Flipping Physics Lecture Notes:

Introduction to Work with Examples

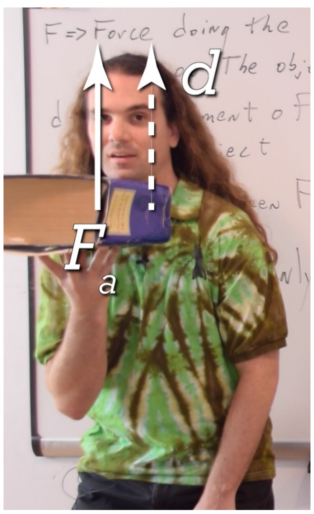
$W = Fd\cos\theta$

- W is Work
- F is the force doing the work on the object
- d is the displacement of the object
- θ is the angle between F and d
- When using this equation use the Magnitude of F and d, it is the cosine of the angle between F and D that determines if work is positive or negative.

Work is a scalar, which means it has magnitude only, it does not have direction.

Dimensions for Work: $W = Fd\cos\theta \Rightarrow N \cdot m = Joules, J$

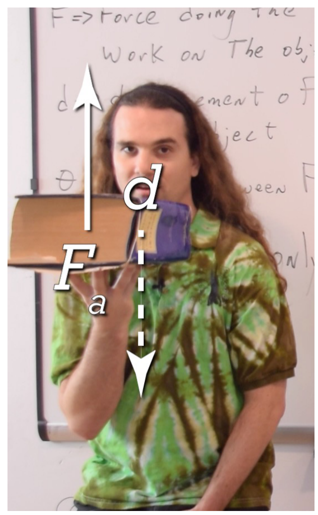
Example #1: Lifting the book up.



$$W_{F_a} = F_a d\cos\theta$$
$$\Rightarrow W_{F_a} = F_a d\cos(0)$$
$$\Rightarrow W_{F_a} = F_a d(1) > 0$$

I do positive work on the book via the force applied as I lift the book upward.

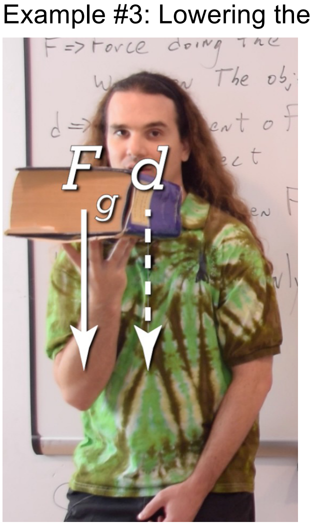
Example #2: Lowering the book down.



$$W_{F_a} = F_a d\cos\theta$$
$$\Rightarrow W_{F_a} = F_a d\cos(180^\circ)$$
$$\Rightarrow W_{F_a} = F_a d(-1) < 0$$

I do negative work on the book via the force applied as I lower the book down.

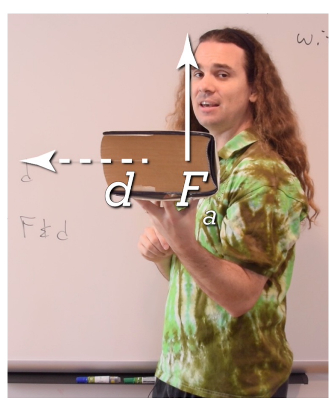
Example #3: Lowering the book down.



$$W_{F_g} = F_g d\cos\theta$$
$$\Rightarrow W_{F_g} = F_g d\cos(0)$$
$$\Rightarrow W_{F_g} = F_g d(1) > 0$$

The Earth does positive work on the book via the force of gravity as I lower the book down.

Example #4: Walking to the left at a constant velocity while holding the book.



$$W_{F_a} = F_a d\cos\theta$$
$$\Rightarrow W_{F_a} = F_a d\cos(90^\circ)$$
$$\Rightarrow W_{F_a} = F_a d(0) = 0$$

I do zero work on the book via the force applied as I walk to the left at a constant velocity while holding the book.

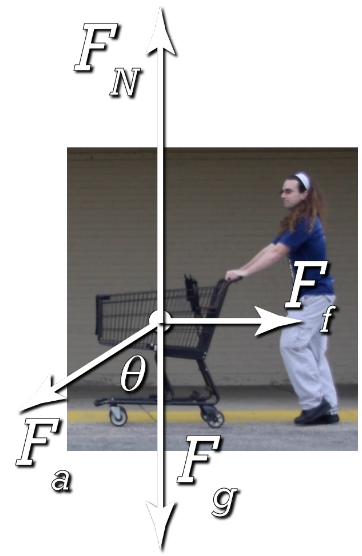
Flipping Physics Lecture Notes:

Introductory Work Problem

A shopper pushes a shopping cart with a force of 6.9 N at an angle of 59° to the left of the negative y-axis. While the cart moves a horizontal distance of 7.0 m, what is the work done by the shopper on the shopping cart?

$$F_a = 6.9N\,@\,59^\circ\ left\ of - y\ axis;\ \Delta x = -7.0m;\ W_{F_a} = ?$$

Draw the Free Body Diagram:



We do not need to break the Force Applied into its components, because the Work equation includes the direction of the force in it.

$$W_{F_a} = F_a d\cos\theta = (6.9)(7)\cos(90^\circ - 59^\circ) = 41.401\ N\cdot m \approx \boxed{41\ J}$$

- The angle in the work equation is the angle between the direction of the force doing the work and the displacement of the object. In this example, the angle is actually 90° minus the angle given in the problem.
- Always use the magnitude of the Force and Displacement in the work equation.

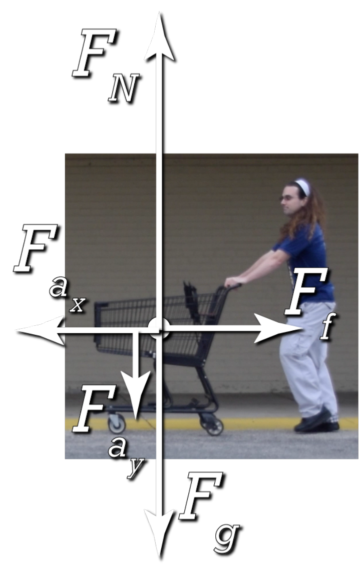
We can also find the work done by the Force Normal and the Force of Gravity:

$$W_{F_g} = F_g d\cos\theta = (mg)d\cos\theta = (mg)d\cos(90) = 0$$

$$W_{F_N} = F_N d\cos\theta = F_N d\cos(90) = 0$$

Because I felt it would make the video too long, the following was not in the video, however, why not include it here, eh?:

If we assume the cart is moving at a constant velocity, which is a pretty good approximation if you look at the video, we *can* determine the work done by the force of friction. We need the horizontal component of the Force Applied:



$$\sin\theta = \frac{O}{H} = \frac{F_{a_x}}{F_a} \Rightarrow F_{a_x} = F_a \sin\theta = 6.9\sin(59^\circ) = 5.91445N$$

Redraw the Free Body Diagram with components of the Force Applied.

$$\sum F_x = F_f - F_{ax} = ma_x = m(0) = 0 \Rightarrow F_f = F_{ax} = 5.91445N$$

$$W_{F_f} = F_f d\cos\theta = (5.91445)(7)\cos(180^\circ) = -41.401 = \boxed{-41\ J}$$

Note: The total work done on the shopping cart while it is moving at a constant velocity is zero. (This is called foreshadowing.)

Flipping Physics Lecture Notes:

Introduction to Kinetic Energy with Example Problem

Kinetic Energy, KE, is the energy associated with the motion of an object: $KE=\frac{1}{2}mv^2$

m = mass of the object and v = the velocity of the object.

$$KE=\frac{1}{2}mv^2 \Rightarrow \left(kg\right)\left(\frac{m}{s}\right)^2=\frac{kg\cdot m^2}{s^2}=\left(\frac{kg\cdot m}{s^2}\right)\left(m\right)=N\cdot m=J$$

Note: Kinetic Energy can*not* be negative. Mass can't be negative and even if velocity is negative, it is square and the square of a negative number is positive.

Example: $m_{prius}=1400\,kg;\ m_{bike}=86\,kg;\ v_{bike}=25\frac{mi}{hr};\ v_{prius}=?$

$$KE_{bike}=KE_{prius}\Rightarrow \frac{1}{2}m_{bike}\left(v_{bike}\right)^2=\frac{1}{2}m_{prius}\left(v_{prius}\right)^2\Rightarrow m_{bike}\left(v_{bike}\right)^2=m_{prius}\left(v_{prius}\right)^2$$

$$\Rightarrow\left(v_{prius}\right)^2=\frac{m_{bike}\left(v_{bike}\right)^2}{m_{prius}}\Rightarrow v_{prius}=v_{bike}\sqrt{\frac{m_{bike}}{m_{prius}}}=25\frac{mi}{hr}\sqrt{\frac{86\,kg}{1400\,kg}}=6.1962\approx\boxed{6\frac{mi}{hr}}$$

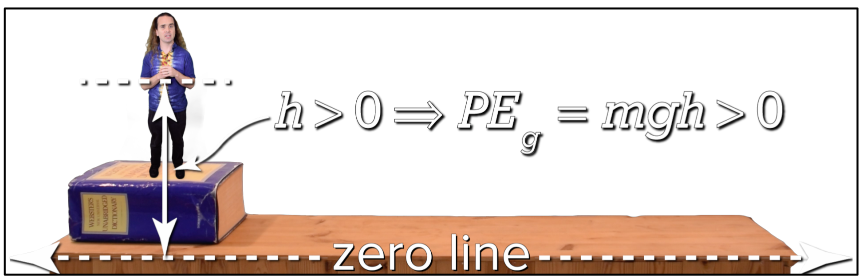
Flipping Physics Lecture Notes:
Introduction to Gravitational Potential Energy with Zero Line Examples

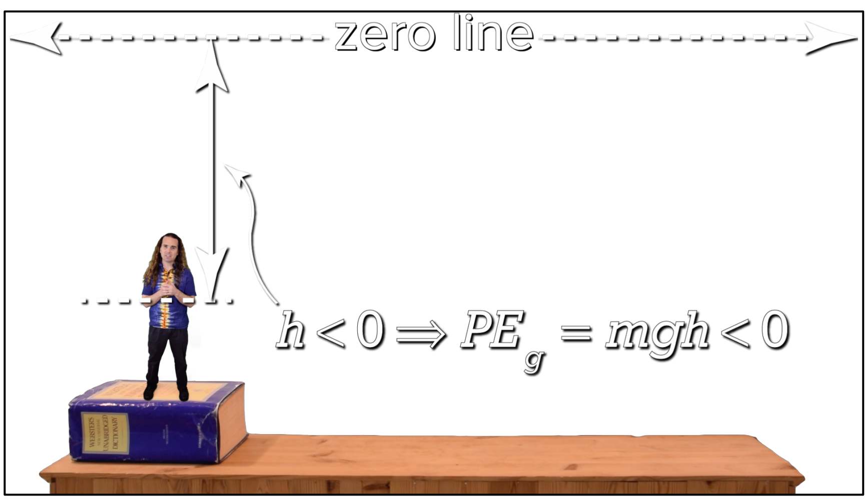
Gravitational Potential Energy is the energy stored in an object as the result of the elevation of that object.

