Flipping Physics Lecture Notes:

You Can't Run from Momentum

- Symbol for momentum is a lowercase p.
    - p is for the Latin word "petere" which means "to make for", "to travel to", "to seek", or "to pursue". It's pretty clear this word is where the letter p for momentum comes from.
    - Do not confuse lowercase "p" for momentum with:
        - Uppercase P, which is for Power.
        - ρ which is for density. (The lowercase Greek symbol ρ is called rho.)
- Equation for momentum is $\boxed{\vec{p} = m\vec{v}}$
    - m is for mass
    - v is for velocity
- Momentum is a vector. So momentum has both magnitude and direction.
- Units for momentum are $\frac{kg \cdot m}{s}$
    - $\vec{p} = m\vec{v} \Rightarrow (kg)\left(\frac{m}{s}\right) = \frac{kg \cdot m}{s}$
    - $\frac{kg \cdot m}{s}$ have no special name.
    - Not to be confused with $\frac{kg \cdot m}{s^2}$ which is a newton.
- If the velocity of the object is zero, then the momentum of the object is zero.
    - $\vec{p} = m\vec{v} = m(0) = 0$

Flipping Physics Lecture Notes:

Force of Impact Equation Derivation

Newton's Second Law is $\sum \vec{F} = m\vec{a}$.

The equation for acceleration is $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$ which we can substitute into Newton's Second Law.

And knowing the equation for momentum is $\vec{p} = m\vec{v}$. Therefore:

$$\sum \vec{F} = m\vec{a} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} \Rightarrow \boxed{\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}}$$

This is the equation for the force of impact during a collision. The net force acting on an object equals the change in momentum of the object divided by the change in time while that net force is acting on the object. Both force and momentum are vectors.

This gets us closer to Newton's original second law which is $\sum \vec{F} = \frac{d\vec{p}}{dt}$. The net force acting on an object equals the derivative of the momentum of that object with respect to time. If you ever take a calculus based physics course, like AP Physics C, you will get an opportunity to work with this equation.

Note: This equation for the force of impact acting on an object during a collision marks a paradigm shift in our physics learning. Before we had this equation, we only looked at objects before they ran into one another. Now, using this equation, we can determine forces during collisions. Which means dropping the medicine ball onto the ground actually has two parts. Part 1, when the medicine ball is in free fall and part 2, when the medicine ball strikes the ground.

Flipping Physics Lecture Notes:

Calculating the Force of Impact when Stepping off a Wall

Example: A 73 kg mr.p steps off a 73.2 cm high wall. If mr.p bends his knees such that he stops his downward motion and the time during the collision is 0.28 seconds, what is the force of impact caused by the ground on mr.p?

With the exception of the mass, m = 73 kg, the known values for this problem need to be split into two parts.

Part 1 – Free Fall: $v_{1iy} = 0;\ h_{1i} = 73.2cm \times \frac{1m}{100cm} = 0.732m$

Part 2 – Collision: $v_{1fy} = 0;\ \Delta t_2 = 0.28\sec$

We are solving for the Force of Impact during part two, therefore we can use the Force of Impact equation $\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ during part two.

Part 2: $\sum \vec{F}_2 = \frac{\Delta \vec{p}_2}{\Delta t_2} = \frac{\vec{p}_{2f} - \vec{p}_{2i}}{\Delta t_2} = \frac{m\vec{v}_{2f} - m\vec{v}_{2i}}{\Delta t_2} = \frac{(73)(0) - (73)(\vec{v}_{2i})}{0.28}$

Therefore, we need the velocity for part 2 initial. Because the beginning of part 2 is the same as the end of part 1, $\vec{v}_{1f} = \vec{v}_{2i}$, therefore, we need to find the final velocity for part 1.

Part 1 – Conservation of Energy: Zero line at the ground, initial point at the start of part 1, final point at the end of part 1.

$$ME_{1i} = ME_{1f} \Rightarrow PE_{g1i} = KE_{1f} \Rightarrow mgh_{1i} = \frac{1}{2}m(v_{1f})^2 \Rightarrow gh_{1i} = \frac{1}{2}(v_{1f})^2 \Rightarrow v_{1f} = \sqrt{2gh_{1i}}$$

$$\Rightarrow v_{1f} = \sqrt{(2)(9.81)(0.732)} = \pm 3.7897 = -3.7897\frac{m}{s} = v_{2i}$$

And now back to part 2: $\sum \vec{F}_2 = \frac{m\vec{v}_{2f} - m\vec{v}_{2i}}{\Delta t_2} = \frac{(73)(0) - (73)(-3.7897)}{0.28} = 988.03 \approx \boxed{990N}$

$$\sum \vec{F}_2 = 988.03N \times \frac{1lb}{4.448N} = 222.13 \approx \boxed{220lb}$$

Flipping Physics Lecture Notes:
Impulse Introduction or
If You Don't Bend Your Knees When Stepping off a Wall

This video is an extension of “Calculating the Force of Impact when Stepping off a Wall”[♥]. The idea is to figure out how much force would be exerted on mr.p’s body if he didn’t bend his knees. I am unwilling to demonstrate this; instead I dropped a tomato. The time of impact for the tomato was 6 frames in a video filmed at 240 frames per second: $\Delta t = 6 frames \times \frac{1 \sec}{240 frames} = 0.025 \sec$

The idea is that we can approximate the time during the collision if I did *not* bend my knees to be the same as the collision for the tomato. Do I know this to be true? No. However, again, I am unwilling to demonstrate stepping off a wall and not bending my knees, so this is a good approximation.

Because I fell 73.2 cm, we determined last time my velocity right before striking the ground is 3.7897 m/s down and my velocity after striking the ground is zero because I stop. My mass is 73 kg. That means the force of impact during the collision is:

Unbent knees: $\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{(73)(0)-(73)(-3.7897)}{0.025} = 11065.93N \times \frac{1lb}{4.448N} = 2487.84 \approx 2500lb$

Bent knees: $\sum \vec{F} = \frac{(73)(0)-(73)(-3.7897)}{0.28} = 988.03N \times \frac{1lb}{4.448N} = 222.13 \approx 220lb$

(The time of impact when bending my knees was 0.28 seconds.)

Not bending my knees decreases the time of impact from 0.28 seconds to 0.025 seconds and:

$\frac{\sum F_{not\,bent}}{\sum F_{bent}} = \frac{11065.83}{988.03} = 11.2 \approx 11$ Makes the force of impact roughly 11 times what it is when I bend my knees.

The key here is that the only thing which is different between the two examples is the time during the collision. The mass, final velocity and initial velocity are all the same in both examples. In other words, the change in momentum during both examples is exactly the same. For this reason, the change in momentum is given a specific name, it is called Impulse. The symbol for Impulse is usually a capital J and sometimes a capital I; I will usually just write out the word Impulse. Note that we can rearrange Newton’s second law to solve for impulse.

