



## Flipping Physics Lecture Notes:

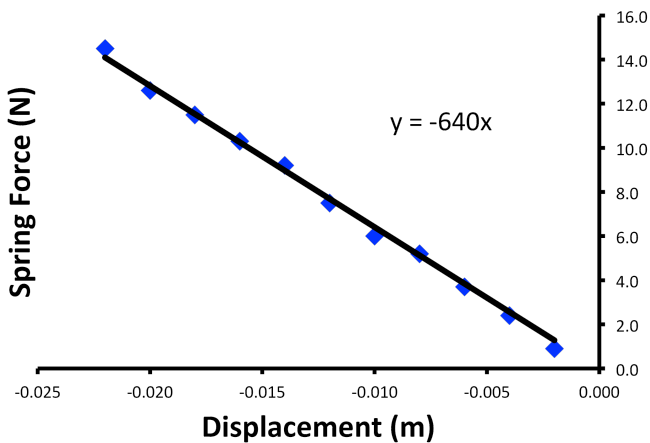
### Hooke's Law Introduction - Force of a Spring

Robert Hooke\* (1635–1703) was an English scientist whose “research and experiments ranged from astronomy to biology to physics; he is particularly recognized for the observations he made while using a microscope and for ‘Hooke's Law’ of elasticity.” † Hooke is also credited being the first to apply “the word ‘cell’ to describe the basic unit of life.” ‡



In our demonstration we are taking a spring and measuring the force it takes to compress the spring a certain distance.

When we graph this data with the force of the spring on the y-axis and the displacement of the spring on the x-axis, we get this result.

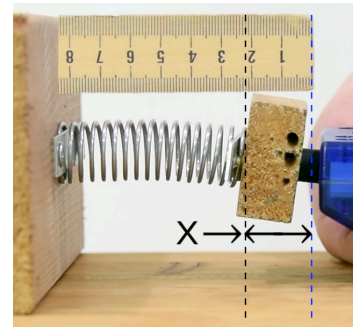


Push	
Spring Force (N)	Position (m)
0.9	-0.002
2.4	-0.004
3.7	-0.006
5.2	-0.008
6.0	-0.010
7.5	-0.012
9.2	-0.014
10.3	-0.016
11.5	-0.018
12.6	-0.020
14.5	-0.022

Hopefully you recognize that there is a linear relationship between the force and displacement of the spring.

This relationship is called “Hooke’s Law”:  $\vec{F}_s = -k\vec{x}$

- $F_s$  is the force caused by the spring.
- $k$  is called the “spring constant”.
  - The spring constant usually has units of newtons per meter:  $\vec{F}_s = -k\vec{x} \Rightarrow k = -\frac{\vec{F}_s}{\vec{x}} \Rightarrow \frac{N}{m}$ 
    - Note: It could be in newtons per cm, dynes per mm, pounds per furlong, etc.
  - $k$  is a measure of how much force it takes to compress or expand a spring per linear meter.
  - Sometimes I have seen it called “force constant”, however, I will not call it that.
  - The spring constant is always positive.
- $x$  is the displacement from equilibrium position or rest position.
  - Equilibrium position or rest position is where the spring is located without any external force causing the spring to compress or expand. It is where the spring is at “rest”.

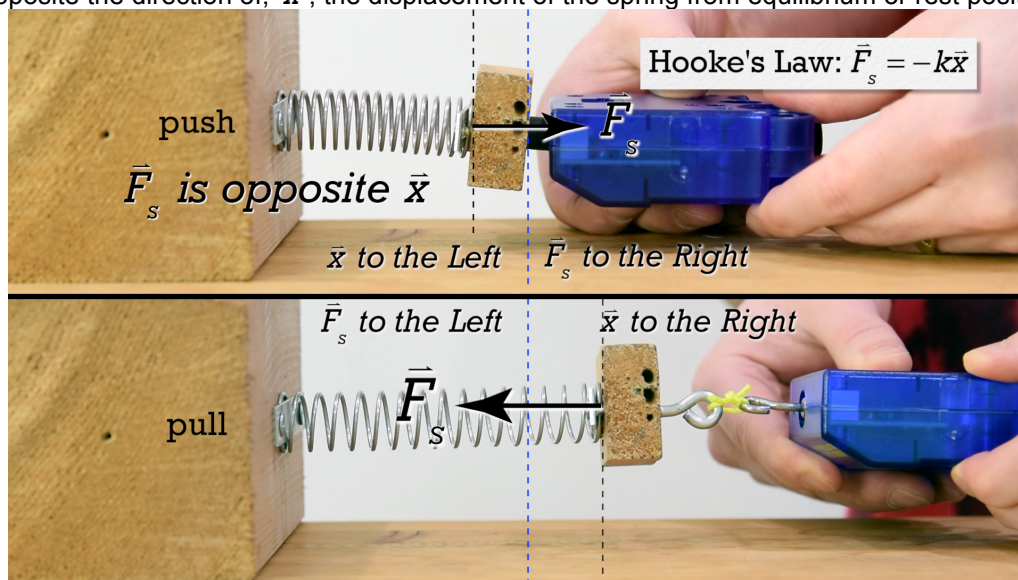


\* [https://en.wikipedia.org/wiki/File:13\\_Portrait\\_of\\_Robert\\_Hooke.JPG](https://en.wikipedia.org/wiki/File:13_Portrait_of_Robert_Hooke.JPG) Interesting tidbit of information: “As no contemporary portrait