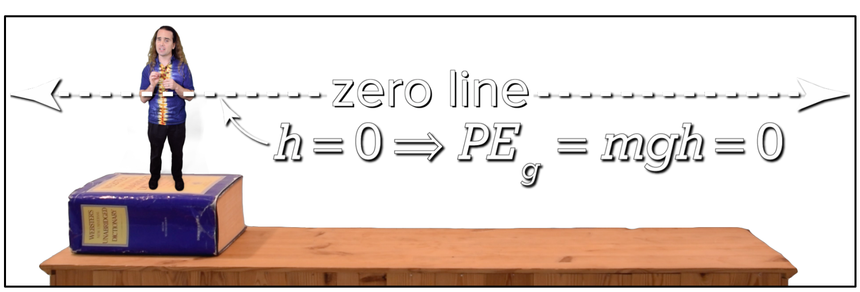
$PE_g = mgh$ (sometimes the symbol is U_g)

- m is the mass of the object.
- g is the acceleration due to gravity where $g_{Earth} = +9.81\frac{m}{s^2}$.
- h is the vertical height above the horizontal zero line.
    - The horizontal zero line is a reference line which you, the person solving the problem, get to decide the location of.
    - You, the person solving the problem, always have to identify the location of the horizontal zero line whenever you are working with gravitational potential energy, every time.
- $PE_g = mgh \Rightarrow kg \cdot \frac{m}{s^2} \cdot m = \left(kg \cdot \frac{m}{s^2}\right) \cdot m = N \cdot m = Joules,\ J$

Three Examples of Zero Line Locations:







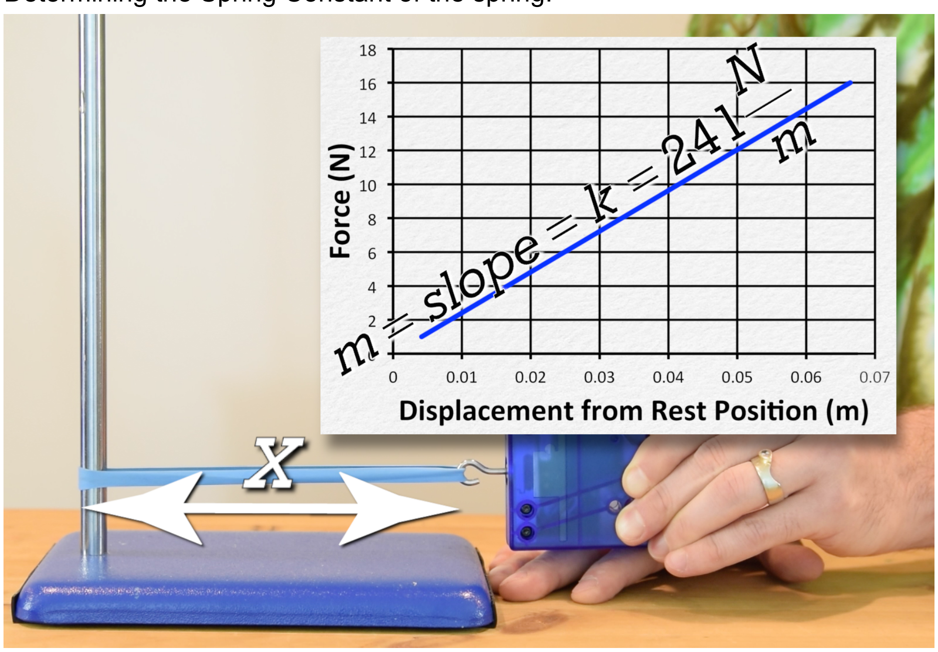
Flipping Physics Lecture Notes:

Introduction to Elastic Potential Energy with Examples

Elastic Potential Energy:

- The symbol is PE_e (or U_e).
- The energy stored in an object due to the temporary deformation of that object.
- $PE_e = \frac{1}{2}kx^2$
  - Spring Constant, k, usually in $\frac{N}{m}$, is how much force it takes to compress or expand the spring per meter.
  - x is displacement from equilibrium position (or rest position).
    - Equilibrium position (or rest position) is where the force of the spring equals zero.
- Because k can't be negative and x is squared, PE_e can never be negative.
- Like Kinetic Energy and Gravitational Potential Energy, PE_e is a scalar.
- The dimensions for Elastic Potential Energy:
  - $PE_e = \frac{1}{2}kx^2 \Rightarrow \left(\frac{N}{m}\right)m^2 = N \cdot m = joules,\ J$

Determining the Spring Constant of the spring:



Example: $k = 241\frac{N}{m}$ & $x = 0.12\,m$; $PE_e = ?$

$$PE_e = \frac{1}{2}kx^2 = \frac{1}{2}(241)(0.12)^2 = 1.7352 \approx \boxed{1.7\,J}$$

Flipping Physics Lecture Notes:

Introduction to Conservation of Mechanical Energy

When the 3.6 kg object is dropped from a height of 2.00 meters:

$$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.6)(0)^2 = 0$$

Set the horizontal zero line at the ground where the object lands …

$$PE_{gi} = mgh_i = (3.6)(9.81)(2) = 70.632 \approx 70.6\,J$$

Determining the velocity of the object after having fallen 1/3 of a meter:

$$v_{iy} = 0;\ a_y = -g = -9.81\frac{m}{s^2};\ \Delta y = -0.\overline{3}m;\ v_f = ?$$

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y = 0^2 + 2a_y\Delta y \Rightarrow v_{fy} = \sqrt{2a_y\Delta y} = \sqrt{(2)(-9.81)(-0.\overline{3})} = \pm 2.55734 = -2.55734\frac{m}{s}$$

$$a = \frac{\Delta v}{\Delta t} \Rightarrow \Delta t = \frac{\Delta v}{a} = \frac{v_f - v_i}{a} = \frac{-2.55734 - 0}{-9.81} = 0.26069s$$

And now determining Kinetic Energy and Gravitational Potential Energy of the object after having fallen 1/3 of a meter:

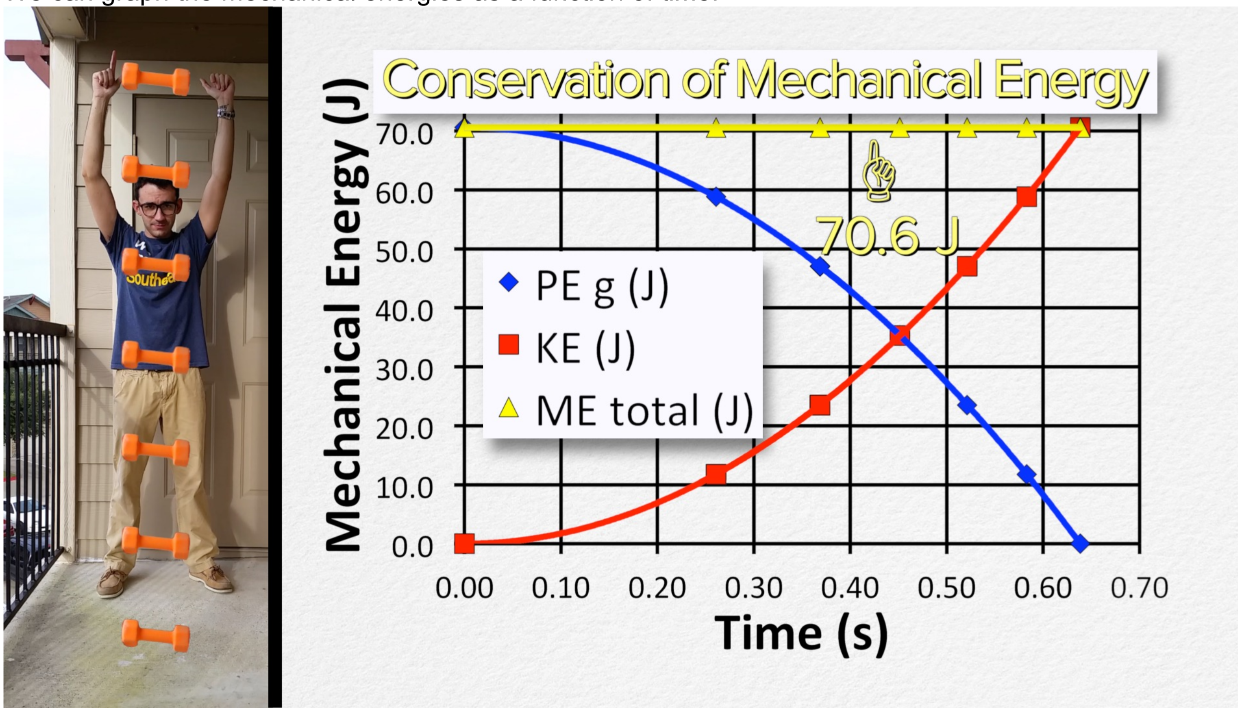
$$PE_h = mgh = (3.6)(9.81)(2 - 0.\overline{3}) = 58.860\,J$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(3.6)(-2.55734)^2 = 11.772\,J$$

And we can continue at each 1/3 of a meter interval to create this data table:

Height (m)	Time (s)	Velocity (m/s)	PE_g (J)	KE (J)	ME (J)
2.00	0	0	70.63	0	70.6
1.67	0.261	-2.56	58.86	11.77	70.6
1.33	0.369	-3.62	47.09	23.54	70.6
1.00	0.452	-4.43	35.32	35.32	70.6
0.67	0.521	-5.11	23.54	47.09	70.6
0.33	0.583	-5.72	11.77	58.86	70.6
0	0.639	-6.26	0	70.63	70.6

We can graph the mechanical energies as a function of time:



The total mechanical energy at every point adds up to 70.6 joules. This is because of Conservation of Mechanical Energy, the idea that energy is neither created nor destroyed; it simply changes forms. As the object falls it’s gravitational potential energy because kinetic energy.