$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \Rightarrow \sum \vec{F}\Delta t = \Delta \vec{p} = Impulse$ Note: Impulse is a vector!

The dimensions for Impulse are $N \cdot s$, which is the same thing as $\frac{kg \cdot m}{s}$ because:

$$N \cdot s = \left(\frac{kg \cdot m}{s^2}\right)s = \frac{kg \cdot m}{s}$$

Point of confusion for students: Impulse is equal to the change in momentum of an object *and it is also equal to* the force of impact times the change in time.

$$Impulse = \sum \vec{F}\Delta t = (988.03)(0.28) = (11065.93)(0.025) = 276.65N \cdot s \approx 280N \cdot s$$

$$\Rightarrow Impulse = \Delta \vec{p} = m\vec{v}_f - m\vec{v}_i = (73)(0)-(73)(-3.7897) = 276.65 \approx 280\frac{kg \cdot m}{s}$$

[♥] http://www.flippingphysics.com/impact-force-problem.html

Flipping Physics Lecture Notes:

Proving and Explaining Impulse Approximation

This video is an extension of “Impulse Introduction or If You Don't Bend Your Knees When Stepping off a Wall”. We determined the force of impact when stepping off the wall for two different cases:

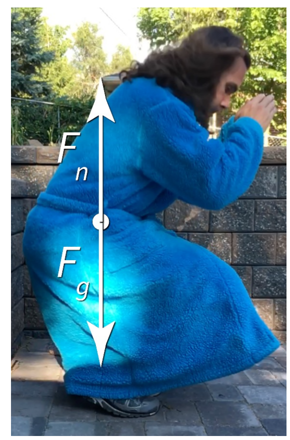
1) Bent knees: $\sum \vec{F}_{bent} = \frac{\Delta \vec{p}}{\Delta t} = 988.03N \approx 990N$ ♥

2) Not bent knees: $\sum \vec{F}_{not\ bent} = \frac{\Delta \vec{p}}{\Delta t} = 11065.93N \approx 11000N$ ♠

In order to say the force of impact during the collision was equal to the net force during the collision, we needed to use the Impulse Approximation.

Impulse Approximation: During the short time interval of a collision, the force of impact is much larger than all the other forces, therefore we can consider the other forces to be negligible when compared to the impact force and the net force is approximately equal to the force of impact.

This begs the question, was this actually true in these two examples? Let’s find out:



In order to determine the force during impact we need to draw a free body diagram:

The force of impact is caused by the ground on my body and is the Force Normal:

$$\sum F_y = F_N - F_g \Rightarrow F_N = \sum F_y + F_g = \sum F_y + mg$$

We can solve for the force of impact (the force normal) during both instances.

Bent knees: $F_N = 988.03 + (73)(9.81) = 1704.16N$

Not bent knees: $F_N = 11065.93 + (73)(9.81) = 11782.06N$

And how far off were these forces of impact from when we used the Impulse Approximation?

Bent knees: $E_r = \frac{O - A}{A} \times 100 = \frac{988.03 - 1704.16}{1704.16} \times 100 = -42.022 \approx -42\%$

Not bent knees: $E_r = \frac{O - A}{A} \times 100 = \frac{11065.93 - 11782.06}{11782.06} \times 100 = -6.0781 \approx -6.1\%$

In other words, with a time interval of 0.28 seconds when bending my knees, the Impulse Approximation was 42% off, which is, in my opinion too much! And we probably shouldn’t have done so!

Also, the shorter the time interval, the larger the force of impact relative to the net force, and therefore the more appropriate it is to use the Impulse Approximation.

♥ If you want to see all of the numbers behind this calculation, please visit: http://www.flippingphysics.com/impact-force-problem.html

♠ If you want to see all of the numbers behind this calculation, please visit: http://www.flippingphysics.com/impulse-introduction.html

Flipping Physics Lecture Notes:
How to Wear a Helmet
A Public Service Announcement from Flipping Physics

Newton's 2nd Law: $\sum \vec{F} = m\vec{a} = m\dfrac{\Delta\vec{v}}{\Delta t} = m\left(\dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = \dfrac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$ & $\sum \vec{F} = \dfrac{\Delta\vec{p}}{\Delta t} = \dfrac{\vec{p}_f - \vec{p}_i}{\Delta t} = \dfrac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$

Impulse Approximation: during the short time interval of the collision, the force of impact is much larger than all of the other forces, therefore we can consider the other forces to be negligible when compared to the impact force and the net force is approximately equal to the force of impact.

$$\vec{F}_{impact} \approx \sum \vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} \Rightarrow \vec{F}_{impact}\Delta t = m\vec{v}_f - m\vec{v}_i$$

Looking at the variables on the right hand side of the equation. Mass, final velocity, and initial velocity of my head: none of these variables depend on whether the helmet is on my head or not. In other words the right hand side of the equation is constant. This is the concept of Impulse and is also equal to the change in momentum of my head.

$$\vec{F}_{impact}\Delta t = m\vec{v}_f - m\vec{v}_i = \Delta\vec{p} = Impulse$$

Wearing a helmet during a collision will increase the time it takes for my head to stop and therefore decrease the force of impact on my head, even though the impulse is constant. That is why you should buckle your helmet, so that it stays on your head, increases the change in time during the collision and reduces the force of impact on your head.

Flipping Physics Lecture Notes:

Introduction to Conservation of Momentum

- Remember, the equation for momentum is $\vec{p} = m\vec{v}$.
- Momentum is conserved in an "isolated system".
    - A system is isolated when the net force on the system equals zero.
    - $\sum \vec{F}_{system} = 0 = \frac{\Delta \vec{p}_{system}}{\Delta t} \Rightarrow 0 \cdot \Delta t = \left(\frac{\Delta \vec{p}_{system}}{\Delta t}\right)\Delta t \Rightarrow 0 = \Delta \vec{p}_{system} = \vec{p}_{f\,system} - \vec{p}_{i\,system} \Rightarrow \vec{p}_{i\,system} = \vec{p}_{f\,system}$
- In an algebra based class this means *momentum is conserved during all collisions and explosions*.
- Conservation of Momentum means the sum of the initial momentums of the system before the collision or explosion equals the sum of the final momentums of the system after the collision or explosion.
- The equation for Conservation of Momentum is $\boxed{\sum \vec{p}_i = \sum \vec{p}_f}$

The skateboard example:

- The velocity of mr.p before the explosion is zero; therefore mr.p's initial momentum is zero.
- The velocity of the ball before the explosion is zero; therefore the ball's initial momentum is zero.
- The total momentum of the system initial is zero. $\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow 0 = \sum \vec{p}_f = \vec{p}_{hf} + \vec{p}_{bf}$
    - "h" is for "human" because "p" for mr.p would be too confusing.
- The ball has a velocity to the right after the explosion; therefore the ball has a positive momentum after the explosion.
- Because the ball has positive final momentum and the total momentum is zero, mr.p must have negative momentum after the explosion. This is why he moves to the left.
    - $\vec{p}_{bf} > 0 \Rightarrow \vec{p}_{hf} < 0$

