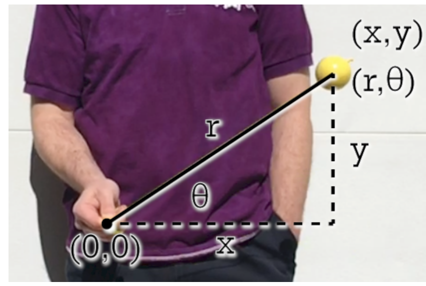
Flipping Physics Lecture Notes:

Introduction to Circular Motion and Arc Length

Circular Motion simply takes what you have learned before and applies it to objects which are moving along a circular path. Let's begin with a drawing of an object which is moving along a circle with a constant radius. We have paused the object at one point in time to discuss how we identify the location of an object which is moving along a circle.

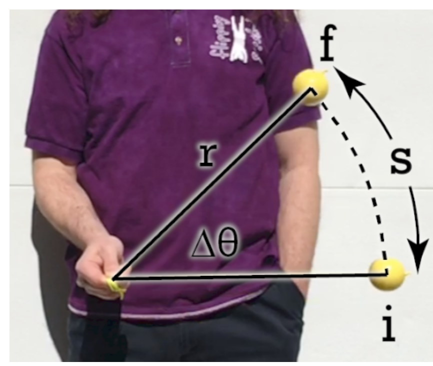


The x-y coordinate system which locates an object in two or three dimensional space relative to an origin was introduced by René Descartes in the 1600s. When we identify the location of an object which is moving along a circle using Cartesian coordinates, notice that both the x and y position values of the object change as a function of time. We can also identify the objects location using polar coordinates which use the radius and angular position to identify the location of the object.

Notice when we use radius and angular position to identify the location of an object moving along a circle, the angular position changes, however, the radius stays constant. Having only one variable change as a function of time while describing the location of an object is much easier to work with than when two variables change.

We can relate Cartesian and polar coordinates using trig functions:

$$\sin\theta = \frac{O}{H} = \frac{y}{r} \Rightarrow y = r\sin\theta \ \ \& \ \ \cos\theta = \frac{A}{H} = \frac{x}{r} \Rightarrow x = r\cos\theta$$



Now let's discuss how we describe an object moving from one location to another when it is moving along a circle with a constant radius. In the drawing the object has moved from the initial position to the final position while moving through an angular displacement:

$$\Delta\theta = \theta_f - \theta_i$$

Arc Length:

- The linear distance traveled when an object is moving along a curve.
- The symbol for arc length is s.
- The equation for arc length is $s = r\Delta\theta$
- You must use radians for the angular displacement in the arc length equation.
  - We will demonstrate why soon in a future video.
- The arc length when an object moves through a full circle is called Circumference.
  - The angular displacement when an object moves through a full circle is 2π radians.
  - The equation for circumference is $C = 2\pi r = r(2\pi)$, which you can see is a special case of the equation for arc length when the angular displacement is one revolution.

Flipping Physics Lecture Notes:

Defining Pi for Physics

Common student answers to the question, “What is π?”

- A number
- 3
- 3.1
- 3.14
- ~3.141592653589793238462643383279502884197169399375105820974944592307816406286
- An irrational number
- Something good to eat

By definition pi is the ratio of a circle’s circumference to its diameter:

- $\pi = \frac{C}{D} = 3.14159...$
- Which we can rearrange $\Rightarrow C = \pi D = \pi(2r) \Rightarrow C = 2\pi r$ to get the equation for circumference
- The equation for circumference is just a restatement of the definition of π

Frisbee example: $\pi = \frac{C}{D} = \frac{86.9cm}{27.5cm} = 3.16 \approx 3.14159...$

The units for π are …

- $\pi = \frac{C}{D} \Rightarrow \frac{meters}{meters} = 1$
    - In other words π has no units, it is dimensionless
    - We give this ratio a specific name, it is called *radians*
- $\frac{C}{D} = \pi\ radians$
- π is in radians and radians are dimensionless.
- π radians represent the ratio of the circumference to the diameter of every circle.
- Radians are a placeholder and we will use this fact repeatedly in physics.

$1\ revolution = 360° = 2\pi\ radians$

- Know this!!
- Note: $1\ revolution \neq 2\ radians$
    - For some reason students often simply leave the π out, don’t be *that* student.

Abbreviations:

- r = radius
- rad = radians
    - do ***NOT*** use r for radians, r is for radius, rad is for radians.
    - $s = r\Delta\theta = (1.5m)(2\pi r)$ leads to r confusion, $s = r\Delta\theta = (1.5m)(2\pi\,\mathrm{rad})$ does not.

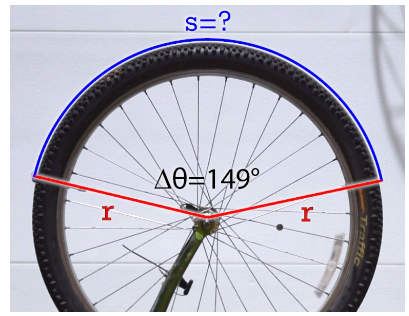
Flipping Physics Lecture Notes:

Introductory Arc Length Problem
Gum on a Bike Tire

Example Problem: How far does a piece of gum stuck to the outside of a 67 cm diameter wheel travel while the wheel rotates through 149°?

Knowns: $D = 67cm;\ r = \frac{D}{2} = \frac{67cm}{2} = 33.5cm;\ \Delta\theta = 149°\left(\frac{2\pi\ radians}{360°}\right) = 2.6005\ radians;\ s = ?$

Suggestion: Whenever the diameter is given in a physics problem, immediately determine the radius as well. Too often I have seen students use the diameter as the radius.



$$s = r\Delta\theta = (33.5)(2.6005) = 87.118 \approx \boxed{87cm}$$

Units: $s = r\Delta\theta \Rightarrow cm \bullet rad = cm$

Radians have no units and are just a placeholder. The radians drop out because we no longer need them as a placeholder. If we had left the angular displacement in degrees, the units for arc length would work out to be in $cm \bullet °$ which makes no sense.

Flipping Physics Lecture Notes:

Angular Velocity Introduction

The equation for average *linear* velocity is: $\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t}$

- Average *linear* velocity equals change in *linear* position over change in time.

Therefore the equation for average *angular* velocity is: $\vec{\omega}_{avg} = \frac{\Delta \vec{\theta}}{\Delta t}$

- Average *angular* velocity equals change in *angular* position over change in time.
- The symbol for angular velocity is the lowercase Greek letter omega, ω .
- The two most common units for angular velocity are:
    - $\frac{rad}{s}$ used most often physics.
    - $\frac{revolutions}{minunte} = \frac{rev}{min} = rpm$ used most often in the “real world”.