The equation for Conservation of Mechanical Energy is $\boxed{ME_i = ME_f}$

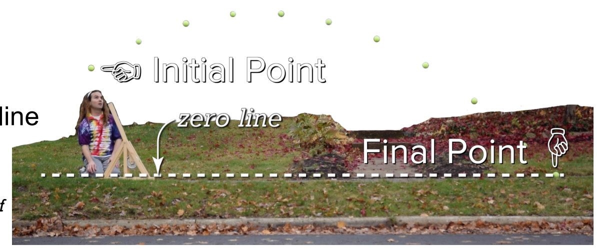
- True when $W_{friction} = 0$ & $W_{F_a} = 0$

Flipping Physics Lecture Notes:

Introductory Conservation of Mechanical Energy Problem

Example: A tennis ball with a mass of 58 grams is launched from a trebuchet with an initial speed of 6.8 m/s and an initial height of 1.3 meters. Assuming level ground, what is the final speed of the ball right before it strikes the ground?

$$m = 58g;\ v_i = 6.8\frac{m}{s};\ h_i = 1.3m;\ v_f = ?$$



Identify initial and final points and set the horizontal zero line

$$ME_i = ME_f \Rightarrow KE_i + PE_{gi} + PE_{ei} = KE_f + PE_{gf} + PE_{ef}$$

$$\Rightarrow \frac{1}{2}mv_i^{\,2} + mgh_i + \frac{1}{2}kx_i^{\,2} = \frac{1}{2}mv_f^{\,2} + mgh_f + \frac{1}{2}kx_f^{\,2}$$

$$\Rightarrow \frac{1}{2}mv_i^{\,2} + mgh_i + \cancel{\frac{1}{2}kx_i^{\,2}} = \frac{1}{2}mv_f^{\,2} + \cancel{mgh_f} + \cancel{\frac{1}{2}kx_f^{\,2}}$$

No Elastic Potential Energy at all because there is no spring and no Gravitational Potential Energy final because the vertical height final above the horizontal zero line is zero.

$$\Rightarrow \frac{1}{2}\cancel{m}v_i^{\,2} + \cancel{m}gh_i = \frac{1}{2}\cancel{m}v_f^{\,2} \Rightarrow \frac{1}{2}v_i^{\,2} + gh_i = \frac{1}{2}v_f^{\,2}$$

Everybody brought mass to the party!

$$\Rightarrow v_i^{\,2} + 2gh_i = v_f^{\,2} \Rightarrow v_f = \sqrt{v_i^{\,2} + 2gh_i} = \sqrt{6.8^2 + (2)(9.81)(1.3)} = 8.4703 \approx \boxed{8.5\frac{m}{s}}$$

Note: This is the final *speed* of the ball, not the final velocity. Mechanical Energy is a scalar and therefore we can only solve for the magnitude of the final velocity.

Also note: We couldn't solve this problem using projectile motion because we did not have an initial angle for the projectile.

In case you were curious, I actually used projectile motion equations to determine the initial speed for this problem. I measured the vertical and horizontal displacements and, if you count frames, there are 57 frames from launch until landing, at 60 frames per second:

$$\Delta t_t = 57\,frames \times \frac{1\sec}{60\,frames} = \frac{57}{60}s;\ \Delta x_t = 5.64m;\ \Delta y_t = -1.34m$$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 \Rightarrow -1.34 = v_{iy}\left(\frac{57}{60}\right) + \frac{1}{2}(-9.81)\left(\frac{57}{60}\right)^2 \Rightarrow v_{iy} = 3.24922\frac{m}{s}$$

$$v_x = \frac{\Delta x}{\Delta t} = \frac{5.64}{57/60} = 5.93684\frac{m}{s}\ \&\ a^2 + b^2 = c^2 \Rightarrow v_i^{\,2} = v_{iy}^{\,2} + v_{ix}^{\,2} \Rightarrow v_i = \sqrt{v_{iy}^{\,2} + v_{ix}^{\,2}}$$

$v_i = \sqrt{3.24922^2 + 5.93684^2} = 6.767832 \approx 6.8\frac{m}{s}$ is the magnitude of the initial velocity.

Yes, we could use $\tan\theta = \frac{O}{H} = \frac{v_{iy}}{v_{ix}}$ to find the initial launch angle.

Flipping Physics Lecture Notes:

Conservation of Energy Problem with Friction, an Incline and a Spring

by Billy

Example: A block with a mass of 11 grams is used to compress a spring a distance of 3.2 cm. The spring constant of the spring is 14 N/m. After the block is released, it slides along a level, frictionless surface until it comes to the bottom of a 25° incline. If μ_k between the block and the incline is 0.30, to what maximum height does the block slide?

Givens: $k = 14\dfrac{N}{m};\ \theta = 25°;\ m = 11g;\ x_i = 3.2cm;\ \mu_k = 0.30;\ h_{\max} = ?$

Convert knowns to base SI units:

$$m = 11g \times \frac{1kg}{1000g} = 0.011kg \ \ \&\ \ x_i = 3.2cm \times \frac{1m}{100cm} = 0.032m$$

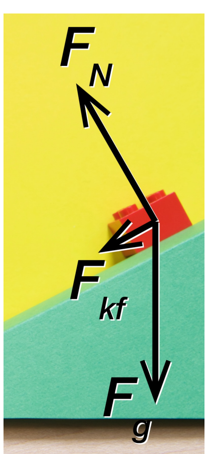
On the level surface, there is no work done by friction or the force applied; therefore we can use Conservation of Mechanical Energy. Set the initial point where the block is completely compressing the spring, the final point at the base of the incline and the zero line at the center of mass of the block while it is on the incline.

$$ME_i = ME_f \Rightarrow \frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

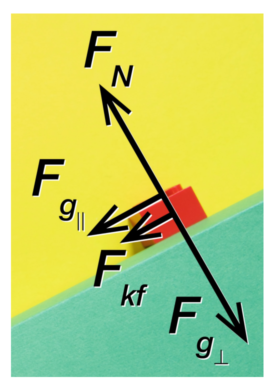
$$\Rightarrow \underbrace{\cancel{\frac{1}{2}mv_i^2}}_{v_i = 0} + \underbrace{\cancel{mgh_i}}_{h_i = 0} + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \underbrace{\cancel{mgh_f}}_{h_f = 0} + \underbrace{\cancel{\frac{1}{2}kx_f^2}}_{\text{not on spring}} \Rightarrow \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 \Rightarrow kx_i^2 = mv_f^2$$

$$\Rightarrow v_f^2 = \frac{kx_i^2}{m} \Rightarrow v_f = \sqrt{\frac{kx_i^2}{m}} = \sqrt{\frac{(14)(0.032)^2}{0.011}} = 1.14161\frac{m}{s} = v_{i_{\parallel}}$$

This the final velocity at the end of the level surface which is also the initial velocity on the incline.



On the incline, we can not use Conservation of Mechanical Energy because there is work done by friction. We need to draw a free body diagram, break the force of gravity into its parallel and perpendicular components, redraw the free body diagram, sum the forces and use the uniformly accelerated motion equations.



$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg\cos\theta$$

$$\sum F_{\parallel} = -F_{g_{\parallel}} - F_{kf} = ma_{\parallel} \Rightarrow -mg\sin\theta - \mu_k F_N = -mg\sin\theta - \mu_k mg\cos\theta = ma_{\parallel}$$

Everybody brought mass to the party!!

$$\Rightarrow a_{\parallel} = -g\sin\theta - \mu_k g\cos\theta = -(9.81)\sin(25) - (0.3)(9.81)\cos(25) = -6.81315\frac{m}{s^2}$$

Now we can use a uniformly accelerated motion equation:

$$v_{f_{\parallel}}{}^2 = v_{i_{\parallel}}{}^2 + 2a_{\parallel}\Delta d_{\parallel} \Rightarrow 0 = v_{i_{\parallel}}{}^2 + 2a_{\parallel}\Delta d_{\parallel} \Rightarrow -v_{i_{\parallel}}{}^2 = 2a_{\parallel}\Delta d_{\parallel} \Rightarrow \Delta d_{\parallel} = \frac{-v_{i_{\parallel}}{}^2}{2a_{\parallel}}$$

$$\Rightarrow \Delta d_{\parallel} = \frac{-(1.14161)^2}{(2)(-6.81315)} = 0.095644m$$

$$\sin\theta = \frac{O}{H} = \frac{h_{\max}}{\Delta d_{\parallel}} \Rightarrow h_{\max} = \Delta d_{\parallel}\sin\theta = (0.095644)\sin(25) = 0.040421m \times \frac{100cm}{1m} \approx \boxed{4.0cm}$$

Flipping Physics Lecture Notes:

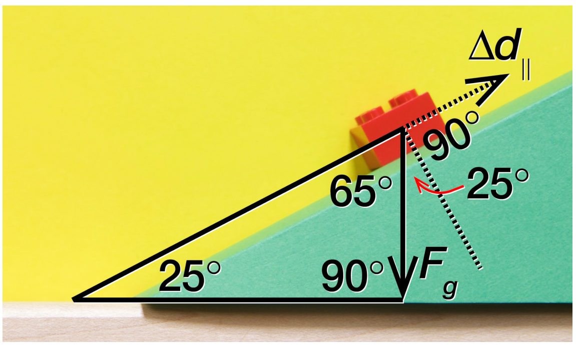
Work due to the Force of Gravity on an Incline
by Billy

This is a continuation of a previous video done by Billy. Please view that video before attempting this one. http://www.flippingphysics.com/coe-incline-problem.html

Find the work done by the force of gravity.

On the level surface: $W_{F_g} = F_g d\cos\theta = (mg)d\cos\theta = mgd\cos(90) = 0$

On the incline:



$$W_{F_g} = F_g d\cos\theta = (mg)\Delta d_{\parallel}\cos(90+25) = (0.011)(9.81)(0.095644)\cos(115)$$

$$\Rightarrow W_{F_g} = -0.0043618J \times \frac{1000mJ}{J} \approx \boxed{-4.4mJ}$$

Flipping Physics Lecture Notes:

Introduction to Mechanical Energy with Friction

Recall that Conservation of Mechanical Energy or $ME_i = ME_f$ is true when $W_{friction} = 0$ and $W_{F_a} = 0$

If $W_{friction} \neq 0$ and $W_{F_a} = 0$ then we use the equation $W_f = \Delta ME$

We can expand the equation: $W_f = \Delta ME \Rightarrow F_f d\cos\theta = ME_f - ME_i$

Note: Because the direction of the force of friction and the displacement of the object are always♥ opposite to one another, the angle in the work equation is 180°.