Flipping Physics Lecture Notes:

Introductory Conservation of Momentum Explosion Problem Demonstration

Knowns:

$$m_b = 0.066kg;\ m_n = 1.791kg;\ \Delta x_b = x_{bf} - x_{bi} = 0.015m - 0.451m = -0.436m;\ \Delta t_b = 0.11\sec$$

$$v_{bf} = \frac{\Delta x_b}{\Delta t_b} = \frac{-0.436m}{0.11\sec} = -3.9\overline{6}\overline{3}\frac{m}{s}$$

Conservation of momentum:

$$\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow \vec{p}_{bi} + \vec{p}_{ni} = \vec{p}_{bf} + \vec{p}_{nf} \Rightarrow m_b\vec{v}_{bi} + m_n\vec{v}_{ni} = m_b\vec{v}_{bf} + m_n\vec{v}_{nf}$$

Note: The initial velocity of everything is zero, therefore the initial momentum of the system is zero.

$$\Rightarrow 0 = m_b\vec{v}_{bf} + m_n\vec{v}_{nf} \Rightarrow m_n\vec{v}_{nf} = -m_b\vec{v}_{bf} \Rightarrow \vec{v}_{nf} = -\frac{m_b\vec{v}_{bf}}{m_n}$$

$$\Rightarrow \vec{v}_{nf} = -\frac{(0.066)(-3.9\overline{6}\overline{3})}{1.791} = 0.14606\frac{m}{s} \approx 0.15\frac{m}{s}(predicted)$$

$$(observed)\vec{v}_{nf} = \frac{\Delta\vec{x}}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{0.416m - 0.400m}{0.11\sec} = 0.1\overline{4}\overline{5}\frac{m}{s}$$

$$E_r = \frac{O-A}{A}\times 100 = \frac{(0.1\overline{4}\overline{5}) - (0.14606)}{0.14606}\times 100 = -0.4170 \approx -0.42\%$$

Flipping Physics Lecture Notes:

Introduction to Elastic and Inelastic Collisions

Let's begin with two different types of collisions:

- Elastic
    - The two objects bounce off of one another.
    - Total momentum is conserved.
        - Remember momentum is conserved in *all* collisions and explosions.
    - Total kinetic energy is conserved.
    - Examples: Billiard balls, air hockey pucks.
- Inelastic
    - Total momentum is conserved.
        - Why?
    - Total kinetic energy is ***not*** conserved.
        - Kinetic energy is converted to heat and sound.
            - When the objects collide, they deform. That deformation causes friction inside the objects to increase the internal energy of the objects. (Internal friction increases the object's temperature.)
    - *Perfectly* Inelastic: The two objects stick together after the collision.
        - Examples: Clay object sticking to another object, two football players colliding (and holding one another close!), two railroad cars coupling.
    - Inelastic Examples: All real world "bounce" collisions.
        - At the atomic level collisions are often elastic, however, in the macroscopic world we live in, elastic collisions are an "ideal case" which is never quite achieved. There is always some deformation of the objects and therefore some kinetic energy converted to internal energy of the objects. Sadly, even billiard balls do not collide elastically though physicists do approximate the collisions as elastic and so do we, for the sake of this class.

Type of Collision	Is Momentum Conserved?	Is Kinetic Energy Conserved?
Elastic	Yes	Yes
Inelastic	Yes	No

Just so you know, collisions between hard spheres are "nearly" elastic and therefore are generally considered to be elastic in physics classes.

Also, sometimes "Perfectly Inelastic" Collisions are called "Completely Inelastic" or "Totally Inelastic". These terms all mean the same thing.

Flipping Physics Lecture Notes:

Introductory Perfectly Inelastic Collision Problem Demonstration

Knowns: $m_c = 0.599kg;\ m_b = 0.066kg;\ \Delta t_b = 0.16\sec;$

$$\vec{v}_{bi} = \frac{\Delta \vec{x}_b}{\Delta t_b} = \frac{x_f - x_i}{\Delta t_b} = \frac{0.313 - 0.566}{0.16} = -1.58125\frac{m}{s}$$

Conservation of momentum:

$$\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow \vec{p}_{bi} + \vec{p}_{ci} = \vec{p}_{bf} + \vec{p}_{cf} \Rightarrow m_b\vec{v}_{bi} + m_c\vec{v}_{ci} = m_b\vec{v}_{bf} + m_c\vec{v}_{cf} \Rightarrow m_b\vec{v}_{bi} = \left(m_b + m_c\right)\vec{v}_f$$

Note: $\vec{v}_{ci} = 0$ & $\vec{v}_{cf} = \vec{v}_{bf} = \vec{v}_f$

$$\Rightarrow \vec{v}_f = \frac{m_b\vec{v}_{bi}}{m_b + m_c} = \frac{\left(0.066\right)\left(-1.58125\right)}{0.066 + 0.599} = -0.15694\frac{m}{s} \approx -0.16\frac{m}{s}\left(predicted\right)$$

$$\left(observed\right)\vec{v}_f = \frac{\Delta \vec{x}_c}{\Delta t_c} = \frac{x_{cf} - x_{ci}}{\Delta t_c} = \frac{0.267 - 0.306}{0.25} = -0.156\frac{m}{s}$$

$$E_r = \frac{O - A}{A} \times 100 = \frac{\left(-0.15694 - \left(-0.156\right)\right)}{-0.156} \times 100 = 0.6001 \approx 0.60\%$$

Flipping Physics Lecture Notes:

Introductory Elastic Collision Problem Demonstration

Example: Cart 1 has a mass of 2m and cart 2 has a mass of m. Cart 2 is initially at rest. Cart 1 is moving at 40.9 cm/s when it collides elastically with cart 2. If the speed of cart 1 after the collision is 13.4 cm/s, what is the speed cart 2 after the collision?