Note: For quite a while we will not discuss the direction of angular quantities like angular displacement and angular velocity. In my experience it is much easier for students to get to know the angular variables without direction first and then introduce direction. Have patience, direction will come soon enough.

- For those of you who are not satisfied with that, understand that clockwise and counterclockwise are observer dependent and therefore we will be using the Right Hand Rule instead of clockwise and counterclockwise, but not yet.

Examples of objects with angular velocity:

Flipping Physics Lecture Notes:

Introductory Angular Velocity Problem
A Turning Bike Tire

Example: The wheel of a bike rotates exactly 3 times in 12.2 seconds. What is the average angular velocity of the wheel in (a) radians per second and (b) revolutions per minute?

Knowns: $\Delta\theta = \text{"exactly"}\,3\,rev;\ \Delta t = 12.2\,\text{sec};\ \omega_{avg} = ?\ (a)\left(\frac{rad}{s}\right) \,\&\, (b)\left(\frac{rev}{\text{min}}\right)$

Note: Unfortunately the word "exactly" is sometimes used in physics problems and it means the number referred to has an infinite number of significant digits. Hopefully you recognize this is impossible.

(a) $\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{3\,rev}{12.2\,sec} = 0.24590\frac{rev}{s}\left(\frac{2\pi\,rad}{1\,rev}\right) = 1.54505 \approx \boxed{1.55\frac{rad}{s}}$

(b) $\omega_{avg} = 1.54505\frac{rad}{s}\left(\frac{60\,s}{1\,\text{min}}\right)\left(\frac{1\,rev}{2\pi\,rad}\right) = 14.7541 \approx \boxed{14.8\frac{rev}{\text{min}}}$

Typical mistakes with this conversion:

1) Forget to include the parenthesis around 2π in your calculator and therefore actually divide by two and then multiply by π results in 145.617 which is not correct.
2) Forget to include the number π in your calculations which results in 46.351 which is also not correct.
3) Add π to your answer even though you typed it in to your calculator and therefore already used its value results in 14.8 π which is, you guessed it, also not correct.

Flipping Physics Lecture Notes:

Angular Acceleration Introduction

The equation for average *linear* acceleration is: $\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$

- Average *linear* acceleration equals change in *linear* velocity over change in time.

The equation for average *angular* acceleration is: $\vec{\alpha}_{avg} = \frac{\Delta \vec{\omega}}{\Delta t}$

- Average *angular* acceleration equals change in *angular* velocity over change in time.
- The symbol for angular acceleration is the lowercase Greek letter alpha, α.
    - We often call it "fishy thing" instead.
- Units for angular acceleration are: $\frac{rad}{s^2}$
- Just like I mentioned when learning about angular velocity, we are not going to discuss direction until later.

Flipping Physics Lecture Notes:

Angular Accelerations of a Record Player

Example Problem: A record player is plugged in, uniformly accelerates to 45 revolutions per minute, and then is unplugged. The record player (a) takes 0.85 seconds to get up to speed, (b) spends 3.37 seconds at 45 rpms, and then (c) takes 2.32 seconds to slow down to a stop. What is the average angular acceleration of the record player during all three parts?

Part (a) knowns: $\omega_{ai} = 0; \omega_{af} = 45\frac{rev}{\min}\left(\frac{1\min}{60\sec}\right)\left(\frac{2\pi rad}{1rev}\right) = 1.5\pi\frac{rad}{\sec}; \Delta t_a = 0.85\sec; \alpha_a = ?$

$$\alpha_a = \frac{\Delta\omega_a}{\Delta t_a} = \frac{\omega_{af} - \omega_{ai}}{\Delta t_a} = \frac{1.5\pi - 0}{0.85} = 5.54399 \approx \boxed{5.5\frac{rad}{s^2}}$$

Part (b) knowns: $\omega_b = constant \Rightarrow \Delta\omega_b = 0$

$$\alpha_b = \frac{\Delta\omega_b}{\Delta t_b} = \frac{0}{\Delta t_b} = \boxed{0}$$

Part (c) knowns: $\omega_{ci} = 1.5\pi\frac{rad}{\sec}; \omega_{cf} = 0; \Delta t_c = 2.32\sec; \alpha_c = ?$

$$\alpha_c = \frac{\Delta\omega_c}{\Delta t_c} = \frac{\omega_{cf} - \omega_{ci}}{\Delta t_c} = \frac{0 - 1.5\pi}{2.32} = -2.03120 \approx \boxed{-2.0\frac{rad}{s^2}}$$

Flipping Physics Lecture Notes:

Introduction to Uniformly Angularly Accelerated Motion

Just like an object can have Uniformly Accelerated Motion or UAM and object can have Uniformly Angularly Accelerated Motion, UαM. This table compares the two:

If the following is constant…	linear acceleration, a	angular acceleration, α
we can use the ______ equations	UAM	UαM
There are 5 variables	$v_f, v_i, a, \Delta x, \Delta t$	$\omega_f, \omega_i, \alpha, \Delta\theta, \Delta t$
There are 4 equations	$v_f = v_i + a\Delta t$ $\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$ $v_f^2 = v_i^2 + 2a\Delta x$ $\Delta x = \frac{1}{2}\left(v_f + v_i\right)\Delta t$	$\omega_f = \omega_i + \alpha\Delta t$ $\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2$ $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$ $\Delta\theta = \frac{1}{2}\left(\omega_f + \omega_i\right)\Delta t$ **Use Radians!**

If we know 3 of the variables we can find the other 2. Which leaves us with 1 Happy Physics Student!

When you use the Uniformly Angularly Accelerated Motion equations please use radians for your angular quantities. Most of the time you have to use radians in the UαM equations and you always can use radians in the UαM equations. Therefore, please, always use radians in the UαM equations.

Flipping Physics Lecture Notes:

Introductory Uniformly Angularly Accelerated Motion Problem

Example Problem: What is the angular acceleration of a compact disc that turns through 3.25 revolutions while it uniformly slows to a stop in 2.27 seconds?

Knowns: $\alpha = ?;\ \Delta\theta = 3.25rev\left(\frac{2\pi rad}{1rev}\right) = 6.5\pi\,\mathrm{rad};\ \omega_f = 0;\ \Delta t = 2.27\,\mathrm{sec}$

A compact disc will slow with a constant angular acceleration so we can use the Uniformly Angularly Accelerated Motion (UαM) equations. There is no UαM equation that has all four of our known variables in it, so we first need to solve for angular velocity initial.

$$\Delta\theta = \frac{1}{2}\left(\omega_f + \omega_i\right)\Delta t \Rightarrow 6.5\pi = \frac{1}{2}\left(0 + \omega_i\right)2.27 \Rightarrow \omega_i = \frac{\left(2\right)\left(6.5\pi\right)}{2.27} = 17.9915\frac{rad}{s}$$

And now that we have the initial angular velocity, we can solve for the angular acceleration.

