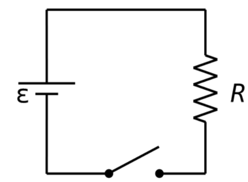
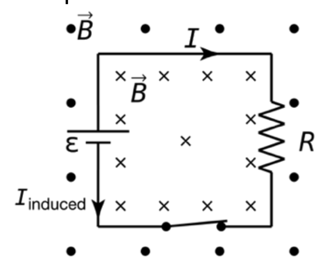
Flipping Physics Lecture Notes:
Inductance
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/apcem-inductance.html



Let's look at a basic circuit. Before time t = 0, the switch in the circuit is open and zero current flows through the open loop. At time t = 0, the switch is closed and remains closed. From this perspective, a clockwise current, I, is now in the circuit. Up to this point we have assumed the current appears instantaneously in the circuit. You should realize that, in the real world, nothing changes instantaneously. So, let's look at what really happens when the switch closes.



According to the alternate right-hand rule, the clockwise current, I, in the circuit causes a magnetic field which is out of the page outside the loop and a magnetic field which is into the page inside the loop. In other words, this circuit is a loop which initially, before time t = 0, has zero magnetic flux in it and, as soon as the switch is closed, the loop has magnetic flux in it. We know, according to Faraday's law, that a changing magnetic flux induces an emf and can induce a current. We can use Lenz' law to determine the direction the induced current would be in the loop:

$$\varepsilon_{induced} = -N\frac{d\Phi_B}{dt}$$

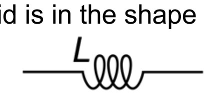
- Initially, there is zero magnetic flux.
- Finally, there is a B field which is into the page inside the loop.
  - Note: Only the magnetic field inside the loop causes a magnetic flux inside the loop.
- Therefore, the magnetic flux is increasing.
- Lenz's law states that an induced magnetic field is created to counteract the change in magnetic flux.
- Therefore, the induced magnetic field is out of the page.
- According to the alternate right-hand rule, an induced current would be counterclockwise in the loop from this perspective.
- This means the current in the circuit does not instantly change from 0 to I. The current in the circuit takes time to transition from 0 to I, because, the circuit itself opposes the change in current.
- This opposition of a circuit to a change in current in that same circuit is called *self-inductance*.
- In general, opposition to a change in current in a conductor is called *inductance.*

To get to the equation for inductance, we need to return to the simple circuit example and the basic concept of Faraday's law.

- Induced emf is proportional to change in magnetic flux with respect to time. $\varepsilon_{induced} \propto \frac{d\Phi_B}{dt}$
- The magnitude of magnetic flux equals the magnetic field times the area of the loop times the cosine of the angle between the direction of the magnetic field and the direction of the area. $\Phi_B = BA\cos\theta$
- Assuming the area and angle are not changing with respect to time, the induced emf is proportional to the change in the magnetic field with respect to time. $\varepsilon_{induced} \propto \frac{dB}{dt}$
- An example of a magnetic field around a current carrying wire is the one which surrounds an infinitely long current carrying wire which we have derived previously. $B = \frac{\mu_0 I}{2\pi a}$
  - "a" is the straight-line distance perpendicular out from the wire to the location of the B field.
- This means the induced emf in a conductor is proportional to change in current in the conductor with respect to time. $\varepsilon_{induced} \propto \frac{dI}{dt}$

An inductor is a circuit element with a known inductance.
The equation for the inductance of an inductor is: $\varepsilon_L = -L\frac{dI}{dt}$

- "L" is the inductance of the inductor.
- The simplest version of an inductor is a small, ideal solenoid. Because a solenoid is in the shape of a coil, the symbol for an inductor looks like the coils of a miniature solenoid.



- The units for inductance are henrys, H.
  - $$\varepsilon_L = -L\frac{dI}{dt} \Rightarrow L = -\frac{\varepsilon_L}{dI/dt} \Rightarrow \frac{V}{A/s} \Rightarrow \text{henry}, H = \frac{V \cdot s}{A}$$

It is important to understand the difference between resistance, resistivity, resistors, inductance, self-inductance, and inductors.

- *Resistance* is an opposition to current. *(concept)*

  $$R = \frac{\Delta V}{I}$$

  - The units for resistance are ohms, Ω.
  - The resistance of a circuit is often assumed to be zero. (self-resistance?)
  - A *resistor* is a circuit element with a specific resistance. *(physical object)*

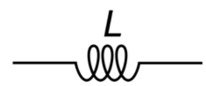
    $$\rho = \frac{RA}{L}$$

    - "R" is the resistance of a resistor.
    - A resistor is made of a material with a material property called *resistivity*, ρ.
      - The units for resistivity are ohm meters, Ω•m.

    - A resistor can be added to a circuit to change the resistance of the circuit.
    - A resistor can be added to a circuit diagram to model the resistance of the circuit itself.
- *Inductance* is an opposition to changes in current. *(concept)*
  - The units for inductance are henrys, H.
  - The opposition of a circuit to the change in the current in that circuit is called *self-inductance*. *(concept)*
    - The self-inductance of a circuit is often assumed to be zero.

    $$L = -\frac{\varepsilon_L}{dI/dt}$$

  - An *inductor* is a circuit element with a specific inductance. *(physical object)*
    - "L" is the inductance of an inductor.
    - A typical shape for an inductor is a small, ideal solenoid.



      - There is no material property called "inductivity" because the inductance of an inductor is mostly caused by the shape, not the material, of the inductor. A magnetic material in its core can affect the inductance through its magnetic permeability, but not the material of the wire coil.
    - An inductor can be added to a circuit to change the inductance of the circuit.
    - An inductor can be added to a circuit diagram to model the self-inductance of the circuit itself.