Knowns: $m_1 = 2m;\ m_2 = m;\ \vec{v}_{1i} = 40.9\frac{cm}{s};\ \vec{v}_{1f} = 13.4\frac{cm}{s};\ \vec{v}_{2i} = 0;\ \vec{v}_{2f} = ?$

Momentum is conserved during all collisions so: $\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

$$\Rightarrow (2m)(40.9) + (m)(0) = (2m)(13.4) + (m)\vec{v}_{2f} \Rightarrow (2)(40.9) = (2)(13.4) + \vec{v}_{2f}$$

$$\Rightarrow \vec{v}_{2f} = (2)(40.9) - (2)(13.4) = 55\frac{cm}{s} \Rightarrow v_{2f} \approx \boxed{55.0\frac{cm}{s}}$$ (predicted)

Measured is the slope of the line: $\vec{v}_{2f} = 52.8\frac{cm}{s}$ (measured)

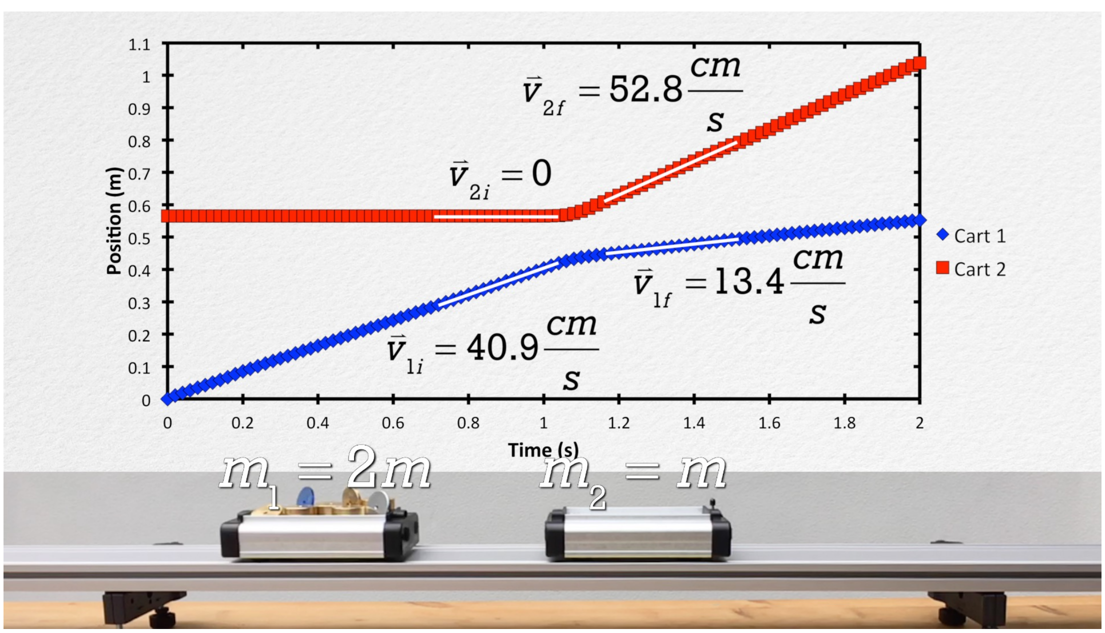
Relative error for our velocity measurement: $E_r = \frac{O - A}{A} \times 100 = \frac{52.8 - 55}{55} \times 100 = -4 \approx -4.00\%$

Is Kinetic Energy conserved? In other words: $\sum KE_i = \sum KE_f \Rightarrow \frac{\sum KE_f}{\sum KE_i} = 1$

$$\sum KE_i = \frac{1}{2}m_1\left(\vec{v}_{1i}\right)^2 + \frac{1}{2}m_2\left(\vec{v}_{2i}\right)^2 = \frac{1}{2}(2m)(40.9)^2 + \frac{1}{2}(m)(0)^2 = 1672.81m$$

$$\sum KE_f = \frac{1}{2}m_1\left(\vec{v}_{1f}\right)^2 + \frac{1}{2}m_2\left(\vec{v}_{2f}\right)^2 = \frac{1}{2}(2m)(13.4)^2 + \frac{1}{2}(m)(52.8)^2 = 1573.48m$$

$\frac{\sum KE_f}{\sum KE_i} = \frac{1573.48m}{1672.81m} = 0.94062 \Rightarrow 94.1\%$ of the Kinetic Energy remains.



Mr. Becke's Point:

With the mass of the cart in base SI units of kilograms: $m = 517g \times \dfrac{1kg}{1000kg} = 0.517kg$

When we substitute that into the equation I gave for kinetic energy initial, we get strange units which are not joules:

$$\sum KE_i = 1672.81m = (1672.81)(0.517) = 864.84J \Rightarrow \left(1672.81\frac{cm^2}{s^2}\right)(0.517kg)$$

Remember joules are $J = N \cdot m = \left(\dfrac{kg \cdot m}{s^2}\right)(m) = \dfrac{kg \cdot m^2}{s^2}$

Mr.p points out it does not matter in this particular case because the dimensions cancel out:

$$\frac{\sum KE_f}{\sum KE_i} = \frac{1573.48m}{1672.81m} \Rightarrow \frac{\cancel{kg \cdot cm^2 / s^2}}{\cancel{kg \cdot cm^2 / s^2}} \Rightarrow \%$$

However, Mr. Becke is correct that it is better to get in to the habit of converting to base SI units when dealing with energy.

Flipping Physics Lecture Notes:

Demonstrating Impulse is Area Under the Curve

Previously we derived Impulse from the force of impact equation:

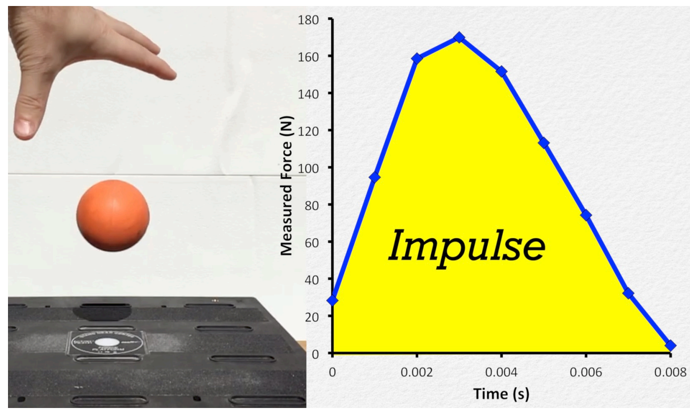
$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \Rightarrow \Delta \vec{p} = \sum \vec{F} \Delta t = Impulse$$

Now we need to do something similar, only using calculus:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \sum \vec{F} dt \Rightarrow \int_{p_i}^{p_f} d\vec{p} = \int_{t_i}^{t_f} \sum \vec{F}\, dt \Rightarrow \Delta \vec{p} = \int_{t_i}^{t_f} \sum \vec{F}\, dt = Impulse$$

And the integral is the "Area Under the Curve".