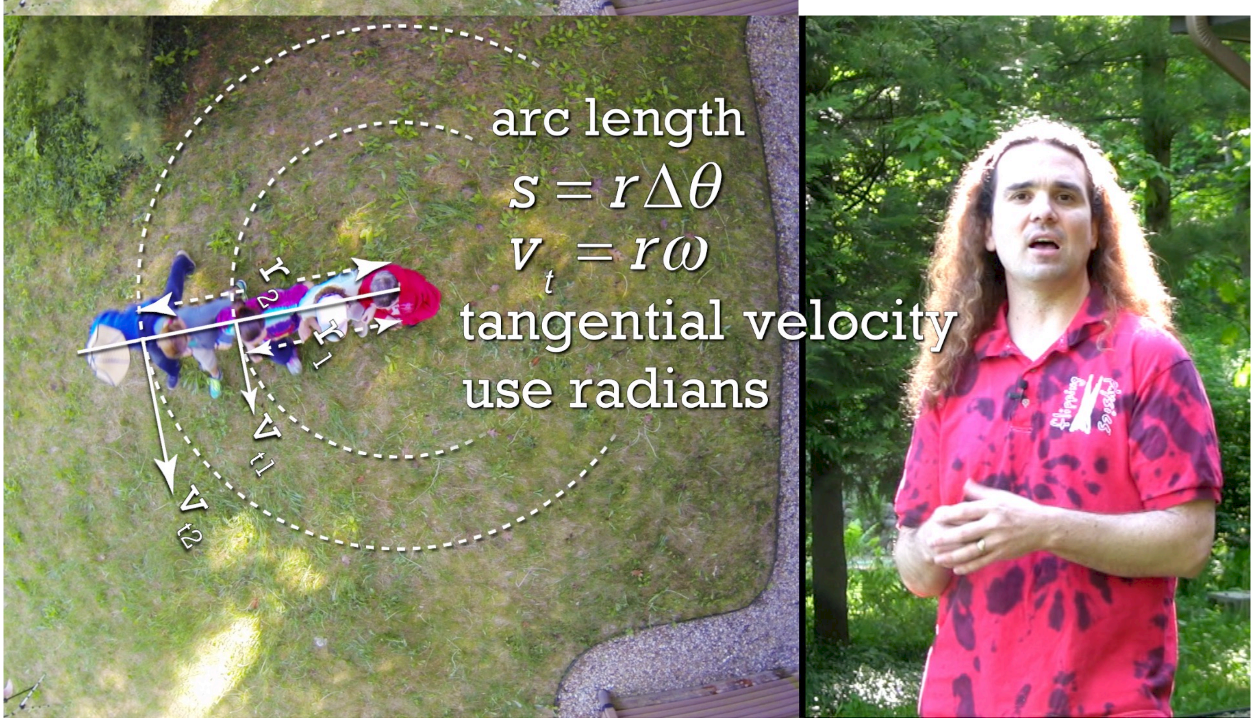
$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 17.9915}{2.27} = -7.9258 \approx \boxed{-7.93\frac{rad}{s^2}}$$

Flipping Physics Lecture Notes:

Human Tangential Velocity Demonstration

In the demonstration:

- Each person has the same angular velocity.
- A larger radius means a larger arc length or linear distance travelled.
- Therefore a larger radius means a larger linear velocity called *tangential velocity*.



Flipping Physics Lecture Notes:

Introductory Tangential Velocity Problem

Example Problem: Three mints are sitting 3.0 cm, 8.0 cm, and 13.0 cm from the center of a record player that is spinning at 45 revolutions per minute. What are the tangential velocities of each mint?

Knowns:

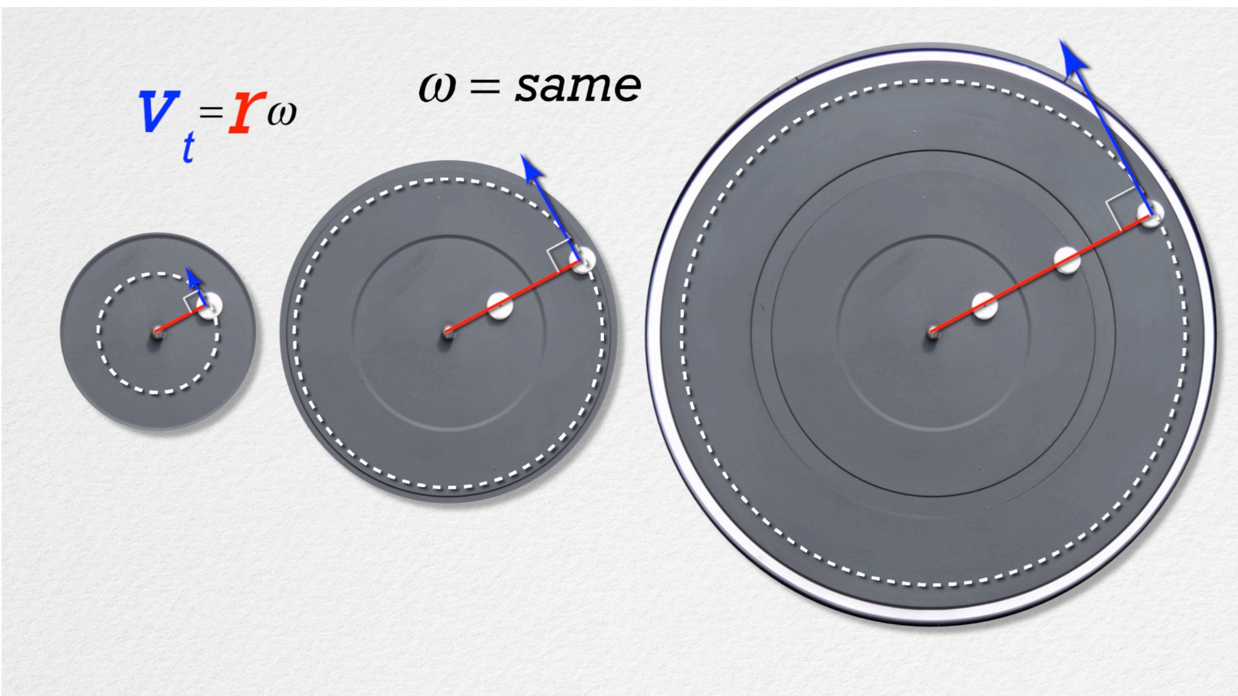
$$r_1 = 3.0cm;\ r_2 = 8.0cm;\ r_3 = 13.0cm;\ \omega = 45\frac{rev}{\min}\left(\frac{2\pi rad}{1rev}\right)\left(\frac{1\min}{60\sec}\right) = 1.5\pi\frac{rad}{s};\ v_t = ?\ (\text{each})$$

$$v_{t1} = r_1\omega = (3)(1.5\pi) = 14.1372\frac{cm\cdot rad}{s} \approx 14\frac{cm}{s}$$

$$v_{t2} = r_2\omega = (8)(1.5\pi) = 37.6991 \approx 38\frac{cm}{s}$$

$$v_{t3} = r_3\omega = (13)(1.5\pi) = 61.2611 \approx 61\frac{cm}{s}$$

The tangential velocity of an object is, by definition, tangent to the circle the object is describing. This means tangential velocity is, by definition, at a 90° angle to the radius of the circle.