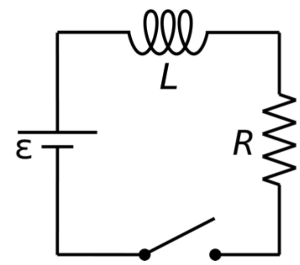
Considering the most common shape for an inductor is a small, ideal solenoid, let's look at that case specifically. We have two different equations for induced emf which we can set equal to one another:

- $$\varepsilon_{\text{induced}} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \Rightarrow N d\Phi_B = L dI$$
  - N is the total number of loops or coils in the solenoid shaped inductor.
  - We can cancel out *dt* on both sides of the equation
- $$\Rightarrow \int N d\Phi_B = \int L dI \Rightarrow N\int_0^{\Phi_B} d\Phi_B = L\int_0^{I} dI \Rightarrow N\Phi_B = LI \Rightarrow L_{\text{solenoid}} = \frac{N\Phi_B}{I} = \frac{N(BA\cos\theta)}{I}$$
  - Take the integral of the whole equation.
  - Both N and L are constants and can be taken out from their integrals.
  - Substitute in the equation for the magnitude of magnetic flux.
- $$\Rightarrow L_{\text{solenoid}} = \frac{NBA\cos(0°)}{I} = B\left(\frac{NA}{I}\right) \quad \& \quad B_{\text{solenoid}} = \mu_0 nI = \frac{\mu_0 NI}{\ell}$$
  - In an ideal solenoid, angle between magnetic field and loop area vector is always 0°.
  - We have the equation for an ideal solenoid which we derived earlier.

    $$n = \frac{N}{\ell}$$

    - n is the turn density of the solenoid.
    - We already defined N as the total number of loops in the solenoid,
    - Therefore, the curly ℓ, is the entire length of the ideal solenoid.
      - Note L ≠ ℓ. (Inductance does not equal solenoid length.)
      - (L for a resistor is its length not its inductance. ☺)
- $$\Rightarrow L_{\text{solenoid}} = \left(\frac{\mu_0 NI}{\ell}\right)\left(\frac{NA}{I}\right) \Rightarrow L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{\ell}$$
  - The inductance of an ideal solenoid is determined by:

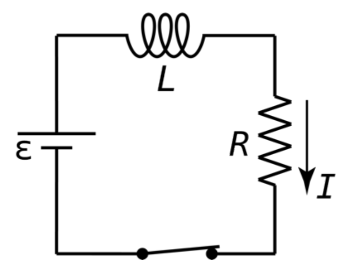
- N, the number of turns: A, the cross-sectional area: ℓ, solenoid length.
- μ, the magnetic permeability of the space inside the solenoid. For an ideal solenoid with nothing inside it, that equals the magnetic permeability of free space.
- μ, the magnetic permeability of the core material, replaces μ_0 when the solenoid has a core material such as iron.
  - Inductance does *not* depend on current through the solenoid!
    - Resistance does *not* depend on current either!



Next, we are going to derive the equation for the energy stored in the magnetic field generated in an inductor as charges move through the inductor. To do that, we need to discuss an LR circuit. A circuit with an inductor and a resistor in it. Initially, at time t < 0, the switch is open. At time t = 0, the switch is closed. The current will increase from zero to some steady-state current, I. We are not going to derive the time-dependent equations for LR circuits today, we will do that in a future lesson.

Using Kirchhoff's Loop Rule, starting from the lower left-hand corner we get:

- $$\Delta V = 0 = \varepsilon - \Delta V_L - \Delta V_R = \varepsilon - L\frac{dI}{dt} - IR \Rightarrow \varepsilon = L\frac{dI}{dt} + IR$$
  - Electric potential across the battery goes up because the battery is adding electric potential energy to the circuit.
  - Electric potential across the inductor goes down because electric potential energy is being stored in the magnetic field of the inductor.
  - Electric potential across the resistor goes down because the resistor dissipates electric potential energy from the system.
  - We can now multiply this whole equation by the circuit current, I.



- $$\Rightarrow P = I\varepsilon = LI\frac{dI}{dt} + I^2R$$
  - We get the equation for power for each circuit element:
    - The rate at which energy is being added to the circuit by the battery.
    - The rate at which energy is being stored in the magnetic field of the inductor.
    - The rate at which energy is being dissipated by the resistor.
- We can now look specifically at the rate at which energy is being stored in the magnetic field of the inductor.
- $$\Rightarrow P = \frac{dU}{dt} = LI\frac{dI}{dt} \Rightarrow dU = LI\,(dI) \Rightarrow \int_0^{U_L} dU = \int_0^I LI\,(dI) = L\int_0^I I\,(dI)$$
  $$\Rightarrow U_L = \left[L\left(\frac{I^2}{2}\right)\right]_0^I \Rightarrow U_L = \frac{1}{2}LI^2$$
- We now have an equation for the energy stored in the magnetic field generated in an inductor as charges move through the inductor.
  - That energy is only present when current is passing through the inductor. This is because the magnetic field generated in the inductor is due to the charges moving through the inductor. If the charges are not moving, there is no magnetic field in the inductor.

A capacitor functions differently:

- The energy stored in a capacitor is stored in the electric field of the capacitor.
  - The energy stored in a capacitor can remain when a capacitor is disconnected from a circuit because charges can remain separated on the plates of the capacitor which would maintain the electric field between the plates of the capacitor.

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}Q\Delta V$$