In other words, if we drop a ball onto a force plate, we get a force curve that looks like this:



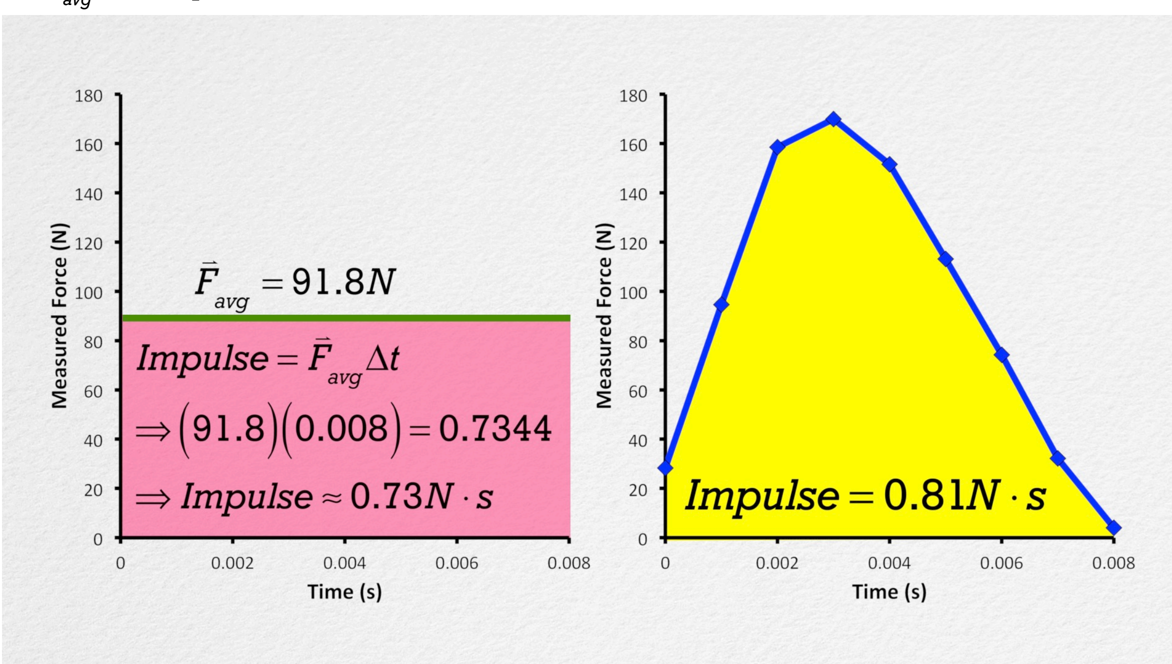
On a Force vs. Time graph, the area between the curve and the time axis is Impulse. In this particular case the impulse is 0.81 N·s

Note the force changes as a function of time. In an algebra based physics class like this one, we use the average force and the change in time to create a rectangle with the same area as under the curve.

$$Impulse = \vec{F}_{avg} \Delta t = (91.8)(0.008) = 0.7344 \approx 0.73 N \cdot s$$

Note: The two values for the Impulse, Area under Curve and $F_{average}$ times Δt, should be the same. However, the PASCO Force Platform does not quite show that correctly.

$\Delta \vec{p} = \vec{F}_{avg} \Delta t = Impulse$ (area under the force vs. time curve)



Flipping Physics Lecture Notes:

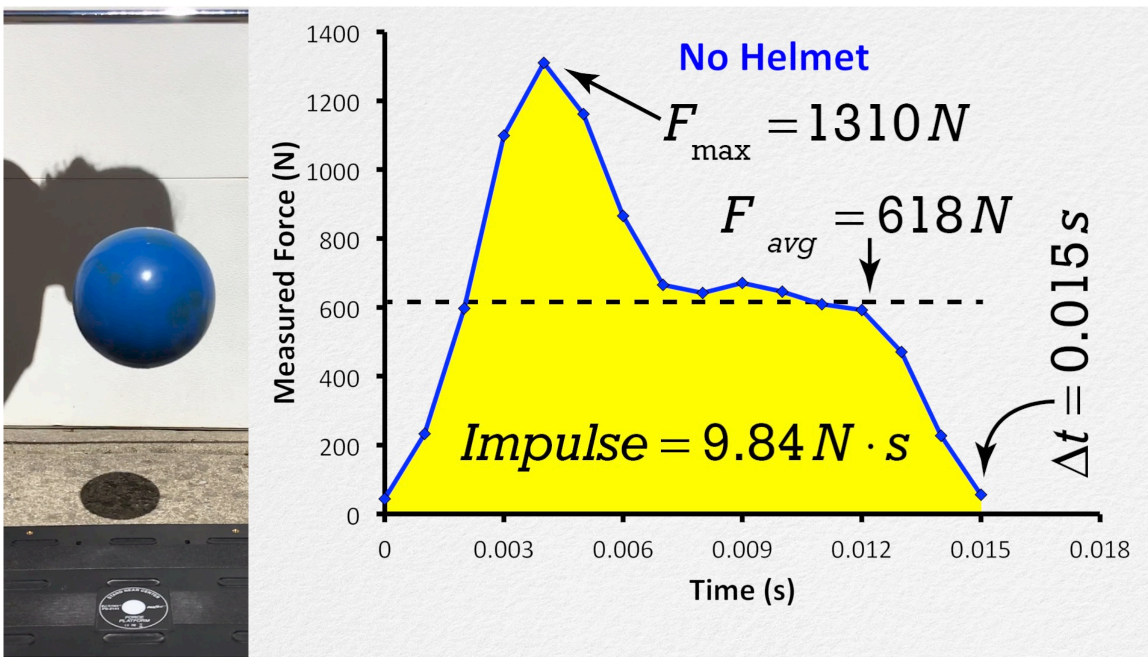
Demonstrating How Helmets Affect Impulse and Impact Force

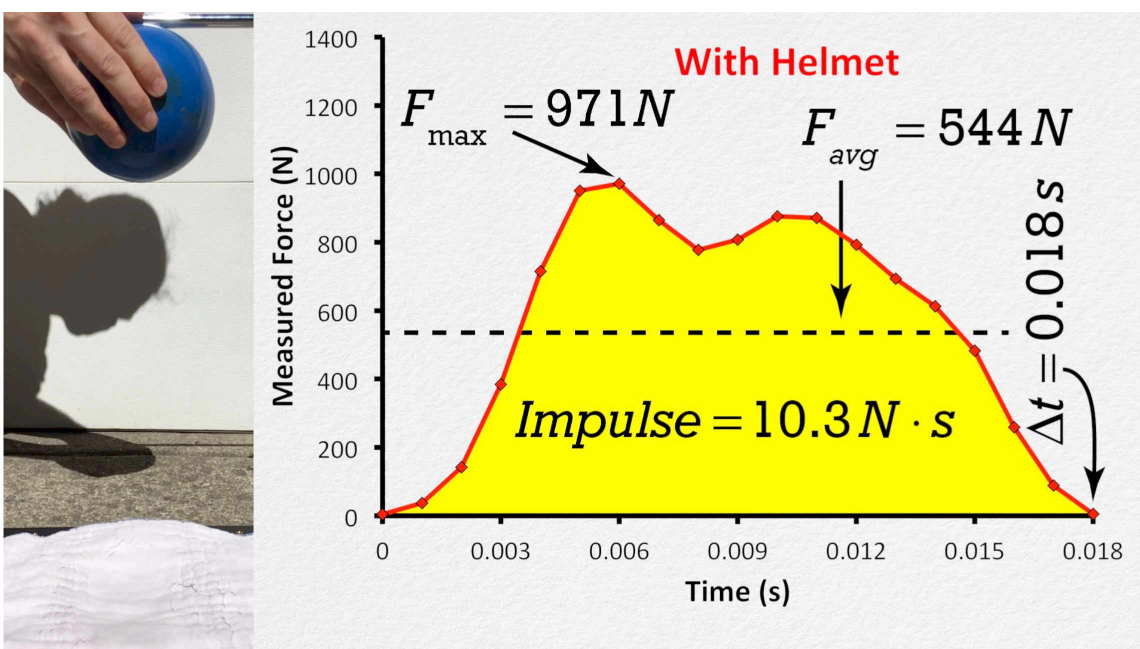
A medicine ball is dropped on to a force platform twice from the same height.
(a) Without a "helmet" and (b) with a "helmet".