Flipping Physics Lecture Notes:

Introduction to Tangential Acceleration with Record Player Example Problem

When an object is moving in a circle…

- it travels a linear distance which is called arc length.
  - $s = r\Delta\theta$
- it has a linear velocity which is called tangential velocity.
  - $v_t = r\omega$
- it has a linear acceleration which is called tangential acceleration.
  - $a_t = r\alpha$
- You must use radians in all three of these equations!

Example Problem: A record player is plugged in and uniformly accelerates to 45 revolutions per minute in 0.85 seconds. Mints are located 3.0 cm, 8.0 cm, and 13.0 cm from the center of the record. What is the magnitude of the tangential acceleration of each mint?

Knowns: $\omega_i = 0;\ \omega_f = 45\frac{rev}{\min}\left(\frac{2\pi rad}{1rev}\right)\left(\frac{1\min}{60\sec}\right) = 1.5\pi\frac{rad}{s};\ \Delta t = 0.85\sec;$

$r_1 = 3.0cm;\ r_2 = 8.0cm;\ r_3 = 13.0cm;\ a_t = ?\ (\text{each})$

First we need to solve for the angular acceleration of the record player and therefore each of the mints:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{1.5\pi - 0}{0.85} = 5.54399\frac{rad}{s^2}$$

Now we can solve for the tangential acceleration of each mint:

$$a_{t1} = r_1\alpha = (3)(5.54399) = 16.63196\frac{cm \cdot rad}{s^2} \approx \boxed{17\frac{cm}{s^2}}$$

$$a_{t2} = r_2\alpha = (8)(5.54399) = 44.35190 \approx \boxed{44\frac{cm}{s^2}}$$

$$a_{t3} = r_3\alpha = (13)(5.54399) = 72.07183 \approx \boxed{72\frac{cm}{s^2}}$$

Tangential velocity and tangential acceleration are by definition tangent to the circle through which the object is moving, that is what the word tangential means. This also means the tangential velocity and acceleration are perpendicular to the radius of the circle.

If you understand the derivative, you can see the relationship between the arc length, tangential velocity, and tangential acceleration equations:

$$s = r\Delta\theta \Rightarrow \frac{d}{dt}(s = r\Delta\theta) \Rightarrow \frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow v_t = r\omega$$

$$v_t = r\omega \Rightarrow \frac{d}{dt}(v_t = r\omega) \Rightarrow \frac{dv_t}{dt} = r\frac{d\omega}{dt} \Rightarrow a_t = r\alpha$$

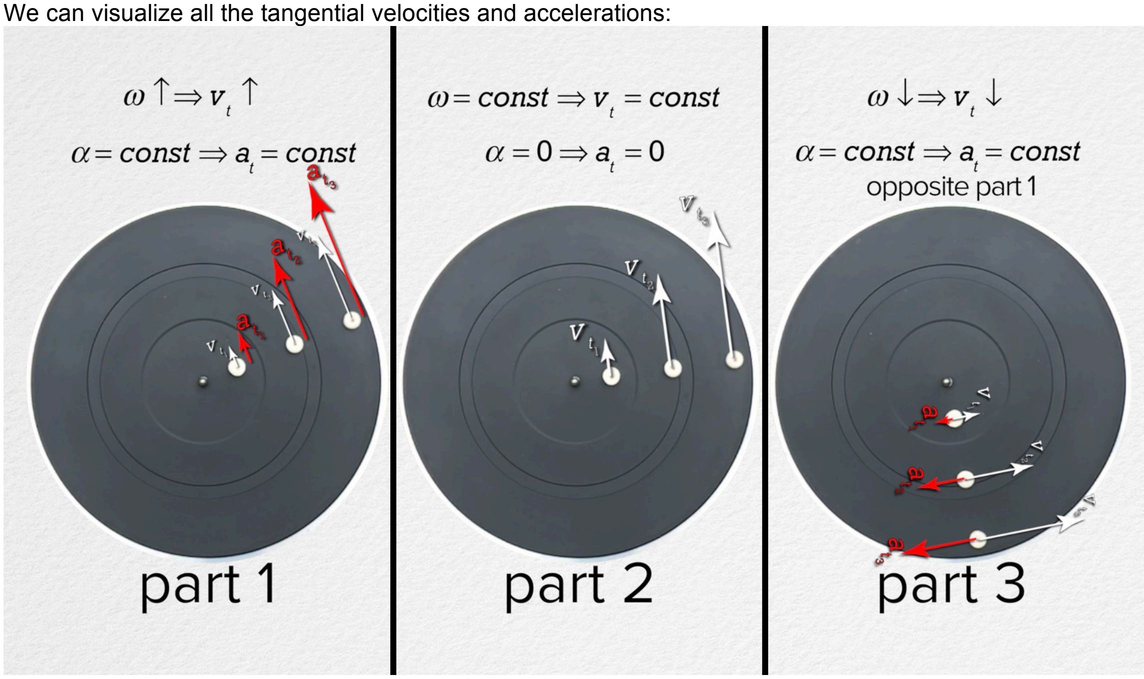
Flipping Physics Lecture Notes:

Demonstrating the Directions of Tangential Velocity and Acceleration

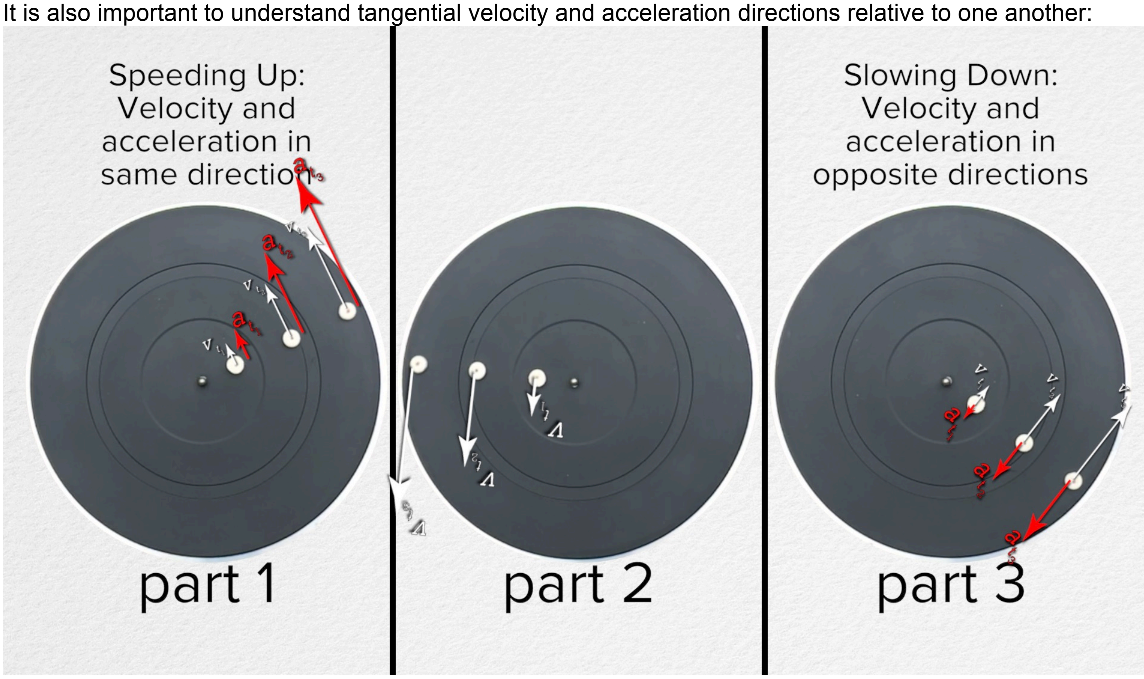
There are three different parts to the demonstration.

1. The turntable is plugged in and angularly accelerates at 4.2 rad/s^2 up to 33 rev/min in less than one second.
2. The turntable rotates at 33 rev/min for around one and a half seconds.
3. The turntable is unplugged and angularly accelerates at -1.5 rad/s^2 to a stop in slightly more than two seconds.

We can visualize all the tangential velocities and accelerations:



It is also important to understand tangential velocity and acceleration directions relative to one another:



And $\Delta\theta, \omega, \alpha$ refer to the whole object, however, s, v_t, a_t refer to a specific point on the object.

Flipping Physics Lecture Notes:

Centripetal Acceleration Introduction

When an object is rotating at a constant angular velocity, the whole object has a constant angular velocity. Therefore, every mint on the turntable has the same, constant angular velocity.

Looking at a single mint on the turntable:

- $\omega = \text{constant}$
- Because the angular velocity is constant, there is no angular acceleration.
  - $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0}{\Delta t} = 0$
- Because the angular acceleration is zero, the tangential acceleration of the mint is zero.
  - $a_t = r\alpha = r(0) = 0$
- The angular velocity of the mint is constant, however, the tangential velocity of the mint is *not* constant. Remember tangential velocity is a vector.
  - The *magnitude* of the tangential velocity of the mint *is* constant.
  - The *direction* of the tangential velocity of the mint is *not* constant.
- Because the tangential velocity of the mint is changing, the mint must have a linear acceleration.
  - $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ (If velocity is changing, there must be a linear acceleration.)
  - As shown above, this linear acceleration is not a tangential acceleration.
  - It also is not an angular acceleration.
    - Angular acceleration is angular, not linear.
    - Also, it's zero anyway.
- The acceleration which causes the tangential velocity to change direction is called Centripetal Acceleration.

Centripetal Acceleration:

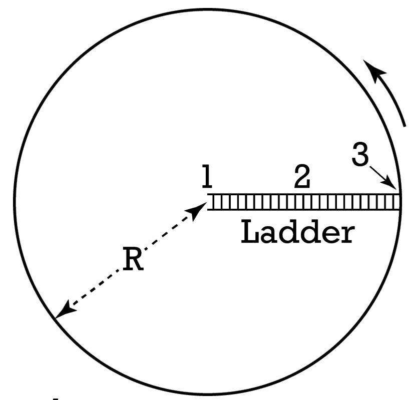
- The acceleration that causes circular motion.
- "Centripetal" means "Center Seeking".
  - Centripetal acceleration is always in toward the center of the circle.
  - Coined by Sir Isaac Newton. Combination of the Latin words "centrum" which means center and "petere" which means "to seek".
- Is a *linear* acceleration.
- $a_c = \frac{v_t^2}{r} = \frac{(r\omega)^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2 \Rightarrow a_c = \frac{v_t^2}{r} = r\omega^2$
- Base S.I. units for centripetal acceleration are $\frac{m}{s^2}$
  - $a_c = r\omega^2 \Rightarrow (m)\left(\frac{rad}{s}\right)^2 = \frac{m \bullet rad^2}{s^2} = \frac{m}{s^2}$

Flipping Physics Lecture Notes:

Introductory Centripetal Acceleration Problem: Cylindrical Space Station

Example: A cylindrical space station with a radius of 115 m is rotating at 0.292 rad/s. A ladder goes from the rim to the center. What is the magnitude of the centripetal acceleration at (1) the top of the ladder, (2) the middle of the ladder, and (3) the base of the ladder?

$$\omega = 0.292\frac{rad}{s};\ r_1 = 0m;\ r_2 = \frac{115m}{2} = 57.5m;\ r_3 = 115m;\ a_c = ?\ (each)$$



(1) $a_c = \frac{v_t^2}{r} = r\omega^2 \Rightarrow a_{c1} = r_1\omega^2 = (0)\omega^2 = 0$

(2) $a_{2c} = r_2\omega^2 = (57.5)(0.292)^2 = 4.90268\frac{m\cdot rad^2}{s^2} \approx 4.90\frac{m}{s^2}$

(3) $a_{3c} = r_3\omega^2 = (115)(0.292)^2 = 9.80536 \approx 9.81\frac{m}{s^2}$

Rim tangential velocity calculation:

$$v_{t3} = r_3\omega = (115)(0.292) = 33.58\frac{m}{s}\left(\frac{3600s}{1hr}\right)\left(\frac{1km}{1000m}\right) = 120.888\frac{km}{hr}$$

$$\Rightarrow v_{t3} = 120.888\frac{km}{hr}\left(\frac{1mile}{1.609km}\right) = 75.13238 \approx 75\frac{mi}{hr}$$

Flipping Physics Lecture Notes:

Centripetal Force Introduction and Demonstration

Newton's Second Law states: $\sum \vec{F} = m\vec{a}$

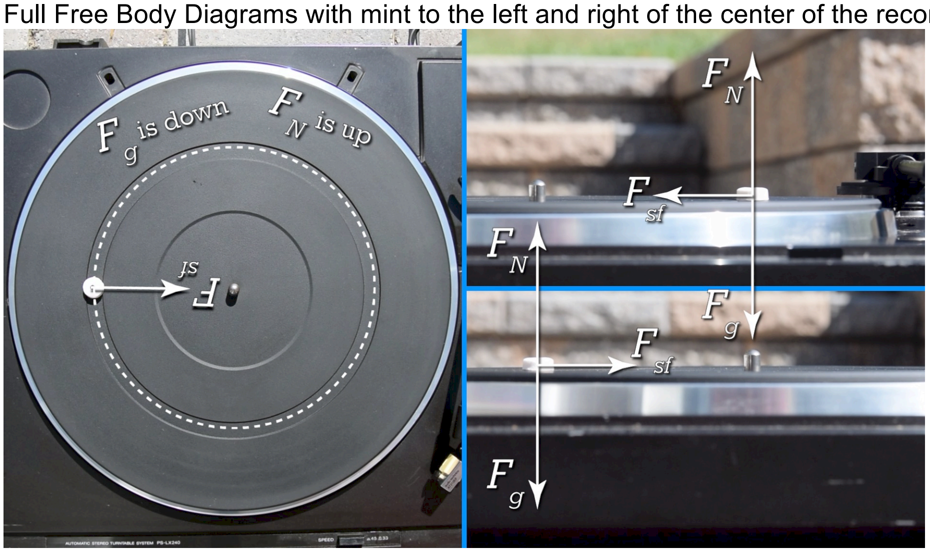
Previously we showed that an object moving along a curved path must have a centripetal acceleration that is inward. Which means: $\sum \vec{F}_{in} = m\vec{a}_c$

The net force in the in-direction or $\sum \vec{F}_{in}$ is called the Centripetal Force.

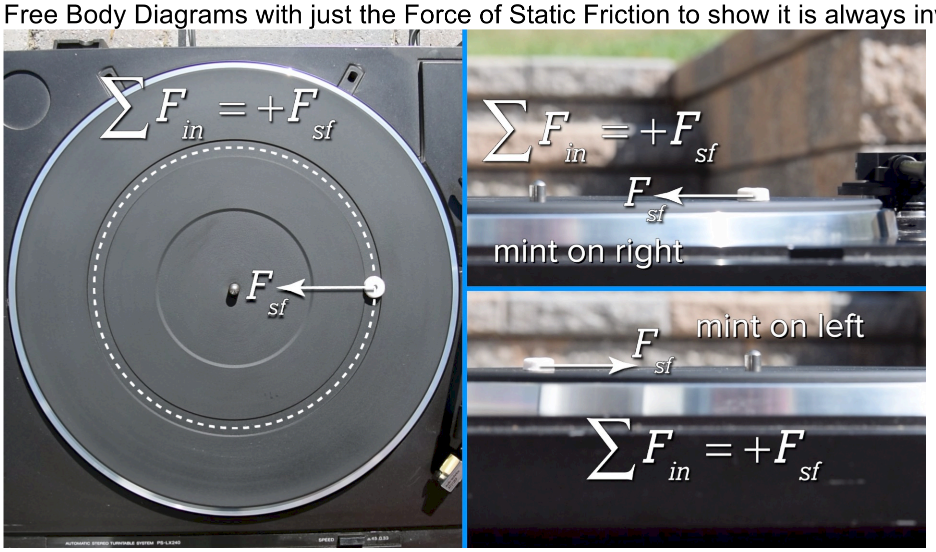
There are three things I need you to remember about the Centripetal Force:

1) Centripetal Force is not a new force.
2) Centripetal Force is never in a free body diagram.
3) When summing the forces in the in-direction, the in-direction is positive and the out-direction is negative.

Full Free Body Diagrams with mint to the left and right of the center of the record player:



Free Body Diagrams with just the Force of Static Friction to show it is always inward:

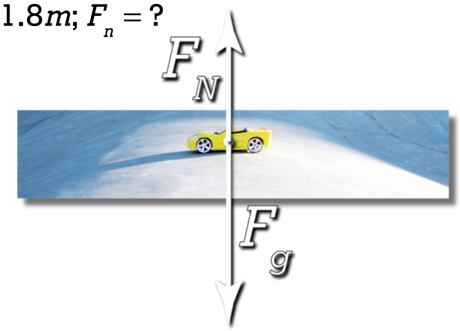


Flipping Physics Lecture Notes:

Introductory Centripetal Force Problem
Car over a Hill

Example: A 453 g toy car moving at 1.05 m/s is going over a semi-circular hill with a radius of 1.8 m. When the car is at the top of the hill, what is the magnitude of the force from the ground on the car?

Knowns: $m = 453g\left(\frac{1kg}{1000g}\right) = 0.453kg; v_t = 1.05\frac{m}{s}; r = 1.8m; F_n = ?$



$$\sum F_{in} = F_g - F_N = ma_c \Rightarrow mg - F_N = m\frac{v_t^2}{r}$$

$$\Rightarrow -F_N = -mg + m\frac{v_t^2}{r} \Rightarrow F_N = mg - m\frac{v_t^2}{r}$$

$$F_n = (0.453)(9.81) - (0.453)\left(\frac{1.05^2}{1.8}\right) = 4.1665 \approx \boxed{4.2N}$$

Note: The force causing the circular motion, the Centripetal Force, or the net force in the in-direction, in this case is the Force of Gravity minus the Force Normal. $\sum F_{in} = F_g - F_N$

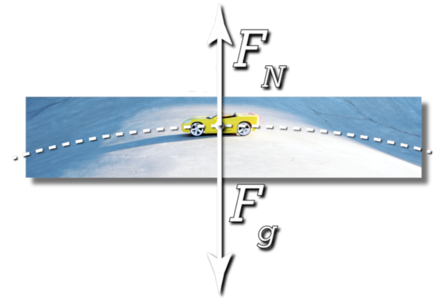
Also note: $F_g = mg = (0.453)(9.81) = 4.444 \approx 4.4N \Rightarrow F_N < F_g$

In other words, as you go over a hill in a car, you feel as if you weigh less. And the faster you move, the smaller the force normal, and the lighter you feel.

Flipping Physics Lecture Notes:

What is the Maximum Speed of a Car at the Top of a Hill?

Example: What is the maximum linear speed a car can move over the top of a semi-circular hill without its tires lifting off the ground? The radius of the hill is 1.8 meters.



The faster the car moves, the smaller the Force Normal. When we increase the speed of the car to the point where the Force Normal is zero, then the only force causing circular motion is the Force of Gravity acting on the car. Any faster and the wheels of the car will leave the road.

