Flipping Physics Lecture Notes:
Inductance of an Ideal Solenoid
http://www.flippingphysics.com/inductance-solenoid.html

Considering the most common shape for an inductor is a small, ideal solenoid, let's look at that case specifically. We have two different equations for induced emf which we can set equal to one another:

- $\varepsilon_{induced} = -N\dfrac{d\Phi_B}{dt} = -L\dfrac{dI}{dt} \Rightarrow Nd\Phi_B = LdI$
    - N is the total number of loops or coils in the solenoid shaped inductor.
    - We can cancel out *dt* on both sides of the equation
- $\Rightarrow \int Nd\Phi_B = \int LdI \Rightarrow N\int_0^{\Phi_B} d\Phi_B = L\int_0^{I} dI \Rightarrow N\Phi_B = LI \Rightarrow L_{solenoid} = \dfrac{N\Phi_B}{I} = \dfrac{N(BA\cos\theta)}{I}$
    - Take the integral of the whole equation.
    - Both N and L are constants and can be taken out from their integrals.
    - Substitute in the equation for the magnitude of magnetic flux.
- $\Rightarrow L_{solenoid} = \dfrac{NBA\cos(0°)}{I} = B\left(\dfrac{NA}{I}\right)$ & $B_{solenoid} = \mu_0 nI = \dfrac{\mu_0 NI}{\ell}$
    - In an ideal solenoid, angle between magnetic field and loop area vector is always 0°.
    - We have the equation for an ideal solenoid which we derived earlier. $n = \dfrac{N}{\ell}$
        - n is the turn density of the solenoid.
        - We already defined N as the total number of loops in the solenoid,
        - Therefore, the curly ℓ, is the entire length of the ideal solenoid.
            - Note L ≠ ℓ. (Inductance does not equal solenoid length.)
            - (L for a resistor is its length not its inductance.)
- $\Rightarrow L_{solenoid} = \left(\dfrac{\mu_0 NI}{\ell}\right)\left(\dfrac{NA}{I}\right) \Rightarrow L_{solenoid} = \dfrac{\mu_0 N^2 A}{\ell}$
    - The inductance of an ideal solenoid is determined by:
        - N, the number of turns: A, the cross-sectional area: ℓ, solenoid length.
        - μ, the magnetic permeability of the space inside the solenoid. For an ideal solenoid with nothing inside it, that equals the magnetic permeability of free space.
        - μ, the magnetic permeability of the core material, replaces μ_0 when the solenoid has a core material such as iron.
            - Inductance does *not* depend on current through the solenoid!
                - Resistance does *not* depend on current either!