If $W_{friction} = 0$ then $W_f = \Delta ME \Rightarrow F_f d\cos\theta = ME_f - ME_i \Rightarrow 0 = ME_f - ME_i \Rightarrow ME_i = ME_f$

Conservation of Mechanical Energy is a special case of $W_f = \Delta ME$ where $W_{friction} = 0$

Therefore, remember to always identify the initial and final points and the location of the horizontal zero line whenever using $W_f = \Delta ME$

♥ This is not actually *always* true, however, at the beginning of learning physics it is. I showed one example in the video when it is not true.

Flipping Physics Lecture Notes:

Introductory
Work due to Friction equals Change in Mechanical Energy
Problem

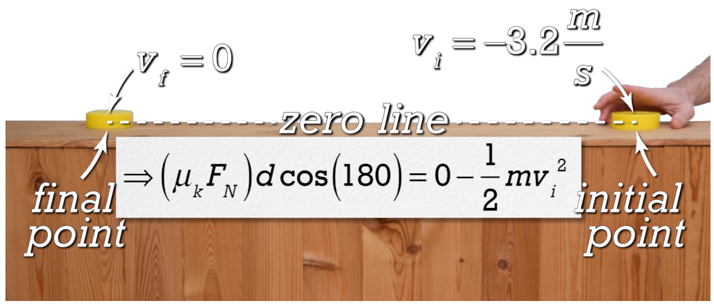
Problem: On a level surface, a street hockey puck is given an initial velocity of -3.2 m/s and slides to a stop. If the coefficient of kinetic friction between the puck and the surfaces is 0.60, how far did the puck slide?

Known values: $v_i = -3.2\frac{m}{s};\ v_f = 0;\ \mu_k = 0.60;\ \Delta x = ?$ (magnitude)

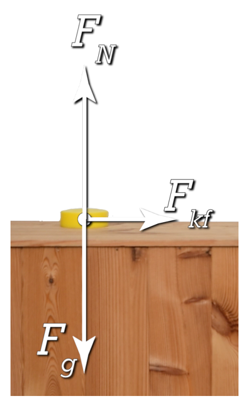
$$W_f = \Delta ME \Rightarrow F_{kf} d\cos\theta = ME_f - ME_i \Rightarrow \left(\mu_k F_N\right) d\cos\left(180\right) = 0 - \frac{1}{2}mv_i^2$$

The equation for force of kinetic friction is: $F_{kf} = \mu_k F_N$

The angle between the force of kinetic friction (to the right) and the displacement (to the left) is 180°. Set the initial point where the puck is released, the final point where it stops and the horizontal zero line at the height of the center of mass of the puck. The height initial and final are both zero, so the gravitational potential energy initial and final are both zero. There is no spring so the initial and final elastic potential energies are zero. The final velocity of the puck is zero, so the kinetic energy final is zero. The on the mechanical energy initial or final is the initial kinetic energy. Where did all the kinetic energy go? It is all converted to heat and sound as the puck slides.



We need the force normal, so we draw a free body diagram:



$$\sum F_y = F_N - F_g = ma_y = m\left(0\right) = 0 \Rightarrow F_N = F_g = mg$$

(and now back to the W_f equation)

$$\Rightarrow \mu_k\left(mg\right)d\left(-1\right) = -\frac{1}{2}mv_i^2 \Rightarrow \mu_k g d = \frac{1}{2}v_i^2$$

(everybody brought *negative* mass to the party!!)

$$\Rightarrow d = \frac{v_i^2}{2g\mu_k} = \frac{\left(-3.2\right)^2}{\left(2\right)\left(9.81\right)\left(0.60\right)} = 0.869861 \approx \boxed{0.87m}$$

Notice we get a positive answer for the distance the puck slid. That is because the work equation has the *magnitude* of the force and the displacement in it, therefore, when we solve for d, we only get the *magnitude* of the displacement of the object.

How I solved for the known variables in this problem:

From the video: $x_f = 32.2cm;\ x_i = 120.8cm;\ v_f = 0;\ \Delta t = 33 frames \times \frac{1\sec}{60 frames} = \frac{33}{60}\sec$

$$\Delta x = x_f - x_i = 32.2 - 120.8 = -88.6cm \times \frac{1m}{100cm} = -0.886m$$

Now we can use a uniformly accelerated motion equation:

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t \Rightarrow \Delta x = \frac{1}{2}\left(v_i + 0\right)\Delta t \Rightarrow 2\Delta x = v_i \Delta t \Rightarrow v_i = \frac{2\Delta x}{\Delta t} = \frac{(2)(-0.886)}{\frac{33}{60}} = -3.221818\frac{m}{s}$$

In the problem we solved the work equation to get:

$$\mu_k g d = \frac{1}{2}v_i^2 \Rightarrow \mu_k = \frac{v_i^2}{2gd} = \frac{(-3.221818)^2}{(2)(9.81)(0.886)} = 0.597131$$

"Why did you use +0.886 meters and not -0.886 meters?" you ask. Remember (again) in the work equation, you use the magnitudes of the force and the displacement and "d" in the above equation came from the work equation.

Another note: The answer in the video is a distance of 0.87 meters and the measurement in the video was 0.89 meters; those two numbers are not the same. This is purely an issue of rounding. I felt my measured values were only good to two significant digits, which is why they are rounded to:

$v_i = -3.2\frac{m}{s}$ & $\mu_k = 0.60$. However, if you use the more precise values found when I originally solved for the variables: $v_i = -3.221818\frac{m}{s}$ & $\mu_k = 0.597131$, you get …

$$d = \frac{v_i^2}{2g\mu_k} = \frac{(-3.221818)^2}{(2)(9.81)(0.597131)} = 0.88600 \approx 0.886m$$

Which was what was measured in the video in the first place.

Flipping Physics Lecture Notes:

Work due to Friction equals Change in Mechanical Energy Problem
by Billy

This is an alternate solution to a problem previously done by Billy.
http://www.flippingphysics.com/coe-incline-problem.html

Example: A block with a mass of 11 grams is used to compress a spring a distance of 3.2 cm. The spring constant of the spring is 14 N/m. After the block is released, it slides along a level, frictionless surface until it comes to the bottom of a 25° incline. If μk between the block and the incline is 0.30, to what maximum height does the block slide?

Last time we solved this problem using Conservation of Energy, Newton's Second Law and Uniformly Accelerated Motion. Mr.P asked me to show you how to solve the problem using the Work due to Friction equation.

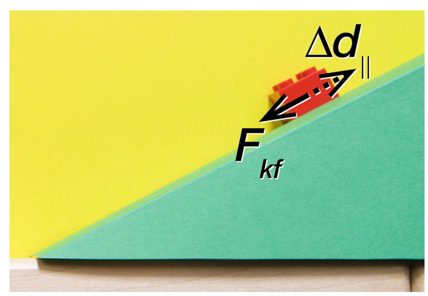
We have the same knowns as last time:

$$k = 14\frac{N}{m};\ \theta = 25°;\ m = 11g;\ x_i = 3.2cm;\ \mu_k = 0.30;\ h_{\max} = ?$$

$$m = 11g \times \frac{1kg}{1000g} = 0.011kg \ \ \&\ \ x_i = 3.2cm \times \frac{1m}{100cm} = 0.032m$$

$$W_f = \Delta ME \Rightarrow F_{kf} d\cos\theta = ME_f - ME_i \Rightarrow \mu_k F_N \Delta d_{\parallel} \cos(180) = mgh_f - \frac{1}{2}kx_i^2$$

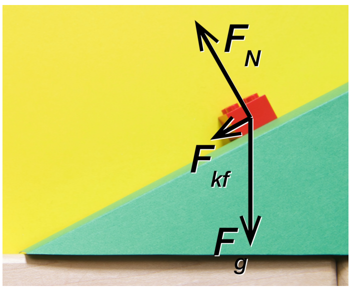
There is no friction on the level surface, so the only place there is Work due to Friction is on the incline. The angle between the Force of Kinetic Friction (down the incline) and the displacement of the block (up the incline) is 180 degrees.



Set the initial point where spring is compressed its maximum distance and the final point at the maximum height of the block. Set the horizontal zero line at the center of mass of the block when it is on the horizontal surface.

The initial velocity of the block is zero, so there is no initial kinetic energy. The initial height of the block is zero, so there is no initial gravitational potential energy. There is an initial compression of the spring, so there *is* kinetic energy initial.

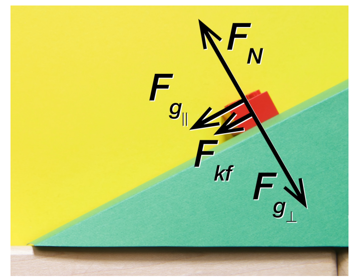
The final height of the block is not zero, so there *is* gravitational potential energy final. The final velocity of the block is zero, so there is no kinetic energy final. The block is not compressing the spring at the end, so there is no elastic potential energy final.



We need to find the force normal, so draw a free body diagram, break the force of gravity into its components, re-draw the free body diagram and sum the forces in the perpendicular direction.

$$\sum F_{\perp} = F_N - F_{g_\perp} = ma_{\perp} = m(0) = 0$$

$$\Rightarrow F_N = F_{g_\perp} = mg\cos\theta$$



Going back to the Work due to Friction equation: $\Rightarrow \mu_k\left(mg\cos\theta\right)\Delta d_{\parallel}\left(-1\right) = mgh_f - \frac{1}{2}kx_i^2$

Draw a triangle to find the $\Delta d_{\parallel}$ in terms of h_f: $\sin\theta = \frac{O}{H} = \frac{h_f}{\Delta d_{\parallel}} \Rightarrow \Delta d_{\parallel} = \frac{h_f}{\sin\theta}$

Substitute that back into the Work due to Friction equation:

$\Rightarrow -\mu_k mg\cos\theta\left(\frac{h_f}{\sin\theta}\right) = mgh_f - \frac{1}{2}kx_i^2$ and solve for h_f:

$\Rightarrow -\frac{\mu_k mg}{\tan\theta}\left(h_f\right) = mgh_f - \frac{1}{2}kx_i^2$ (because $\frac{\sin\theta}{\cos\theta} = \tan\theta$)

$$\Rightarrow \frac{1}{2}kx_i^2 = mgh_f + \frac{\mu_k mgh_f}{\tan\theta} = mgh_f\left(1 + \frac{\mu_k}{\tan\theta}\right) \Rightarrow h_f = \frac{\frac{1}{2}kx_i^2}{mg\left(1 + \frac{\mu_k}{\tan\theta}\right)}$$

$$\Rightarrow h_f = \frac{\frac{1}{2}(14)(0.032)^2}{(0.011)(9.81)\left(1 + \frac{(0.3)}{\tan(25)}\right)} = \frac{0.007168}{(0.011)(9.81)(1.643352)} = 0.040421 \approx \boxed{0.040m = h_{\text{max}}}$$

Flipping Physics Lecture Notes:

Deriving the Work-Energy Theorem using Calculus

Yes, this video uses calculus, which is not a part of an algebra based course, however, sometimes it is useful to do math which is above your pay grade, just to see what it looks like.