Knowns: $r = 1.8m; v_{t\max} = ?$

$$\sum F_{in} = F_g - F_N = ma_c \Rightarrow mg - 0 = m\frac{v_t^{\,2}}{r} \Rightarrow g = \frac{v_t^{\,2}}{r} \Rightarrow v_t^{\,2} = gr \Rightarrow v_t = \sqrt{gr}$$

$$\Rightarrow v_t = \sqrt{(9.81)(1.8)} = 4.20214 \approx \boxed{4.2\frac{m}{s}}$$

Flipping Physics Lecture Notes:

The Scalar Nature of Variables in Rotational Motion Equations
http://www.flippingphysics.com/scalar-rotational-variables.html

We have already discussed the following equations:

$$s = r\Delta\theta;\ v_t = r\omega;\ a_t = r\alpha;\ a_c = \frac{v_t^2}{r} = r\omega^2$$

It is important that the vector symbol is never used in any of those equations. Let's look at one of them specifically and identify what this means.

$$v_t = r\omega$$

Typically, I have said this equation as tangential velocity equals radius times angular velocity. Let's look at what it would mean if this equation did have vector symbols on it.

$$\vec{v}_t = r\vec{\omega}$$

Because radius, r, is a scalar, this equation would imply that the tangential velocity and angular velocity are in the same direction. Hopefully you recognize that the tangential velocity and angular velocity of an object cannot be in the same direction. The tangential velocity is directed tangent to the circle the object is moving along, and the object's angular velocity is normal to the two-dimensional plane the object is moving along and those cannot be the same direction. Therefore, this equation cannot refer to the vector quantities and must refer to the magnitudes of the vector quantities.

In other words, this equation, $v_t = r\omega$, needs to be read as the magnitude of tangential velocity equals radius times the magnitude of angular velocity. Alternatively, it could also be read as tangential speed equals radius times angular speed.

So, realize all of these equations refer to the magnitudes of the vectors. For example:

- $s = r\Delta\theta$ → Arc length equals radius times the magnitude of angular displacement.
- $v_t = r\omega$ → Tangential speed equals radius times angular speed.
- $a_t = r\alpha$ → The magnitude of tangential acceleration equals radius times the magnitude of angular acceleration.
- $a_c = \frac{v_t^2}{r} = r\omega^2$ → The magnitude of centripetal acceleration equals tangential speed squared divided by radius and it also equals radius times angular speed squared.

I know I have not been overly clear about this before, and I apologize for that.
Hopefully this clears this up a bit. ☺

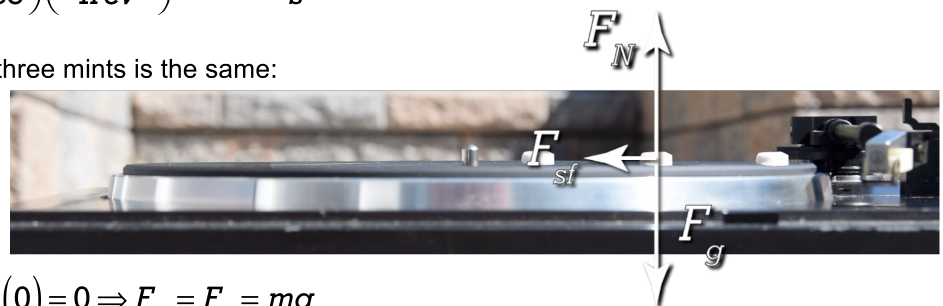
Flipping Physics Lecture Notes:

Mints on a Rotating Turntable
Determining the Static Coefficient of Friction

Example: A turntable is turning 45 revolutions per minute. Mints are located 0.030 m, 0.080 m, and 0.130 m from the center of the record. Determine what you can about the coefficient of static friction between the turntable and the mints.

Knowns: $\omega = 45\frac{rev}{\min}\left(\frac{1\min}{60\sec}\right)\left(\frac{2\pi rad}{1rev}\right) = 1.5\pi\frac{rad}{s};\ r_1 = 0.030m;\ r_2 = 0.080m;\ r_3 = 0.130m;\ \mu_s = ?$

The free body diagram for all three mints is the same:



$$\sum F_y = F_N - F_g = ma_y = m(0) = 0 \Rightarrow F_N = F_g = mg$$

$$\sum F_{in} = F_{sf\,\max} = ma_c \Rightarrow \mu_s F_N = mr\omega^2 \Rightarrow \mu_s mg = mr\omega^2 \Rightarrow \mu_s g = r\omega^2$$

$$\Rightarrow \mu_{s1} = \frac{r_1\omega^2}{g} = \frac{(0.03)(1.5\pi)^2}{9.81} = 0.067910 \approx 0.068$$

$$\Rightarrow \mu_{s2} = \frac{r_2\omega^2}{g} = \frac{(0.08)(1.5\pi)^2}{9.81} = 0.18109 \approx 0.18$$

$$\Rightarrow \mu_{s3} = \frac{r_3\omega^2}{g} = \frac{(0.13)(1.5\pi)^2}{9.81} = 0.29428 \approx 0.29$$

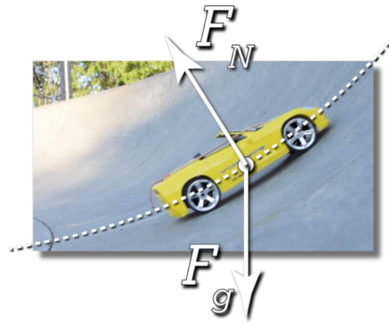
Because the force of static friction is actually less than or equal to the coefficient of static friction times force normal and we used the maximum force of static friction, we actually determined the minimum coefficient of friction necessary to keep each mint on the turntable. Therefore the answer is that the coefficient of static friction must be greater than or equal to 0.29.

$$\boxed{\mu_s \geq 0.29}$$

Flipping Physics Lecture Notes:

Determining the Force Normal on a Toy Car moving up a Curved Hill

Example: A 0.453 kg toy car moving at 1.15 m/s is going up a semi-circular hill with a radius of 0.89 m. When the hill makes an angle of 32° with the horizontal, what is the magnitude of the force normal on the car?