† <https://www.biography.com/people/robert-hooke-9343172>

‡ <http://www.history-of-the-microscope.org/robert-hooke-microscope-history-micrographia.php>

- The negative in the equation  $\vec{F}_s = -k\vec{x}$  indicates that the direction of,  $\vec{F}_s$ , the force of the spring is opposite the direction of,  $\vec{x}$ , the displacement of the spring from equilibrium or rest position.



- Hooke's Law means the force required to compress or expand a spring is linearly proportional to the distance the spring is compressed or expanded.
- Going back to our original spring and data, the equation for the best-fit line of the graph is

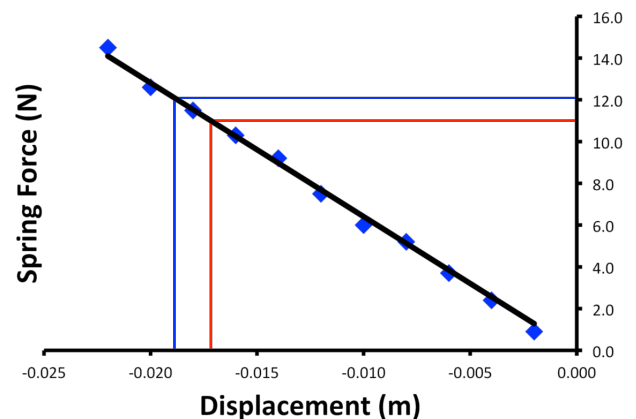
$\vec{F}_s = -640\vec{x}$ , that means the spring constant of that spring,  $k = 640 \frac{N}{m}$ . And we can use that

equation to predict data. For example:

- $\vec{F}_s = -640\vec{x} = -(640)(-0.019) = 12.16 \approx 12N$
- $\vec{F}_s = -640\vec{x} \Rightarrow \vec{x} = -\frac{\vec{F}_s}{k} = -\frac{11}{640} = -0.01719 \approx -0.017m$

- Restoring Force: The spring force is a restoring force, a force that is always towards equilibrium. In other words, the force tends to bring the object back to the rest position.
- Elastic Limit: The maximum displacement before the spring will be permanently deformed. In other words, once the spring reaches beyond its elastic limit, the spring will no longer return to its original shape and Hooke's Law no longer holds true.
- Note: The AP Physics 1 equation sheet has Hooke's Law as:  $|\vec{F}_s| = k|\vec{x}|$ . This gives only the magnitude of the force of the spring and ignores the direction.

- I prefer to use  $\vec{F}_s = -k\vec{x}$  because it includes the direction of the force of the spring.



Flipping Physics Lecture Notes:

Determining the Spring Constant,  $k$ , with a Vertically Hanging Mass

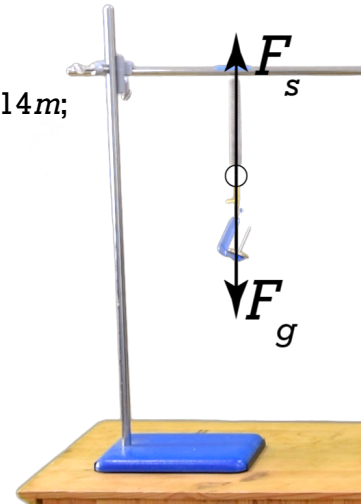
Example: A vertically hanging spring with a natural length of 5.4 cm is extended to a length of 11.4 cm when 25 grams is suspended from it. What is the spring constant of the spring?

Knowns:  $L_i = 5.4\text{cm} \times \frac{1\text{m}}{100\text{cm}} = 0.054\text{m}$ ;  $L_f = 11.4\text{cm} \times \frac{1\text{m}}{100\text{cm}} = 0.114\text{m}$ ;

$m = 25\text{g} \times \frac{1\text{kg}}{1000\text{g}} = 0.025\text{kg}$ ;  $k = ?$