The definition of work using an integral is: $W = \int_{x_i}^{x_f} F\,dx$

Which is read, Work equals the integral from position initial position to final position of force with respect to position. FYI: because x means position, dx means "with respect to position". For our derivation we will be working with the *net* work and the *net* force. So the equation is: $W_{net} = \int_{x_i}^{x_f} F_{net}\,dx$ and we know,

according to Newton's Second Law, that $\sum \vec{F} = m\vec{a}$ therefore $W_{net} = \int_{x_i}^{x_f} \left(ma\right)dx$

(Let's drop the vector symbol over the acceleration because everything here will be in the x direction.)

Mass is a scalar so it can be removed from the integral: $W_{net} = m\int_{x_i}^{x_f} \left(a\right)dx$

We can't take this integral yet because acceleration is not with respect to position.

The equation for average acceleration is $a_{average} = \frac{\Delta v}{\Delta t}$, however, we are interested in the calculus version of instantaneous acceleration which is the derivative of velocity with respect to time or

$a_{instantaneous} = \frac{dv}{dt}$, which we can substitute in. $\Rightarrow W_{net} = m\int_{x_i}^{x_f} \left(\frac{dv}{dt}\right)dx$ However, we still can't integrate this with respect to position.

It turns out that $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$ therefore $\Rightarrow W_{net} = m\int_{x_i}^{x_f} \left(\frac{dv}{dx}\frac{dx}{dt}\right)dx$

And we can do some canceling out and rearranging: $\Rightarrow W_{net} = m\int_{x_i}^{x_f} \left(\frac{dv}{\cancel{dx}}\frac{dx}{dt}\right)\cancel{dx} = m\int_{v_i}^{v_f} \left(\frac{dx}{dt}\right)dv$

When we changed what we were taking the integral with also needed to change the initial and final limits. That is why we are now taking the integral from the initial velocity to the final velocity.

The equation for average velocity is $v_{average} = \frac{\Delta x}{\Delta t}$, however, we need the calculus version of instantaneous velocity which is the derivative of position with respect to time is velocity:

$v_{instantaneous} = \frac{dx}{dt}$

Therefore: $\Rightarrow W_{net} = m\int_{v_i}^{v_f}(v)dv$ and the integral of this equation is: $\Rightarrow W_{net} = m\left[\frac{v^2}{2}\right]_{v_i}^{v_f}$

Read: The net work equals the mass of the object times the velocity of the object squared divided by two from velocity initial to velocity final. Which works out to be: $\Rightarrow W_{net} = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}$

This is where the definition of Kinetic Energy comes from: Kinetic Energy is defined as $\frac{1}{2}mv$

Therefore the net work equation is: $\Rightarrow W_{net} = KE_f - KE_i = \Delta KE$

Notice how, unlike $ME_i = ME_f$ or $W_f = \Delta ME$, we didn't need to specify anything about work done by the force of friction or the force applied. Therefore, $W_{net} = \Delta KE$ is *always* true. And unlike some other alwayses♥, this always is always true. $W_{net} = \Delta KE$ is *always* true.

An aside: Now that we have these two equations $W_f = \Delta ME$ and $W_{net} = \Delta KE$, students often confuse the two of them. Students mistakenly interchange the ΔME and the ΔKE. Don't let this be you!!

♥ Find me a plural of always, I dare you!

Flipping Physics Lecture Notes:

Work-Energy Theorem Problem
by Billy

This video is a second alternate solution to a problem previously done by Billy.
http://www.flippingphysics.com/coe-incline-problem.html

Here is the first alternate solution:
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Example: A block with a mass of 11 grams is used to compress a spring a distance of 3.2 cm. The spring constant of the spring is 14 N/m. After the block is released, it slides along a level, frictionless surface until it comes to the bottom of a 25° incline. If μk between the block and the incline is 0.30, to what maximum height does the block slide?

We have solved this problem using 1) Conservation of Energy, Newton's Second Law and Uniformly Accelerated Motion and 2) $W_f = \Delta ME$. Now we are going to solve the problem using the Work-Energy Theorem or what Mr. P prefers to call the Net Work-Kinetic Energy Theorem, because, evidently we often confuse $W_{net} = \Delta KE$ with $W_f = \Delta ME$.

We have the same knowns as the last two times we solved this problem:

$$k = 14\frac{N}{m};\ \theta = 25°;\ m = 11g;\ x_i = 3.2cm;\ \mu_k = 0.30;\ h_{max} = ?$$

$$m = 11g \times \frac{1kg}{1000g} = 0.011kg \ \ \&\ \ x_i = 3.2cm \times \frac{1m}{100cm} = 0.032m$$

And we now begin with $W_{net} = \Delta KE = KE_f - KE_i$ and we need to identify the locations of the initial and final points. Set the initial point where spring is compressed its maximum distance and the final point at the maximum height of the block. **PICTURE** Because the velocity initial and final are both zero, there is no Kinetic Energy initial or final, in other words:

$W_{net} = KE_f - KE_i = 0 - 0 = 0 \Rightarrow W_{net} = 0$ The net work done on the block is zero.

The equation for work is: $W = Fd\cos\theta$

Because the direction the block is moving is different on the horizontal, frictionless surface than on the incline, let's separate the net work into those two parts:

Horizontal Surface:
We need to draw the **free body diagram** for the forces acting on the block when it is on the horizontal surface. The angle between the direction the block is moving (to the right) and the Force Normal (up) is 90°. The cosine of 90° is zero, so the work done by the Force normal is zero. The same is true for the Force of Gravity. $W_{F_N} = F_N d\cos(90) = 0 = F_g d\cos(90) = W_{F_g}$

Because the force caused by the spring on the block is not constant, we need to find the work done by the spring on the block a little bit differently. As the block is pushed by the spring, the spring loses Elastic Potential Energy, that loss in Elastic Potential Energy equals the Work done by the Force of the Spring on the block. In equation form that is:

$$W_{F_s} = -\Delta PE_e = -\left(PE_{e_f} - PE_{e_i}\right) = -\left(0 - PE_{e_i}\right) = PE_{e_i} = \frac{1}{2}kx_i^2$$

Because the block is not on the spring finally, the final Elastic Potential Energy of the spring is zero. Therefore the Work done by the Spring on the block equals the Elastic Potential Energy initial of the spring.

Therefore the Net Work done on the block on the horizontal surface is: $W_{F_s} = \frac{1}{2}kx_i^2$

Incline:
Again, we need the **free body diagram** on the incline. Again the work done by the Force Normal is zero because the Force Normal is perpendicular to the displacement of the block.

$$W_{F_N} = F_N \Delta d_{\parallel} \cos(90) = 0$$

The work done by the Force of Gravity is: $W_{F_g} = Fd\cos\theta = F_g \Delta d_{\parallel} \cos\theta_1$

The equation for the Force of Gravity is: $F_g = mg$

The displacement of the block on the incline is the displacement in the parallel direction.

There are going to be several angles in this problem, so I have labeled this angle θ_1. This is the angle between the displacement of the block (up the incline) and the Force of Gravity (down), which is 90° + 25° or 115°. 1 minute and 29 seconds in to this previous video, I showed finding this angle in detail: http://www.flippingphysics.com/work-billy.html

We now have: $W_{F_g} = mg\Delta d_{\parallel}\cos\theta_1$ with $\theta_1 = 115°$

We need $\Delta d_{\parallel}$ in terms of h_{max}, so we draw a **triangle** and $\sin\theta_2 = \frac{O}{H} = \frac{h_{max}}{\Delta d_{\parallel}} \Rightarrow \Delta d_{\parallel} = \frac{h_{max}}{\sin\theta_2}$

Therefore: $W_{F_g} = mg\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_1 = \frac{mgh_{max}\cos\theta_1}{\sin\theta_2}$ where $\theta_2 = 25°$ (the incline angle)

The work done by the Force of Kinetic Friction is:

$$W_{F_{kf}} = Fd\cos\theta = F_{kf}\Delta d_{\parallel}\cos\theta_3 = \mu_k F_N\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_3$$

Now we need the work done by the Force of Kinetic Friction.

Remember the equation for the Force of Kinetic Friction is $F_{kf} = \mu_k F_N$

θ_3 is the angle between the direction of the displacement of the block (up the incline) and the force of kinetic friction (down the incline) which is 180°. $\theta_3 = 180°$

We need Force Normal so we need to break the force of gravity into its components, redraw the **free body diagram** and sum the forces in the perpendicular direction:

$$\sum F_\perp = F_N - F_{g_\perp} = ma_\perp = m(0) = 0 \Rightarrow F_N = F_g = mg\cos\theta_2$$

Which we can substitute back into the equation for the Work done by Friction:

$$W_{F_{kf}} = \mu_k\left(mg\cos\theta_2\right)\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_3 = \frac{\mu_k mgh_{max}\cos\theta_3}{\tan\theta_2}$$ (because $\dfrac{\cos\theta_2}{\sin\theta_2} = \dfrac{1}{\tan\theta_2}$)

Going back to the original Net Work-Kinetic Energy Theorem:

$$W_{net} = \Delta KE = 0 = W_{F_s} + W_{F_g} + W_{f_{kf}}$$

Now we can substitute in equations and numbers and solve for h_{max}:

$$\Rightarrow W_{net} = 0 = \frac{1}{2}kx_i^2 + \frac{mgh_{max}\cos\theta_1}{\sin\theta_2} + \frac{\mu_k mgh_{max}\cos\theta_3}{\tan\theta_2}$$

$$\Rightarrow 0 = \frac{1}{2}(14)(0.032)^2 + \frac{(0.011)(9.81)h_{max}\cos(115)}{\sin(25)} + \frac{(0.30)(0.011)(9.81)h_{max}\cos(180)}{\tan(25)}$$

$$\Rightarrow 0 = 0.007168 + (-0.10791)h_{max} + (-0.069424)h_{max} \Rightarrow 0.177334h_{max} = 0.007168$$

$$h_{max} = 0.040421 \approx \boxed{0.040m}$$

(yes, this is the same answer we got the previous two times we solved this problem.)

