{webbers}, Wb$$
- Example #1: Current through a wire loop. Use the right-hand rule to determine the direction of the area vector. (Similar to the right-hand rule for angular velocity direction.) Fingers curl in the direction of the current, thumb points in the direction of the area vector.
  $\Phi_B = BA\cos\theta = BA\cos 90° = 0$
- Example #2: $\Phi_B = BA\cos\theta = BA\cos 0° = \Phi_{B_{max}}$



Gauss's law has to do with electric flux:

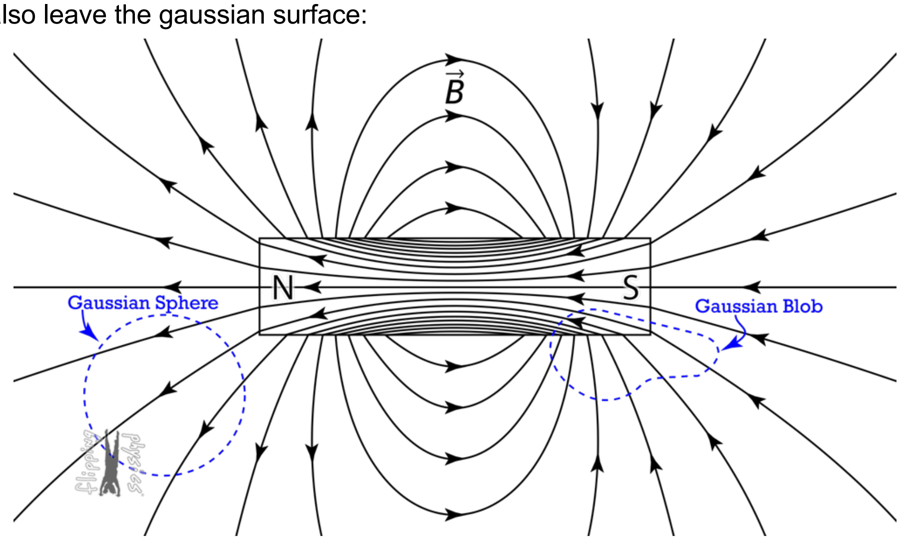
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

Gauss's law for magnetism has to do with magnetic flux:

- Because a magnetic monopole has never been found in nature or in a manmade experiment, every magnetic field line is a closed loop.
- Therefore, no matter what shape the gaussian surface has, every magnetic field line which enters the gaussian surface will also leave the gaussian surface:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

- The above equation, Gauss's law for magnetism, is the second of Maxwell's equations which are a collection of equations which fully describe electromagnetism.



Electromagnetic Induction:

- We have already discussed that moving electric charges create magnetic fields.
- It should be no surprise that moving magnetic poles create electric fields.
    - Notice how these interact with one another!
- When a magnetic field changes over time, this can induce an electric potential difference called an induced emf, this causes charge to flow in a closed loop of wire which is called an induced current. More specifically, the relationship is between a changing magnetic flux and the resulting induced emf in a single closed loop of wire and is described by Faraday's law of electromagnetic induction:

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right|$$

    - Induced emf = the derivative of magnetic flux with respect to time. (magnitudes)
- Substitute in the equation for magnetic flux:

$$|\varepsilon| = N\left|\frac{d\Phi_B}{dt}\right| = N\left|\frac{d\left(\vec{B}\cdot\vec{A}\right)}{dt}\right| = N\left|\frac{d\,(BA\cos\theta)}{dt}\right|$$

    - N is the number of loops
    - An emf can be induced by changing:
        - Magnitude of the magnetic field.
        - Area enclosed by the loop.
        - Angle between magnetic field and loop area. (θ between $\vec{B}$ and $\vec{A}$)
        - Or any combination of the three.
            - In other words, if the only one of those three ($\vec{B}$, $\vec{A}$, and θ) which is changing is the magnitude of the magnetic field, then the magnitude of the induced emf through one loop of wire is:

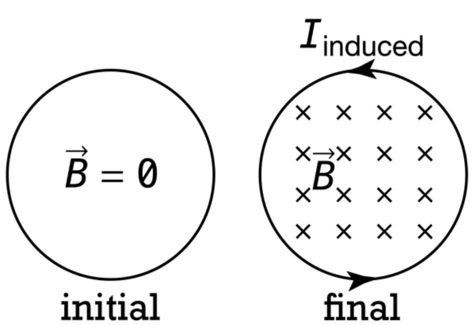
$$|\varepsilon| = \left|A\cos\theta\left(\frac{dB}{dt}\right)\right|$$

- Electromagnetic induction is the process of inducing an electromotive force by a change in magnetic flux.
- Faraday's law is the third of Maxwell's equations which are a collection of equations which fully describe electromagnetism.

Now we need to determine the direction of the induced emf caused by a changing magnetic flux. That is shown by removing the absolute value from the equation, which gives us, assuming only one loop:

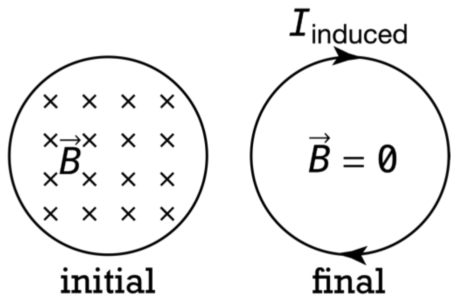
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- The negative in this equation means the induced emf is opposite the direction of the change in magnetic flux.
- The direction of the induced emf is called Lenz' law.
    - Yes, the negative added to Faraday's law is called Lenz' law.
    - Lenz' law: The current induced in a circuit due to a change in a magnetic field is directed to oppose the change in magnetic flux and to exert a mechanical force which opposes the motion.
- We use the right-hand rule[1] to determine the direction of the induced emf. Examples below:



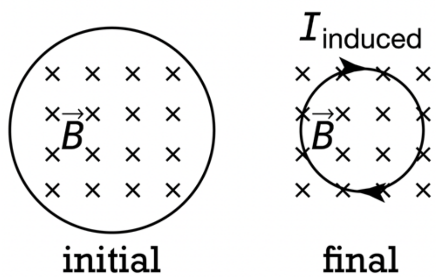
- Zero initial magnetic flux inside the loop.
- Original B field is into the screen and increasing, therefore the original magnetic flux is increasing.
- Induced magnetic field opposes the change in the original magnetic flux and therefore is induced out of the screen to counteract the change in original magnetic flux.
- According to the right-hand rule, fingers curl out of the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is counterclockwise.

[1] This is the "alternate" right-hand rule with the thumb pointing in the direction of the current in the wire and fingers curling in the direction of the magnetic field created by the current in the wire.



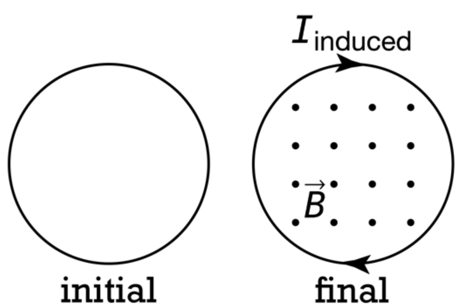
● Original B field in the loop is into the screen and decreasing, which means the original magnetic flux is decreasing.
● B_{induced} opposes this change in magnetic flux and attempts to maintain the original magnetic flux. Therefore, B_{induced} is into the screen.
● According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is clockwise.

Note: Magnetic flux is a dot product, so magnetic flux is a scalar. So, the induced magnetic flux does not have a direction, however, the induced magnetic field does have a direction and the direction of the induced magnetic field in the plane of the loop is always normal to the loop in which the induced current is created.

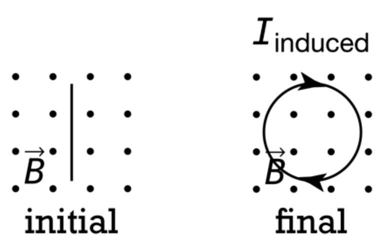


● Original B field inside the loop is into the screen and the area is decreasing which means the original magnetic flux is decreasing.
● B_{induced} opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, B_{induced} is into the screen.
● According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is clockwise.

(The next example was cut out of the video, however, y'all still get to enjoy it here!)

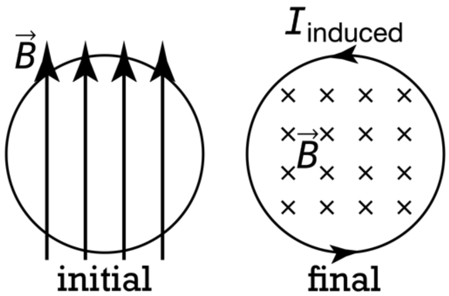


● There is no original B field so no original magnetic flux. The B field is increasing out of the screen so the original magnetic flux is increasing.
● B_{induced} opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, B_{induced} is into the screen.
● According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is counterclockwise.

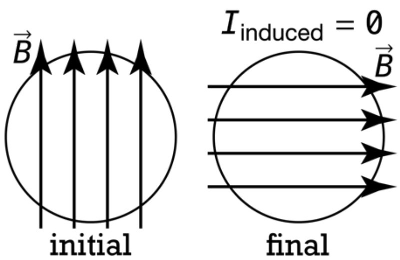


● B field is originally parallel to the loop, so there is zero original magnetic flux through the loop. Loop turns to cause the area of the loop to now be normal to the B field which is out of the screen. So, the original magnetic flux is increasing.
● B_{induced} opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, B_{induced} is into the screen.
● According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is clockwise.

Note: No matter which way the loop is turned, the change in the magnetic flux through the loop is the same and the induced magnetic field is into the screen caused by the induced current which is clockwise from this perspective.



● B field is originally parallel to the loop, so there is no original magnetic flux. B field turns to now be into the screen. So, the original magnetic flux is increasing.
● $B_{induced}$ opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, $B_{induced}$ is out of the screen.
● According to the right-hand rule, fingers curl out of the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is counterclockwise.



● B field is originally parallel to the loop, so there is no original magnetic flux. B field turns to now be … still parallel to the loop. So, the magnetic flux through the loop is still zero.
● No change in the magnetic flux means there is no induced current. ☺️

We now have covered all four of Maxwell's equations which are a collection of equations which fully describe electromagnetism:

1) Gauss' law: $\Phi_E = \oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$
2) Gauss' law in magnetism: $\Phi_B = \oint_{\text{surface}} \vec{B} \cdot d\vec{A} = 0$
3) General form of Maxwell-Faraday's law of induction: $\varepsilon = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
4) The Ampère-Maxwell law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{in}} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

Maxwell's third equation is:

- The Faraday's law of induction we previously learned: $\varepsilon = -\frac{d\Phi_B}{dt}$
  - Which shows that changing magnetic fields create an electric potential difference
- plus the more general addition: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
  - Which shows that a changing magnetic field must also create a nonconservative electric field.

Maxwell's fourth equation is:

- Ampère's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{in}}$
  - Which shows that magnetic fields can be generated by electric currents
- plus Maxwell's addition of $\varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$
  - Which shows that a changing electric field creates a magnetic field.
    - In a similar manner to how a moving charge creates a magnetic field.