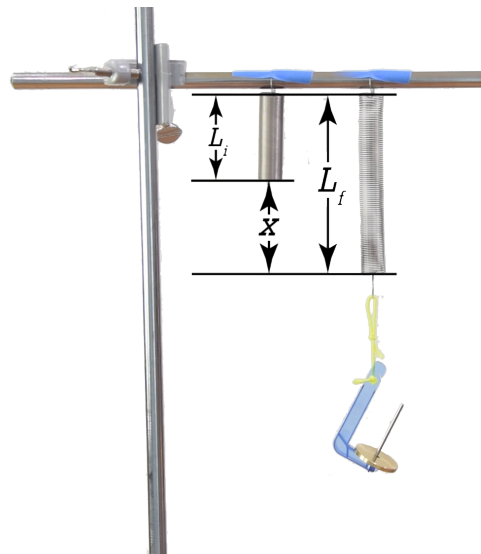
$\sum F_y = F_s - F_g = ma_y = m(0) = 0 \Rightarrow F_s = F_g$

$\Rightarrow kx = mg \Rightarrow k = \frac{mg}{x} = \frac{(0.025)(9.81)}{0.060} = 4.0875 \approx 4.1 \frac{\text{N}}{\text{m}}$



Some things to realize:

- When plugging the equation into Newton's Second Law, use the magnitude of  $F_s$ , the spring force, because we already determined the direction of the spring force in the free body diagram.
- When plugging  $x$ , the displacement from equilibrium position, into the equation for the force of the spring, use the magnitude of  $x$ , because we already determined the direction of the spring force in the free body diagram.
- $x = L_f - L_i = 0.114 - 0.054 = 0.060\text{m}$

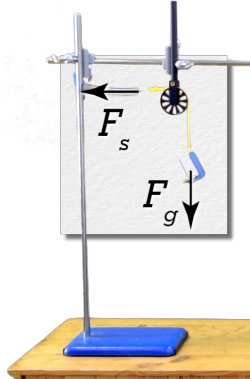




## Flipping Physics Lecture Notes:

### The Human Spine acts like a Compression Spring

Example: A horizontal spring is attached to a cord, the cord goes over a pulley, and a 0.025 kg mass is attached to the cord. If the spring is stretched by 0.045 m, what is the spring constant of the spring?



Knowns:  $m = 0.025\text{kg}$ ;  $x = 0.045\text{m}$ ;  $k = ?$

Add the free body diagram for all the forces acting horizontally on the spring. Yes, there is a force of gravity acting downward on the center of mass of the spring, however, that will not expand the spring.

Let's define the positive direction as to the right, over the pulley, and down.

$$\sum F_+ = F_g - F_s = ma_+ = m(0) = 0 \Rightarrow F_s = F_g \Rightarrow kx = mg$$

$$\Rightarrow k = \frac{mg}{x} = \frac{(0.025)(9.81)}{0.045} = 5.45 \approx 5.4 \frac{\text{N}}{\text{m}}$$

Note: It rounds to 5.4 not 5.5 because of the arcane rounding rule. You always round to an even number when the number to be rounded ends in a perfect 5.

Previously, with the same spring oriented **vertically**, we determined the spring constant to be  $4.1 \frac{\text{N}}{\text{m}}$ .

Why do we get two different spring constants if we take measurements vertically vs. horizontally?

It is because the force of gravity acting down on the center of mass of the spring, along with the corresponding force of tension holding the spring up, expands the spring when it hangs vertically. Notice the natural length of the spring changes based on orientation:

$$L_{\text{vertical}} = 5.4\text{cm} \ \& \ L_{\text{horizontal}} = 5.3\text{cm} \ \text{(expansion spring)}$$

This spring is an expansion spring and therefore has a longer vertical length than horizontal length. A compression spring shows the reverse.

$$L_{\text{vertical}} = 6.1\text{cm} \ \& \ L_{\text{horizontal}} = 6.2\text{cm} \ \text{(compression spring)}$$

This is because the force of gravity acting downward on the center of mass of a compression spring, along with the force normal acting upward on the compression spring, compress the spring when it is vertical.

This same thing happens to humans because our spine acts much like a compression spring.\* Our vertical height when standing is smaller than our horizontal length when lying down. For example:

Mr.p's vertical height = 171.6 mm & Mr.p's horizontal length = 172.5 mm

When humans stand, the intervertebral disks, which are layers of squishy cartilage between the vertebrae of our spine, get squished slightly. These intervertebral "(d)iscs function like coiled springs."\* This decreases our height when standing and, in fact, our height decreases the longer we stand. That means you are taller when you get out of bed in the morning than when you get in bed at night. And while you are sleeping, your spine slowly expands to make you taller.

This is also why astronauts get taller in space. They still have a force of gravity acting on them; however, there is no corresponding force normal to compress their spines. In space, humans can grow up to 3% taller, that means a 6-foot tall astronaut can gain as much as 2 inches in height. When they return back to Earth, however, they shrink back to their usual height.\* I will also point out that there could be deleterious health effects caused by the spinal expansion and compression that are currently being studied.

\* <https://www.ncbi.nlm.nih.gov/pubmed/6600017?dopt=Abstract>

\* <https://www.mayfieldclinic.com/PE-AnatSpine.htm>

\* <https://www.scientificamerican.com/article/astronauts-get-taller-in-space/>