It is important to note that:
The net work put into the system by the Force Applied is the same as the work done by the Force of the Spring which is positive because the spring converts Elastic Potential Energy to Kinetic Energy. $\sum W_{in} = PE_e = W_{F_s} = \frac{1}{2}kx_i^2 = \frac{1}{2}(14)(0.032)^2 = 0.007168J$

The work done by the Force of Kinetic Friction is negative because it converts Kinetic Energy into heat and sound energy.

$$W_{F_{kf}} = (-0.069424)h_{max} = (-0.069424)(0.040421) = -0.0028062J$$

The work done by the Force of Gravity is negative because it converts the Kinetic Energy into Gravitational Potential Energy.

$$W_{F_g} = (-0.10791)h_{\max} = (-0.10791)(0.040421) = -0.0043618J$$

Because: $W_{F_g} = \dfrac{mgh_{\max}\cos\theta_1}{\sin\theta_2} = mgh_{\max}\left(\dfrac{\cos(115)}{\sin(25)}\right) = mgh_{\max}(-1) = -mgh_{\max}$

Which means $\dfrac{W_{F_{kf}}}{\sum W_{in}} \times 100 = \dfrac{-0.0028062}{0.007168} \times 100 = -39.149 \approx -39\%$ of the total energy put into the system by the force applied was dissipated as heat and sound energy.

And $\dfrac{W_{F_g}}{\sum W_{in}} \times 100 = \dfrac{-0.0043618}{0.007168} \times 100 = -0.60851 \approx -61\%$ of the total energy put into the system by the force applied was converted to Gravitational Potential Energy.

Flipping Physics Lecture Notes:

Finding the Force on a Ball from a Dent
http://www.flippingphysics.com/force-ball-dent.html

A 67 N ball is dropped from a height of 79.8 cm above a bag of sand. If the ball makes a 9.0 mm deep dent in the sand, what is the average force the sand applies on the ball during the collision?

$$F_g = 67N;\ h_i = 79.8cm\left(\frac{1m}{100cm}\right) = 0.798m;\ dent = 9.0mm\left(\frac{1m}{1000mm}\right) = 0.009m;\ F_a = ?\left(average\right)$$

We could use free fall to determine the final velocity of the ball right before it strikes the sand, which is the same as the initial velocity of the collision. (Alternatively, we could use conservation of mechanical energy to find this velocity.) Then we could use uniformly accelerated motion equations to determine the acceleration of the ball during the collision because we know the initial velocity during the collision, the final velocity of the ball during the collision is zero, and we know the displacement during the collision is 9.0 mm down. Then we could draw a free body diagram and sum the forces in the y-direction to determine the average force on the ball during the collision. Or ….

We could remember the Work Energy Theorem.
(What I like to call the Net Work equals Change in Kinetic Energy Theorem.)

Setting the initial point at the top where the ball is dropped and the final point at the bottom where the ball momentarily stops before moving back upward, we get:

$$W_{net} = \Delta KE = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - 0 = 0$$

$$W_{net} = W_{F_a} + W_{F_g} = 0 \Rightarrow W_{F_a} = -W_{F_g} \Rightarrow F_a \Delta r_{F_a} \cos\theta_{F_a} = -F_g \Delta r_{F_g} \cos\theta_{F_g}$$

$$\Rightarrow F_a\left(0.009\right)\cos\left(180\right) = -\left(67\right)\left(0.798 + 0.009\right)\cos\left(0\right) \Rightarrow F_a = 6007.\overline{6} \approx 6.0\times10^3 N$$

$$\Rightarrow F_a = 6007.\overline{6}N\left(\frac{1lb}{4.448N}\right) = 1350.64 \approx 1400lb$$

I would argue that remembering the Net Work equals Change in Kinetic Energy Theorem makes the solution much simpler.

But why is the average force applied by the sand on the ball about 90 times greater than the force of gravity?

$$\frac{F_a}{F_g} = \frac{6007.\overline{6}}{67} = 89.\overline{6} \approx 9.0\times10^1$$

Because the force applied acts on the ball for a distance which is about 90 times smaller than the distance during which the force of gravity acts on the ball.

$$F_a \Delta r_{F_a} \cos\theta_{F_a} = -F_g \Delta r_{F_g} \cos\theta_{F_g} \Rightarrow F_a \Delta r_{F_a} \cos\left(180\right) = -F_g \Delta r_{F_g} \cos\left(0\right)$$

$$\Rightarrow -F_a \Delta r_{F_a} = -F_g \Delta r_{F_g} \Rightarrow \frac{F_a}{F_g} = \frac{\Delta r_{F_g}}{\Delta r_{F_a}} = \frac{0.798 + 0.009}{0.009} = 89.\overline{6} \approx 9.0\times10^1$$

Flipping Physics Lecture Notes:

Work-Energy Theorem Problem
by Billy

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We have the same knowns as the last two times we solved this problem:

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And we now begin with $W_{net} = \Delta KE = KE_f - KE_i$ and we need to identify the locations of the initial and final points. Set the initial point where spring is compressed its maximum distance and the final point at the maximum height of the block. **PICTURE** Because the velocity initial and final are both zero, there is no Kinetic Energy initial or final, in other words:

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Therefore the Net Work done on the block on the horizontal surface is: $W_{F_s} = \frac{1}{2}kx_i^{\,2}$

Incline:
Again, we need the **free body diagram** on the incline. Again the work done by the Force Normal is zero because the Force Normal is perpendicular to the displacement of the block.

$$W_{F_N} = F_N \Delta d_{\parallel} \cos(90) = 0$$

The work done by the Force of Gravity is: $W_{F_g} = Fd\cos\theta = F_g \Delta d_{\parallel} \cos\theta_1$

The equation for the Force of Gravity is: $F_g = mg$

The displacement of the block on the incline is the displacement in the parallel direction.

There are going to be several angles in this problem, so I have labeled this angle θ_1. This is the angle between the displacement of the block (up the incline) and the Force of Gravity (down), which is 90° + 25° or 115°. 1 minute and 29 seconds in to this previous video, I showed finding this angle in detail: http://www.flippingphysics.com/work-billy.html

We now have: $W_{F_g} = mg\Delta d_{\parallel} \cos\theta_1$ with $\theta_1 = 115°$

We need $\Delta d_{\parallel}$ in terms of h_{max}, so we draw a **triangle** and $\sin\theta_2 = \frac{O}{H} = \frac{h_{max}}{\Delta d_{\parallel}} \Rightarrow \Delta d_{\parallel} = \frac{h_{max}}{\sin\theta_2}$

Therefore: $W_{F_g} = mg\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_1 = \frac{mgh_{max}\cos\theta_1}{\sin\theta_2}$ where $\theta_2 = 25°$ (the incline angle)

The work done by the Force of Kinetic Friction is:

$$W_{F_{kf}} = Fd\cos\theta = F_{kf}\Delta d_{\parallel}\cos\theta_3 = \mu_k F_N \left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_3$$

Now we need the work done by the Force of Kinetic Friction.

Remember the equation for the Force of Kinetic Friction is $F_{kf} = \mu_k F_N$

θ_3 is the angle between the direction of the displacement of the block (up the incline) and the force of kinetic friction (down the incline) which is 180°. $\theta_3 = 180°$

We need Force Normal so we need to break the force of gravity into its components, redraw the **free body diagram** and sum the forces in the perpendicular direction:

$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_g = mg\cos\theta_2$$

Which we can substitute back into the equation for the Work done by Friction:

$$W_{F_{kf}} = \mu_k\left(mg\cos\theta_2\right)\left(\frac{h_{max}}{\sin\theta_2}\right)\cos\theta_3 = \frac{\mu_k mgh_{max}\cos\theta_3}{\tan\theta_2}$$ (because $\frac{\cos\theta_2}{\sin\theta_2} = \frac{1}{\tan\theta_2}$)

Going back to the original Net Work-Kinetic Energy Theorem:

$$W_{net} = \Delta KE = 0 = W_{F_s} + W_{F_g} + W_{f_{kf}}$$

Now we can substitute in equations and numbers and solve for h_{max}:

$$\Rightarrow W_{net} = 0 = \frac{1}{2}kx_i^2 + \frac{mgh_{max}\cos\theta_1}{\sin\theta_2} + \frac{\mu_k mgh_{max}\cos\theta_3}{\tan\theta_2}$$

$$\Rightarrow 0 = \frac{1}{2}(14)(0.032)^2 + \frac{(0.011)(9.81)h_{max}\cos(115)}{\sin(25)} + \frac{(0.30)(0.011)(9.81)h_{max}\cos(180)}{\tan(25)}$$

$$\Rightarrow 0 = 0.007168 + (-0.10791)h_{max} + (-0.069424)h_{max} \Rightarrow 0.177334h_{max} = 0.007168$$

$$h_{max} = 0.040421 \approx \boxed{0.040m}$$

(yes, this is the same answer we got the previous two times we solved this problem.)

It is important to note that:
The net work put into the system by the Force Applied is the same as the work done by the Force of the Spring which is positive because the spring converts Elastic Potential Energy to Kinetic Energy. $\sum W_{in} = PE_e = W_{F_s} = \frac{1}{2}kx_i^2 = \frac{1}{2}(14)(0.032)^2 = 0.007168J$

The work done by the Force of Kinetic Friction is negative because it converts Kinetic Energy into heat and sound energy.

$$W_{F_{kf}} = (-0.069424)h_{max} = (-0.069424)(0.040421) = -0.0028062J$$

The work done by the Force of Gravity is negative because it converts the Kinetic Energy into Gravitational Potential Energy.

$$W_{F_g} = \left(-0.10791\right)h_{\max} = \left(-0.10791\right)\left(0.040421\right) = -0.0043618J$$

Because: $W_{F_g} = \dfrac{mgh_{\max}\cos\theta_1}{\sin\theta_2} = mgh_{\max}\left(\dfrac{\cos\left(115\right)}{\sin\left(25\right)}\right) = mgh_{\max}\left(-1\right) = -mgh_{\max}$

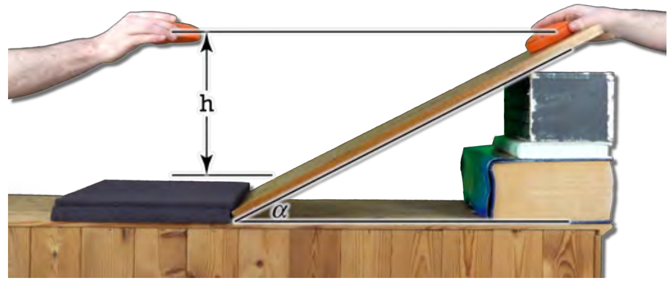
Which means $\dfrac{W_{F_{kf}}}{\sum W_{in}} \times 100 = \dfrac{-0.0028062}{0.007168} \times 100 = -39.149 \approx -39\%$ of the total energy put into the system by the force applied was dissipated as heat and sound energy.