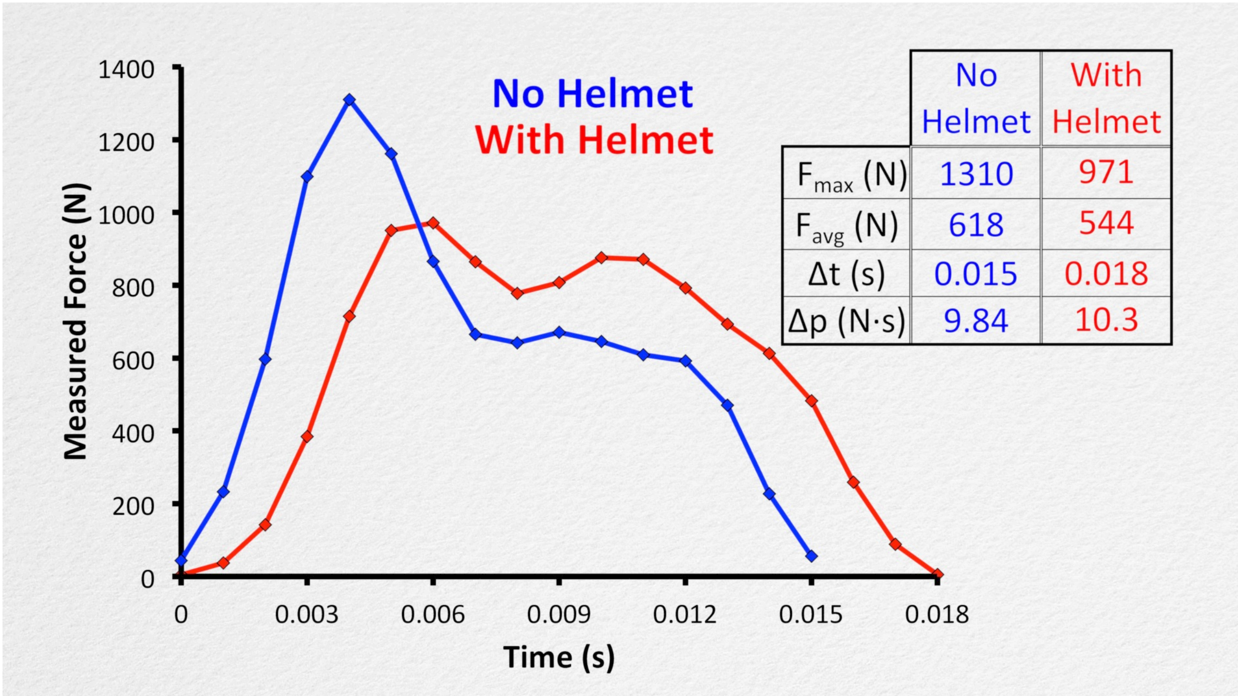
FYI: The "helmet" is a cloth diaper, which serves the exact same function as a helmet for our medicine ball.

$$\Delta\vec{p} = \vec{F}_{avg}\Delta t = Impulse$$

Remember, a helmet will increase the time during the collision to decrease the average force during the collision, however, it should not affect Impulse.






No Helmet
With Helmet
Measured Force (N)
Time (s)
1400
1200
1000
800
600
400
200
0
0
0.003
0.006
0.009
0.012
0.015
0.018
No Helmet
With Helmet
F_{max} (N) 1310 971
F_{avg} (N) 618 544
Δt (s) 0.015 0.018
Δp (N·s) 9.84 10.3

Flipping Physics Lecture Notes:

Review of Momentum, Impact Force, and Impulse

Conservation of Momentum: $\sum \vec{p}_i = \sum \vec{p}_f$

- Remember to write out the full equation before you use it.
  - $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$
- Momentum is conserved when all forces are internal.
  - In other words, during all collisions and explosions.
    - An explosion is a collision moving backwards in time.
- A minimum of two objects in this equation!

Force of Impact: $\sum \vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$

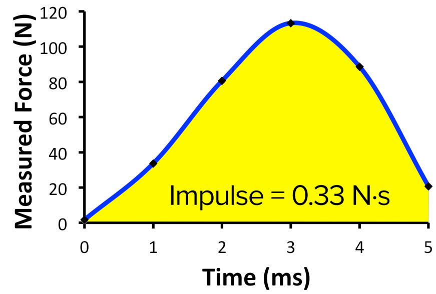
- Clearly we use this equation when we are solving for the force of impact during a collision.
- This equation only deals with the force acting on 1 object!

Impulse: $\sum \vec{F} = \frac{\Delta\vec{p}}{\Delta t} \Rightarrow Impulse = \Delta\vec{p} = \sum \vec{F}_{avg}\Delta t$

- Impulse is the area under the curve.
- Again, this equation only deals with the impulse acting on 1 object!
- Impulse equals three things: $\Delta\vec{p}$ and $\sum \vec{F}_{avg}\Delta t$ and Area under the Force vs. Time curve.

Flipping Physics Lecture Notes:
Using Impulse to Calculate Initial Height

Example Problem: A 66 g beanbag is dropped and stops upon impact with the ground. If the impulse measured during the collision is 0.33 N·s, from what height above the ground was the beanbag dropped?



It is important to recognize there are two parts to this problem:

- Part 1, when the beanbag is in free fall.
- Part 2, when the beanbag is colliding with the ground.

Knowns: $mass = 66g \times \frac{1kg}{1000g} = 0.066kg;\ Impulse = 0.33N \cdot s;\ \vec{v}_{2f} = 0;\ \vec{v}_{1f} = \vec{v}_{2i};\ h_{1i} = ?$

Part 2: $Impulse_2 = \Delta\vec{p}_2 = m\vec{v}_{2f} - m\vec{v}_{2i} = 0 - m\vec{v}_{2i} \Rightarrow \vec{v}_{2i} = -\frac{Impulse_2}{m} = -\frac{0.33}{0.066} = -5\frac{m}{s} = \vec{v}_{1f}$

Part 1: Use conservation of mechanical energy. Set the initial point where the beanbag is dropped, the final point where the beanbag strikes the ground and the zero line at the final point.

