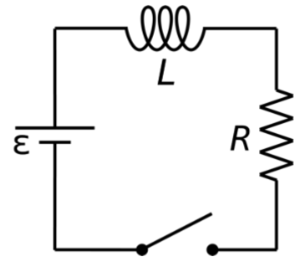
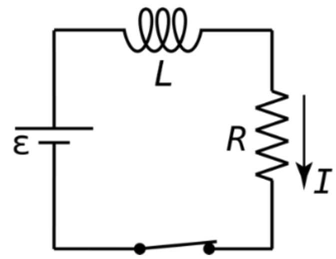
Flipping Physics Lecture Notes:
Energy Stored in an Inductor
http://www.flippingphysics.com/inductor-energy.html



Let's derive the equation for the energy stored in the magnetic field generated in an inductor as charges move through the inductor. To do that, we need to discuss an LR circuit. A circuit with an inductor and a resistor in it. Initially, at time $t < 0$, the switch is open. At time $t = 0$, the switch is closed. The current will increase from zero to some steady-state current, I. We are not going to derive the time-dependent equations for LR circuits today, we will do that in a future lesson.



Using Kirchhoff's Loop Rule, starting from the lower left-hand corner we get:

- $$\Delta V = 0 = \varepsilon - \Delta V_L - \Delta V_R = \varepsilon - L\frac{dI}{dt} - IR \Rightarrow \varepsilon = L\frac{dI}{dt} + IR$$
    - Electric potential across the battery goes up because the battery is adding electric potential energy to the circuit.
    - Electric potential across the inductor goes down because electric potential energy is being stored in the magnetic field of the inductor.
    - Electric potential across the resistor goes down because the resistor dissipates electric potential energy from the system.
    - We can now multiply this whole equation by the circuit current, I.
- $$\Rightarrow P = I\varepsilon = LI\frac{dI}{dt} + I^2R$$
    - We get the equation for power for each circuit element:
        - The rate at which energy is being added to the circuit by the battery.
        - The rate at which energy is being stored in the magnetic field of the inductor.
        - The rate at which energy is being dissipated by the resistor.
- We can now look specifically at the rate at which energy is being stored in the magnetic field of the inductor.
- $$\Rightarrow P = \frac{dU}{dt} = LI\frac{dI}{dt} \Rightarrow dU = LI\,(dI) \Rightarrow \int_0^{U_L} dU = \int_0^I LI\,(dI) = L\int_0^I I\,(dI)$$
  $$\Rightarrow U_L = \left[L\left(\frac{I^2}{2}\right)\right]_0^I \Rightarrow U_L = \frac{1}{2}LI^2$$
- We now have an equation for the energy stored in the magnetic field generated in an inductor as charges move through the inductor.
    - That energy is only present when current is passing through the inductor. This is because the magnetic field generated in the inductor is due to the charges moving through the inductor. If the charges are not moving, there is no magnetic field in the inductor.

A capacitor functions differently:

- The energy stored in a capacitor is stored in the electric field of the capacitor.
    - The energy stored in a capacitor can remain when a capacitor is disconnected from a circuit because charges can remain separated on the plates of the capacitor which would maintain the electric field between the plates of the capacitor.

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}Q\Delta V$$