And $\dfrac{W_{F_g}}{\sum W_{in}} \times 100 = \dfrac{-0.0043618}{0.007168} \times 100 = -0.60851 \approx -61\%$ of the total energy put into the system by the force applied was converted to Gravitational Potential Energy.

Flipping Physics Lecture Notes:

Conservative and Nonconservative Forces

http://www.flippingphysics.com/conservative-nonconservative.html



If an object goes straight down a distance h, the work done by force of gravity on the object equals mgh:

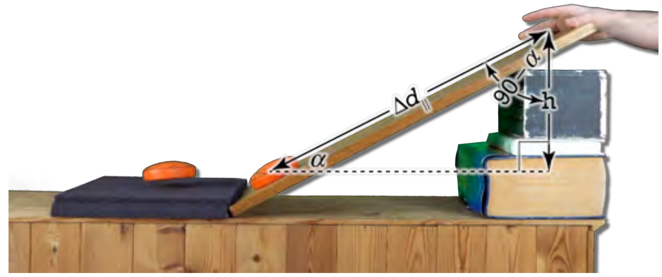
$$W_{F_g} = F_g \Delta r \cos\theta = (mg)(h)\cos(0) = mgh$$

If that object instead slides down an incline of angle alpha through the same vertical distance h, the work done by the force of gravity on the object is still mgh:

$$W_{F_g} = F_g \Delta r \cos\theta = (mg)(\Delta d_{\parallel})\cos(90-\alpha)$$

$$\sin\alpha = \frac{O}{H} = \frac{h}{\Delta d_{\parallel}} \Rightarrow \Delta d_{\parallel} = \frac{h}{\sin\alpha} \;\&\; \sin\alpha = \cos(90-\alpha)$$

$$\Rightarrow W_{F_g} = (mg)\left(\frac{h}{\sin\alpha}\right)(\sin\alpha) = mgh$$



In other words, no matter the angle of the incline, the work done by the force of gravity on the object as the object goes down a distance h is the same. Because the work done by the force of gravity on an object is *independent* of the path taken by the object, the force of gravity is a *conservative force*.

Examples of conservative forces are: gravitational force, spring force, electromagnetic force between two charged particles, and magnetic force between two magnetic poles.

In fact, notice that the work done on the object by the force of gravity equals the negative of the change in gravitational potential energy of the mass:

$$\Delta U_g = U_{gf} - U_{gi} = mgh_f - mgh_i = mg(h_f - h_i) = mg(\Delta h) = mg(-h) = -mgh$$

$$W_{F_g} = mgh = -(-mgh) = -\Delta U_g$$

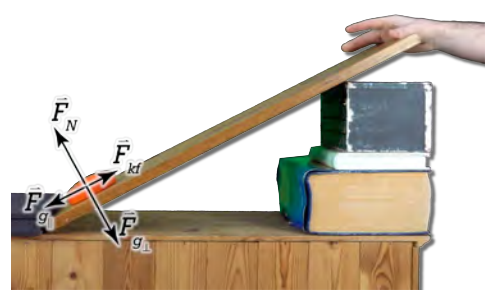
That is because the work done by a conservative force equals the negative of the change in potential energy of the object which is associated with that force.

$$W_{\text{conservative force}} = -\Delta U$$

For example:

- Gravitational potential energy goes with the force of gravity.
- Elastic potential energy goes with spring force.
- Electric potential energy goes with the electromagnetic force.

The work done by the force of friction on an object as the object slides down an incline through a vertical distance h, is *dependent* on the incline angle and therefore the path taken by the object:



$$W_{F_{kf}} = F_{kf}\Delta r\cos\theta = \mu_k F_N \Delta d_{\parallel}\cos(180)$$

$$\sum F_{\perp} = F_N - F_{g_{\perp}} = ma_{\perp} = m(0) = 0 \Rightarrow F_N = F_{g_{\perp}} = mg\cos\alpha$$

$$\Rightarrow W_{F_{kf}} = \mu_k(mg\cos\alpha)\left(\frac{h}{\sin\alpha}\right)(-1) = -\mu_k mgh\cot\alpha$$

Therefore, because the force of friction is *dependent* on the path the object moves through, the force of friction is a *nonconservative force*.

All conservative forces have the following two equivalent properties:

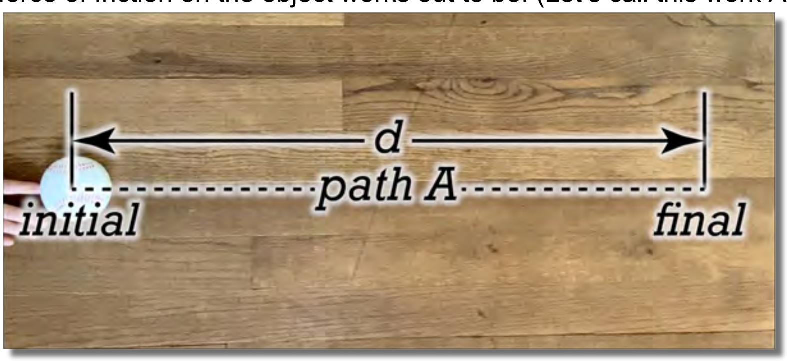
1) The work done by a conservative force on an object moving between any two points is independent of the path taken by the object.
2) The work done by a conservative force on an object moving through any closed path equals zero. (A closed path is a path where the initial and final points are the same location.)

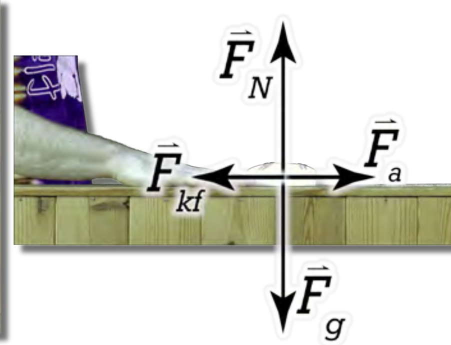
If a force does not have these two equivalent properties, it is a nonconservative force.

Clearly, we have shown the first property to be true, let's look at work done on an object as it moves through a closed path in order to investigate the second property. We have already shown that the work done on an object by the force of gravity equals mass times acceleration due to gravity times height, regardless of the path taken. For a closed path, the initial and final heights will be the same and the work done on the object by the force of gravity will equal zero. That shows that the force of gravity has the second equivalent property necessary to be a conservative force.

$$W_{F_g} = mgh = mg(0) = 0(\text{closed path})$$

Now let's look at *nonconservative* forces in more detail. If we have an object on a level surface and we displace that object a straight-line distance d, from an initial point to a final point, the work done by the force of friction on the object works out to be: (Let's call this work A and path A.)

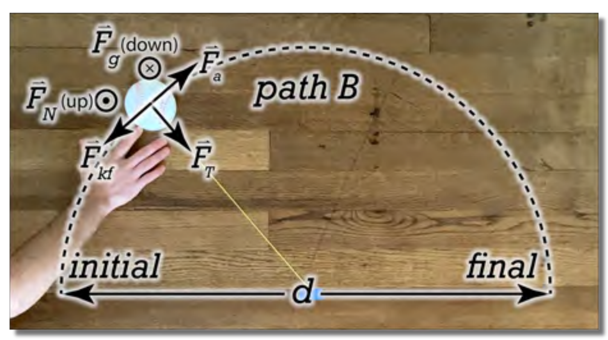




$$W_A = F_{kf}\Delta r_A \cos\theta = (\mu_k F_N)(d)\cos(180) = -\mu_k mgd$$

$$\sum F_y = F_N - F_g = ma_y = m(0) = 0 \Rightarrow F_N = F_g = mg$$

Now instead let's tie a string to the object and move it from the same initial point to the same final point, only this time the path of the object will form half a circle. The work done by the force of friction on the object now is: (Let's call it work B and path B.)



$$W_B = F_{kf}\Delta r_B \cos\theta = (\mu_k F_N)\left(\frac{C}{2}\right)\cos(180) = -\mu_k(mg)\left(\frac{\pi d}{2}\right) = -\frac{\pi}{2}\mu_k mgd \neq -\mu_k mgd = W_A$$

$$C = \pi D = \pi d \Rightarrow \frac{C}{2} = \frac{\pi d}{2}$$

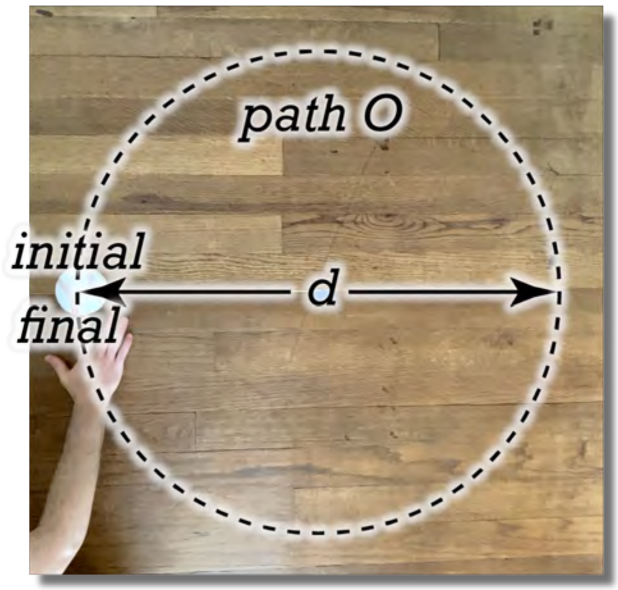
In other words, the work done by the force of friction on the object is different depending on the path taken by the object. That makes the force of friction a nonconservative force because it does not have the first of the two equivalent properties.

