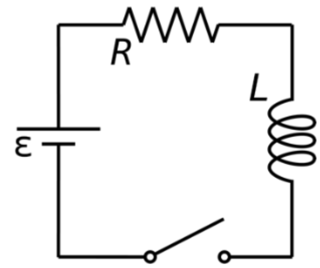
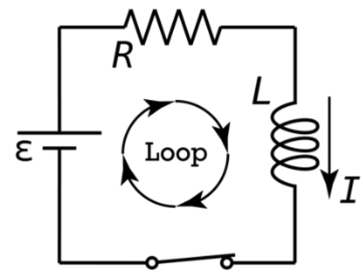
Previously we learned about these basics of an LR circuit:



LR Circuit Limits:

- At $t_{initial} = 0$; $I_{initial} = 0$ & $\left(\frac{dI}{dt}\right)_{initial} = \frac{\varepsilon}{L}$ [max value]
- At $t_{final} \approx \infty$; $I_f = \frac{\varepsilon}{R}$ [max value] & $\left(\frac{dI}{dt}\right)_{final} = 0$

Today we are going to derive the equations for current as a function of time and the time rate of change of current as a function of time. To do this we use with Kirchhoff's Loop Rule starting in the lower left-hand corner of the LR circuit.



$$\Delta V_{Loop} = 0 = \varepsilon - \Delta V_R - \Delta V_L = \varepsilon - IR - L\frac{dI}{dt}$$

$$\Rightarrow L\frac{dI}{dt} = \varepsilon - IR \Rightarrow \frac{L}{R}\frac{dI}{dt} = \frac{\varepsilon}{R} - I$$

$$\text{\& Let } u = \frac{\varepsilon}{R} - I \Rightarrow du = -dI \Rightarrow \frac{L}{R}\frac{-du}{dt} = u \Rightarrow \frac{du}{u} = -\frac{R}{L}dt$$

$$\Rightarrow \int \frac{du}{u} = \int -\frac{R}{L}dt \Rightarrow \int_{u_i}^{u_f} \frac{1}{u}du = -\frac{R}{L}\int_0^t dt \Rightarrow \ln u\Big|_{u_i}^{u_f} = -\frac{R}{L}t\Big|_0^t$$

$$\text{\& } \int \frac{dx}{x-a} = \ln|x-a| \Rightarrow \int \frac{du}{u} = \ln|u| = \ln u$$

- In this problem a = 0 and, because I varies from 0 to $\frac{\varepsilon}{R}$, u is always positive (or zero).

$$\Rightarrow \ln u_f - \ln u_i = \ln\left(\frac{u_f}{u_i}\right) = -\frac{R}{L}t \Rightarrow e^{\left(\ln\left(\frac{u_f}{u_i}\right)\right)} = e^{\left(-\frac{R}{L}t\right)} \Rightarrow \frac{u_f}{u_i} = e^{\left(-\frac{Rt}{L}\right)}$$

$$\Rightarrow u_f = u_i e^{\left(-\frac{Rt}{L}\right)} \text{ \& } u_f = \frac{\varepsilon}{R} - I_f \text{ \& } u_i = \frac{\varepsilon}{R} - I_i = \frac{\varepsilon}{R}$$

$$\Rightarrow \frac{\varepsilon}{R} - I_f = \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)} \Rightarrow -I_f = -\frac{\varepsilon}{R} + \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)} \Rightarrow I_f = \frac{\varepsilon}{R} - \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)}$$

$$\Rightarrow I(t) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{Rt}{L}\right)}\right) = I_{max}\left(1 - e^{\left(-\frac{Rt}{L}\right)}\right)$$

Note, this fits our limits because:

- $I(0) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{R(0)}{L}\right)}\right) = \frac{\varepsilon}{R}\left(1 - e^0\right) = \frac{\varepsilon}{R}(1-1) = 0$
- $I(\infty) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{R(\infty)}{L}\right)}\right) = \frac{\varepsilon}{R}\left(1 - e^{-\infty}\right) = \frac{\varepsilon}{R}(1-0) = \frac{\varepsilon}{R}$

We can also determine the time rate of change of current as a function of time:

$$I_f = \frac{\varepsilon}{R} - \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)} \Rightarrow \frac{dI}{dt} = \frac{d}{dt}\left(\frac{\varepsilon}{R} - \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)}\right) = \left(-\frac{\varepsilon}{R}\right)\frac{d}{dt}e^{\left(-\frac{Rt}{L}\right)} = -\left(\frac{\varepsilon}{R}\right)\left(\frac{R}{L}\right)e^{\left(-\frac{Rt}{L}\right)}$$

$$\Rightarrow \frac{dI}{dt}(t) = \frac{\varepsilon}{L}e^{\left(-\frac{Rt}{L}\right)} = \left(\frac{dI}{dt}\right)_{max} e^{\left(-\frac{Rt}{L}\right)} \;\&\; \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

Again, this fits our limits because:

$$\frac{dI}{dt}(0) = \frac{\varepsilon}{L}e^{\left(-\frac{R(0)}{L}\right)} = \frac{\varepsilon}{L}e^{0} = \frac{\varepsilon}{L} \;\&\; \frac{dI}{dt}(\infty) = \frac{\varepsilon}{L}e^{\left(-\frac{R(\infty)}{L}\right)} = \frac{\varepsilon}{L}e^{-\infty} = 0$$