Flipping Physics Lecture Notes:
Buoyant Force Calculation: A Submerged Wood Cylinder
http://www.flippingphysics.com/buoyant-force-wood.html

In our previous lesson we discussed the Buoyant Force which acts on submerged objects. Now we are going to walk through a specific example and then demonstrate it.

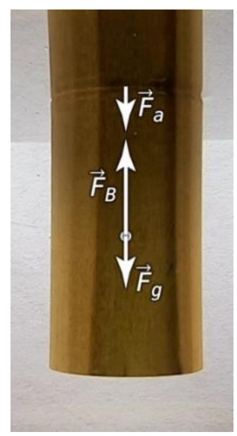
Example: This wood cylinder has a diameter of 50.7 mm and a height of 86.9 mm. If the density of this wood is 540 kg/m^3 and the density of water is 1.00 x 10^3 kg/m^3, what buoyant force acts on the wood cylinder when it is submerged in water?

Knowns: $D = 50.7mm\left(\frac{1m}{1000mm}\right) = 0.0507m \Rightarrow R = \frac{D}{2} = \frac{0.0507}{2} = 0.02535m;$
$H = 86.9mm\left(\frac{1m}{1000mm}\right) = 0.0869m; \rho_{\text{wood}} = \rho_o = 540\frac{kg}{m^3};$
$\rho_{\text{water}} = \rho_f = 1.00 \times 10^3 \frac{kg}{m^3}; F_B = ?$

$$F_B = m_f g \ \& \ \rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow F_B = \rho_f V_f g$$

Sumberged: $V_f = V_o = \pi R^2 H = \pi\left(0.02535\right)^2\left(0.0869\right) = 1.75439 \times 10^{-4} m^3$

$$\Rightarrow F_B = \rho_f V_f g = (1000)\left(1.75439 \times 10^{-4}\right)(9.81) = 1.72105 \approx 1.7N$$



But notice 1.7 N is not the force we measure when the wood cylinder is submerged in the water. What we measure is the downward force applied necessary to hold the cylinder under the water. To check if our answer is correct we need to draw a free body diagram, sum the forces, and solve for the force applied.

Free body diagram: Buoyant Force, up; Force of Gravity, down; Force Applied, down.

$$\sum F_y = F_B - F_a - F_g = m_o a_y = m_o(0) = 0$$

$$\Rightarrow F_g = \rho_o V_o g = (540)\left(1.75439 \times 10^{-4}\right)(9.81) = 0.92937 \approx 0.93N$$

$$\Rightarrow F_a = F_b - F_g = 1.72105 - 0.92937 = 0.79168 \approx 0.79N$$

And the measured Force Applied is really close to 0.79 N. The Physics Works!!

Two common mistakes made by students when using the buoyant force equation:

- The density in the buoyant force equation is the density of the fluid displaced by the object, not the density of the object itself.
- The volume in the buoyant force equation is the volume of the fluid displaced by the object. Only when the object is submerged in a fluid is this the same as the volume of the object.