The difference here is that more energy has been converted to thermal energy when going along path B than when going along path A. The work that went into the system via the force applied moving the object was converted into thermal energy in the object via work done by the force of friction.

Now, we should know, because the two conservative force properties are equivalent, that the force of friction will also not have the second, equivalent property, the work over a closed path for the force of friction will not equal zero, however, let's just confirm that. Let's determine the work done by the force of friction acting on an object moving through one full circle of radius d over 2. In other words, from the original initial point all the way through one circumference back to that same point. The work done by the force of friction on the object works out to be: (Let's call it work O and path O)

$$W_O = F_{kf}\Delta r_O \cos\theta = \left(\mu_k F_N\right)\left(C\right)\cos\left(180\right) = -\mu_k\left(mg\right)\left(\pi d\right) = -\pi\mu_k mgd \neq 0$$

Which, clearly, does not equal zero. So, the work done by the force of friction on an object over a closed path does not equal zero and the force of friction is a nonconservative force. Again, work done by the force of friction converts kinetic energy into thermal energy and the object is warmer after going through the circle than it was before.



Flipping Physics Lecture Notes:

Energy Transferred Into and Out of a System
http://www.flippingphysics.com/energy-transfer-system.html

We have talked a lot about work and energy in this class. Now it is time to discuss the most general equation we have which relates work and energy:

$$\Delta E_{system} = \sum T$$

This equation states that the change in energy of the system equals the net energy transferred into or out of the system. That may seem a little obvious, however, it is a great place to start and we are going to derive some equations from this obvious starting place.

Energy can be transferred into or out of the system in many ways:

- Mechanical Waves – Sound waves are a great example of energy transferring from one system to another through a disturbance of a medium.
- Work – A force applied can transfer energy into or out of a system by doing work on the system.
- Heat – Heat is a way to transfer energy from one system to another via a difference in temperature.
- Electricity – Electrical current can transfer energy from one system to another, think of an electric motor converting electric potential energy to kinetic energy of the rotating motor.
- Radiation – Electromagnetic waves like visible light, microwaves, etc. Consider that a microwave oven transfers energy to food through microwave radiation.

For a long time in this class, the only source of energy transfer into or out of a system we will consider will be a mechanical one. That means the net energy transferred into or out of the system will be done by the work done by a force applied. Also, the change in energy of a system is equal to the change in mechanical energy of the system plus the change in internal energy of the system:

$$\Delta E_{system} = \sum T \Rightarrow \Delta ME + \Delta E_{internal} = W_{F_a}$$

If there is no force applied transferring energy into or out of the system via work, then we get:

$$\Rightarrow \Delta ME + \Delta E_{internal} = 0$$

The change in internal energy of the system is caused by the work done by nonconservative forces like friction. In other words, the work done by nonconservative forces on a system converts mechanical energy into internal energy of the system. You should recognize this because when you rub your hands together, they get warmer. The kinetic energy of the motion of your hands is being converted, via work done by friction, to internal energy in your hands, they get warmer. That equation looks like this:

$\Delta E_{internal} = -W_{NC}$ (where NC stands for nonconservative forces)

In other words, the general equation about energy transfer we started with, if there is zero work done by a force applied on the system, looks like this:

$$\Delta E_{system} = \sum T \Rightarrow \Delta ME + \Delta E_{internal} = W_{F_a} \Rightarrow \Delta ME - W_{NC} = 0 \Rightarrow W_{NC} = \Delta ME$$

Which is the work due to friction equals change in mechanical energy of a system we used before, only now we have derived it. And, now that we have defined nonconservative forces, we can identify this as the work done by nonconservative forces on a system equals the change in mechanical energy of the system.

And if we go one step further and say there is zero work done by nonconservative forces, we get this:

$$\Rightarrow 0 = \Delta ME = ME_f - ME_i \Rightarrow ME_i = ME_f$$

Which is the conservation of mechanical energy we have used before, only now we have derived it. ☺

To review: Starting with the general concept that the change in energy of the system equals the net energy transferred into or out of the system:

$$\Delta E_{system} = \sum T$$

And the fact that the work done by nonconservative forces on a system converts mechanical energy into internal energy of the system:

$$\Delta E_{internal} = -W_{NC}$$

We were able to prove that, when there is no force applied doing work on the system, that the work done by nonconservative forces on a system equals the change in mechanical energy of the system:

$W_{NC} = \Delta ME$ (no work done by force applied)

And further, if there is also no work done by nonconservative forces on the system, mechanical energy is conserved:

$ME_i = ME_f$ (no work done by force applied and no work done by nonconservative forces)

Please, please, please remember whenever you use these equations you have to identify your initial and final points.

You also need to identify which object(s) are a part of your system. The importance of which will be obvious when you watch my next video, "Energy Systems Clarified".
http://www.flippingphysics.com/energy-systems.html

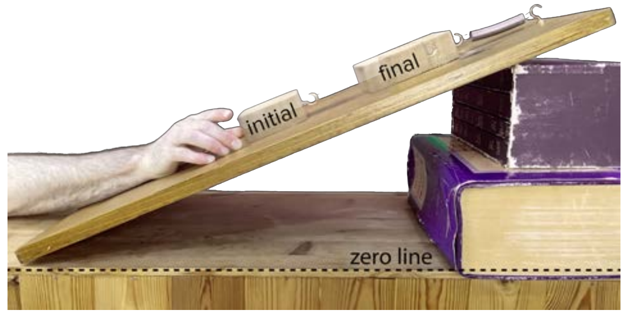
Flipping Physics Lecture Notes:

Energy Systems Clarified

http://www.flippingphysics.com/energy-systems.html

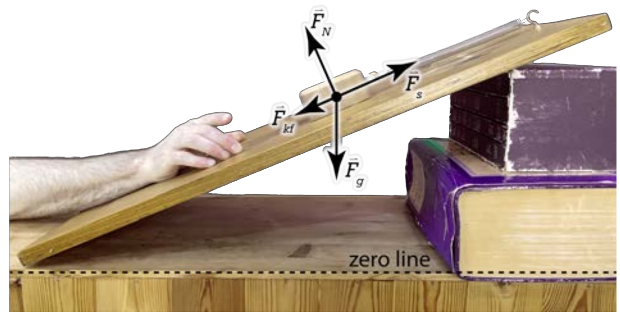
The objects which are a part of the system determine how work and energy are related. Let's start with an example to show how this works.

Example: A block attached to a spring slides up an incline.



Let's define:

- The initial point after the block has started moving
- The final point before the block has stopped moving and before the block gets to the spring's equilibrium position.
- The horizontal zero line at the bottom of the incline. The block is always above the zero line.



We need to draw a free body diagram of all the forces acting on the block. Notice the force of gravity, the force of kinetic friction, and the spring force all do work on the bock, however, the force normal does not do work on the block because it is perpendicular to the direction of the motion of the block.

In order to discuss work and energy of a system, let's start with the most basic equation relating work and energy: $\Delta E_{system} = \sum T$

This equation states that the change in energy of the system equals the net energy transferred into or out of the system. Let's rearrange the equation to make it easier to work with in this situation.

$$\Delta E_{system} = \sum T \Rightarrow E_{system\ f} - E_{system\ i} = \sum T \Rightarrow E_{system\ f} = E_{system\ i} + \sum T \Rightarrow E_{system\ i} + \sum T = E_{system\ f}$$

Remember, energy is transferred into or out of the system via work done on the system by forces external to the system.

If we choose ***the block*** as the system:

- All the forces which act on the block are external to the block.
- The block starts and ends with kinetic energy
- The block does not have gravitational potential energy because gravitational potential energy is caused by the interaction of two masses and the block is just one mass.
- The block does not have elastic potential energy because the spring is not a part of the system.

The equation relating work and energy for the block system looks like this:

$$KE_i + W_s + W_g + W_{F_{kf}} = KE_f \Rightarrow KE_i + W_{net} = KE_f \Rightarrow W_{net} = KE_f - KE_i \Rightarrow W_{net} = \Delta KE$$

Notice this ends up being the Work Energy Theorem
(or the Net Work equals Change in Kinetic Energy Theorem)

If we choose ***the block and the Earth*** as the system:

- The force of gravity is now internal to the system.
- Instead of work done by the force of gravity on the system, we can now look at it in terms of the initial and final gravitational potential energy of the system.

The equation relating work and energy for the block and Earth system looks like this:

$$KE_i + W_s + U_{gi} + W_{F_{kf}} = KE_f + U_{gf}$$

If we choose ***the block, the Earth, and the spring*** as the system:

- The spring force is now internal to the system.
- Instead of work done by the spring force on the system, the system now has initial and final elastic potential energy.

The equation relating work and energy for the block, Earth, and spring system looks like this:

$$KE_i + U_{ei} + U_{gi} + W_{F_{kf}} = KE_f + U_{ef} + U_{gf} \Rightarrow W_{F_{kf}} = \Delta KE + \Delta U_e + \Delta U_g \Rightarrow W_{NC} = \Delta ME$$

And … let's do one more:
If we choose ***the block, the Earth, the spring, and the incline*** as the system:

- The force of friction is now internal to the system.
- Instead of work done by the force of friction on the system, the system now has initial and final internal energy.

The equation relating work and energy for the block, Earth, spring, and incline system looks like this:

$$KE_i + U_{ei} + U_{gi} + U_{Ii} = KE_f + U_{ef} + U_{gf} + U_{If} \Rightarrow 0 = \Delta KE + \Delta U_e + \Delta U_g + \Delta U_I$$

Notice this means energy is conserved. When you include everything in your system, you get conservation of energy because energy is neither created nor destroyed, it just changes forms.

This is just a restatement of $\Delta E_{system} = \sum T$ But, there is no energy transferred into or out of the system because the system contains everything.

Remember that work done by the force of friction goes into the system as internal energy: $W_{F_{kf}} = -\Delta U_I$
And the work done by a conservative force equals the negative of the change in potential energy associated with that force:

$$W_{conservative\ force} = -\Delta U$$

And because the force of gravity and the spring force are both conservative forces:

$$W_g = -\Delta U_g \ \& \ W_s = -\Delta U_e$$

Therefore, if we take the equation we ended with, move the changes in potential and internal energy to the left-hand side, and replace all the negative changes in energy with work, we return to our original Net Work equals Change in Mechanical Energy equation and we are back to just having **the block as the only object in our system**.

$$\Rightarrow -\Delta U_e - \Delta U_g - \Delta U_I = \Delta KE \Rightarrow W_s + W_g + W_{F_{kf}} = \Delta KE \Rightarrow W_{net} = \Delta KE$$