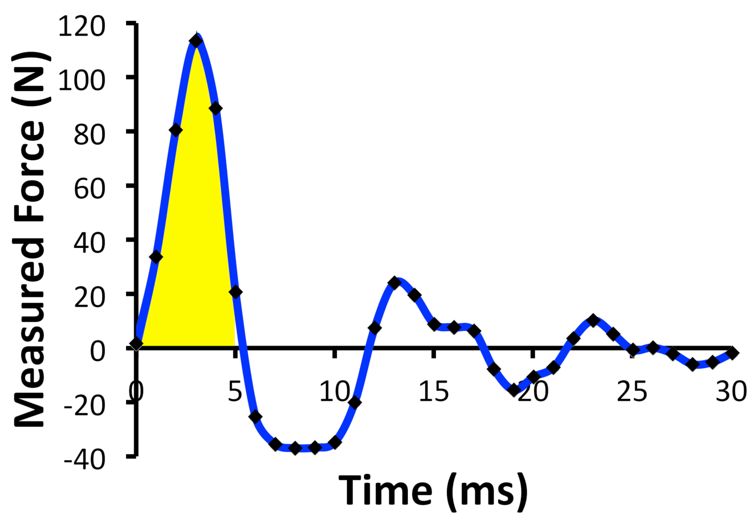
$$ME_{1i} = ME_{1f} \Rightarrow mgh_{1i} = \frac{1}{2}mv_{1f}^{\ 2} \Rightarrow gh_{1i} = \frac{1}{2}v_{1f}^{\ 2} \Rightarrow h_{1i} = \frac{v_{1f}^{\ 2}}{2g} = \frac{(-5)^2}{(2)(9.81)} = 1.27421 \approx \boxed{1.3m}$$

But the actual measured height was 0.50 m.

Therefore our prediction is way, way off.

$$E_r = \frac{O-A}{A} \times 100 = \frac{1.27421 - 0.50}{0.50} \times 100 = 154.842 \approx 150\%$$

We can understand this error if we look at the data from the force sensor *after* the collision. You can see the force measured has a damped oscillation around zero. It goes negative, then positive and continues that pattern, lessening in magnitude each time until it settles down to zero.



A negative force measurement on the force platform makes no sense because the beanbag does not pull upward on the force platform. My best guess is the collision between the beanbag and the force platform causes the force platform itself to enter into simple harmonic motion and therefore causes the force platform to register a larger impulse than it should. I don't think the force platform is intended for such dynamic measurements; it is instead intended for more static measurements.

Flipping Physics Lecture Notes:

Impulse Comparison of Three Different Demonstrations

Example Problem: A racquetball is dropped on to three different substances from the same height above each: water, soil, and wood. Rank the _______ during the collision with each substance in order from least to most. (a) Impulse. (b) Average Force of Impact.
(Assume the racquetball stops during the collision with the water and soil.)

Let's split the demonstrations up in to parts:

Part 1 is the free fall portion. Because the racquetball is dropped from the same height in all three examples, the velocity at the end of part 1 is the same for all three substances.

Part 2 is the collision. The initial velocity for part 2 is the final velocity for part 1 so all three substances have the same initial velocity for part 2. We are assuming the racquetball stops after colliding with the water and soil, therefore the velocity for part 2 final for each is zero. However, after colliding with the wood, the ball rebounds to about 2/3rds its original height, therefore the racquetball has a positive final velocity for part 2.

	Water	Soil	Wood
$\vec{v}_{1f} = \vec{v}_{2i}$	Same	Same	Same
$\vec{v}_{2f}$	0	0	Positive
$m_{racquetball}$	Same	Same	Same

The mass of the racquetball is the same for all three substances.

Part (a) for the water and the soil: $Impulse_2 = \Delta\vec{p}_2 = m\vec{v}_{2f} - m\vec{v}_{2i} = m(0) - m\vec{v}_{2i} = -m\vec{v}_{2i}$
So the impulse for the water and the soil is the same.
Note: This impulse is actually positive because the velocity for part 2 initial is down and therefore negative, which makes the impulse for the collisions with both the water and the soil, positive.
For the wood $Impulse_2 = m\vec{v}_{2f} - m\vec{v}_{2i}$ and we know velocity for part 2 final is positive, so the impulse for the wood is greater than the impulse for the water and the soil.
Answer: $Impulse_{water} = Impulse_{soil} < Impulse_{wood}$

Part (b) because they both have the same impulse, comparing force of impact for water and soil is rather straightforward. We know impulse equals the average force of impact multiplied by the change in time during the collision. From the video, you can see the time of impact during the collision with the water is much greater than the time of impact with the soil, therefore the average force of impact during the collision with the water must be less than the average force of impact with the soil.

$Impulse = \vec{F}_{avg}\Delta t$: Impulse is the same, $\Delta t_{water} > \Delta t_{soil} \Rightarrow \vec{F}_{water} < \vec{F}_{soil}$

We already know $Impulse_{soil} < Impulse_{wood}$, however, in order to compare the average force of impact between the soil and the wood, we need to be able to compare the change in time during each collision. Both of those collisions appear to last for roughly 1 frame. Therefore we can *estimate* that the time during the collision is the same. Because the impulse for the soil is less than the impulse for the wood and the two changes in time are the same, then the force of impact for the soil must be less than the force of impact for the wood. Answer: $\vec{F}_{water} < \vec{F}_{soil} < \vec{F}_{wood}$

Flipping Physics Lecture Notes:

Review of Mechanical Energy and Momentum Equations and When To Use Them!

Mechanical Energy Equations:

- Conservation of Mechanical Energy: $ME_i = ME_f$
  - Use when $W_{friction} = 0$ & $W_{F_a} = 0$
- Work due to friction equation: $W_{friction} = \Delta ME$
  - Use when $W_{friction} \neq 0$ & $W_{F_a} = 0$
- Net Work and Kinetic Energy equation: $W_{net} = \Delta KE$
  - This equation is *always true!*
  - Do not confuse with $W_{friction} = \Delta ME$ even though they look so similar.
- Whenever you use these equations you must first identify: Initial Point, Final Point, and Zero Line

The following is from my video "Review of Momentum, Impact Force, and Impulse". flippingphysics.com/impulse-review.html

Conservation of Momentum: $\sum \vec{p}_i = \sum \vec{p}_f$

- Remember to write out the full equation before you use it.
  - $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$
- Momentum is conserved when all forces are internal.
  - In other words, during all collisions and explosions.
    - An explosion is a collision moving backwards in time.
- A minimum of two objects in this equation!

Force of Impact: $\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$

- Clearly we use this equation when we are solving for the force of impact during a collision.
- This equation only deals with the force acting on 1 object!

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- Impulse is the area under the curve.
- Again, this equation only deals with the impulse acting on 1 object!
- Impulse equals three things: $\Delta \vec{p}$ and $\sum \vec{F}_{avg} \Delta t$ and Area under the Force vs. Time curve.

Three important additions:

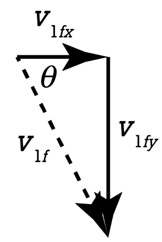
1. Students often tell me the work due to friction needs to be zero for Conservation of Momentum to be true. This is not correct and is probably because they confuse Conservation of Momentum with Conservation of Energy. Conservation of Momentum is true when all the forces are internal or balanced. We translate that to mean during all collisions and explosions.
2. You do not need to identify initial and final points because they are always assumed as:
   a. Initial point is right before the collision/explosion.
   b. Final point is right after the collision/explosion.
3. Impulse and Impact force both start with the letter "I" and often get confused by students.
   a. Don't let this happen to you!

Flipping Physics Lecture Notes:

2D Conservation of Momentum Example using Air Hockey Discs

Example: A 28.8 g yellow air hockey disc elastically strikes a 26.9 g stationary red air hockey disc. If the velocity of the yellow disc before the collision is 33.6 cm/s in the x direction and after the collision it is 10.7 cm/s at an angle 63.4° S of E, what is the velocity of the red disc after the collision?

Knowns: $m_1 = 28.8g;\ m_2 = 26.9g;\ \vec{v}_{2i} = 0;\ \vec{v}_{1iy} = 0;\ \vec{v}_{1ix} = 33.6\frac{cm}{s};\ \vec{v}_{1f} = 10.7\frac{cm}{s} @ 63.4° S\ of\ E;\ \vec{v}_{2f} = ?$



Remember momentum is a vector so we need to break velocities into components in the x & y directions:

$$\cos \ = \frac{A}{H} = \frac{v_{1fx}}{v_{1f}} \Rightarrow v_{1fx} = v_{1f}\cos \ = (10.7)\cos(63.4) = 4.7910\frac{cm}{s}$$

$$\sin\theta = \frac{O}{H} = \frac{v_{1fy}}{v_{1f}} \Rightarrow v_{1fy} = v_{1f}\sin\theta = (10.7)\sin(63.4) = -9.5675\frac{cm}{s}$$ (negative because it is South)

Now we can use conservation of momentum in both the x and y directions:

x-direction: $\sum \vec{p}_{ix} = \sum \vec{p}_{fx} \Rightarrow m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \Rightarrow m_1 v_{1ix} = m_1 v_{1fx} + m_2 v_{2fx}$

(because $v_{2ix} = 0$)

$$\Rightarrow m_1 v_{1ix} - m_1 v_{1fx} = m_2 v_{2fx} \Rightarrow v_{2fx} = \frac{m_1 v_{1ix} - m_1 v_{1fx}}{m_2} = \frac{(28.8)(33.6) - (28.8)(4.7910)}{26.9}$$

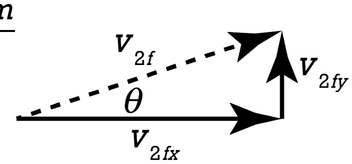
$$\Rightarrow v_{2fx} = 30.8438\frac{cm}{s}$$

note: $v_{2fx} \Rightarrow \frac{(g)\left(\frac{cm}{s}\right) - (g)\left(\frac{cm}{s}\right)}{g} \Rightarrow \frac{cm}{s}$

y-direction: $\sum \vec{p}_{iy} = \sum \vec{p}_{fy} \Rightarrow m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy} \Rightarrow 0 = m_1 v_{1fy} + m_2 v_{2fy}$

(because $v_{1iy} = v_{2iy} = 0$)

$$\Rightarrow -m_1 v_{1fy} = m_2 v_{2fy} \Rightarrow v_{2fy} = -\frac{m_1 v_{1fy}}{m_2} = -\frac{(28.8)(-9.5675)}{26.9} = 10.2432\frac{cm}{s}$$



$$a^2 + b^2 = c^2 \Rightarrow {v_{2f}}^2 = {v_{2fx}}^2 + {v_{2fy}}^2 \Rightarrow v_{2f} = \sqrt{{v_{2fx}}^2 + {v_{2fy}}^2} = \sqrt{30.8438^2 + 10.2432^2} = 32.500 \approx 32.5\frac{cm}{s}$$

$$\tan\theta_{2f} = \frac{O}{A} = \frac{v_{2fy}}{v_{2fx}} \Rightarrow \theta_{2f} = \tan^{-1}\left(\frac{v_{2fy}}{v_{2fx}}\right) = \tan^{-1}\left(\frac{10.2432}{30.8438}\right) = 18.3713 \approx 18.4°$$

$\vec{v}_{2f} \approx 32.5\frac{cm}{s} @ 18.4° N\ of\ E$ (predicted)

Measured final velocity of the red air hockey disc is $\Rightarrow \vec{v}_{2f} \approx 32.0\dfrac{cm}{s} @ 13.0^\circ N\ of\ E$ (pretty close, eh!)

We consider this an elastic collision, so was kinetic energy conserved?
First, in order to work with energy, convert everything to base SI units!

$$m_1 = 28.8g \times \frac{1kg}{1000g} = 0.0288kg;\ m_2 = 26.9g \times \frac{1kg}{1000g} = 0.0269kg$$

$$v_{1i} = 33.6\frac{cm}{s} \times \frac{1m}{100cm} = 0.336\frac{m}{s};\ v_{1f} = 10.7\frac{cm}{s} \times \frac{1m}{100cm} = 0.107\frac{m}{s};\ v_{2f} = 32.5\frac{cm}{s} \times \frac{1m}{100cm} = 0.325\frac{m}{s}$$

$$\sum KE_i = \frac{1}{2}m_1 v_{1i}^{\ 2} + \frac{1}{2}m_2 v_{2i}^{\ 2} = \frac{1}{2}\left(0.0288\right)\left(0.336\right)^2 = 0.0016257J$$ (because $v_{2i} = 0$)

$$\sum KE_i = 0.0016257J \times \frac{1000mJ}{1J} = 1.6257mJ$$

$$\sum KE_f = \frac{1}{2}m_1 v_{1f}^{\ 2} + \frac{1}{2}m_2 v_{2f}^{\ 2} = \frac{1}{2}\left(0.0288\right)\left(0.107\right)^2 + \frac{1}{2}\left(0.0269\right)\left(0.320\right)^2 = 0.0015421J$$

$$\sum KE_f = 0.0015421J \times \frac{1000mJ}{1J} = 1.5421mJ$$

$$E_r = \frac{O-A}{A} \times 100 = \frac{1.5421 - 1.6257}{1.6257} \times 100 = -5.1397 \approx -5.14\%$$

In other words, 5.14% of the kinetic energy was converted to heat and sound during the “elastic” collision.