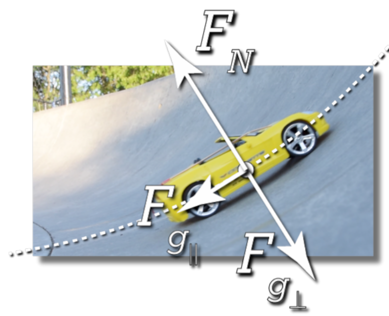
Knowns: $m = 0.453kg;\ \mathrm{v}_t = 1.15\frac{m}{s};\ r = 0.89m;\ \theta = 32°;\ F_N = ?$

Draw FBD:

Break forces into components (in-direction and parallel to in-direction)

$$F_{g_\perp} = mg\cos\theta \ \&\ F_{g_\parallel} = mg\sin\theta$$

Re-draw FBD:



$$\sum F_{in} = F_N - F_{g_\perp} = ma_c \Rightarrow F_N = F_{g_\perp} + ma_c = mg\cos\theta + m\frac{v_t^2}{r}$$

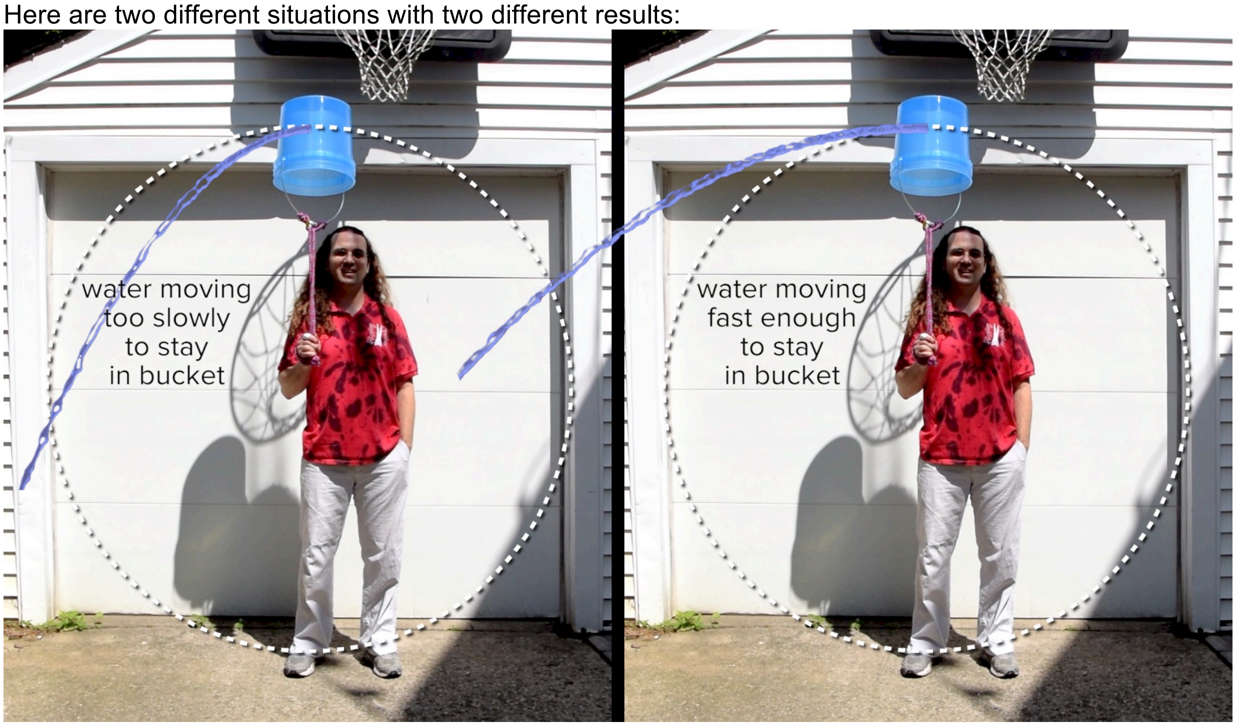
$$\Rightarrow F_N = (0.453)(9.81)\cos(32) + (0.453)\frac{1.15^2}{0.89} = 4.3804 \approx \boxed{4.4N}$$

Flipping Physics Lecture Notes:

Demonstrating Why Water Stays in a Bucket Revolving in a Vertical Circle

Inertia is the tendency of an object to maintain its state of motion. Looking at the top position: The inertia of the water tries to keep the water moving to the left and the force of gravity pulls the water down.

Here are two different situations with two different results:

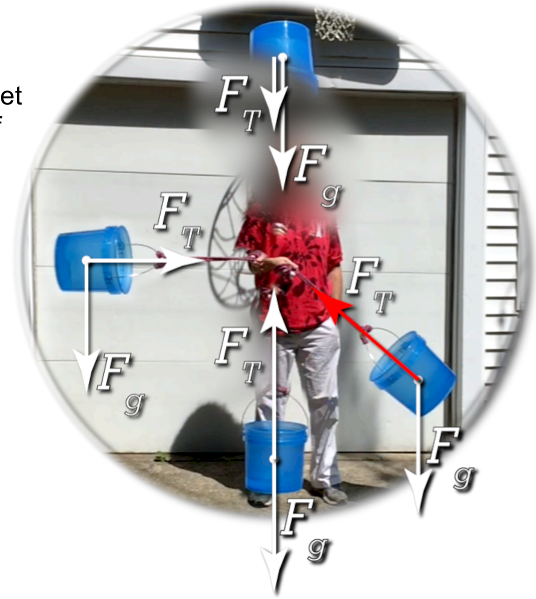


The inertia of the water, or the tendency of the water to maintain its state of motion, is what keeps water in the bucket. In other words, the water is moving and wants to keep moving in a straight line, however, the bucket keeps getting in the way, which is why water stays in the bucket.

Analyzing Water in a Bucket Revolving in a Vertical Circle

Free Body Diagrams:

Notice how all free body diagrams are different at different bucket locations. However, force of gravity is always down and force of tension is always in.



Let's start by analyzing when the bucket is at the bottom.

$$\sum F_{in} = F_T - F_g = ma_c$$
$$\Rightarrow F_T - mg = mr\omega^2$$
$$\Rightarrow F_T = mg + mr\omega^2$$

A couple of things to notice:

1) The centripetal force is Force of Tension minus Force of Gravity.
2) The faster the bucket spins the larger the force of tension and therefore …
3) The faster the bucket spins the larger the centripetal force necessary to keep the bucket moving in circular motion.

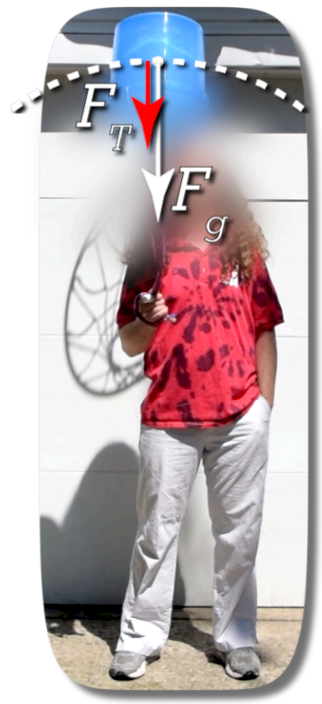
If we knew the radius of the path, the mass and angular speed of the bucket and water we could solve for the force of tension.[♥] We could do this at any bucket location: Top, Bottom, Side, etc. Notice at a location which is not the bottom, top, or side, we would need to break the force of gravity into its components in the in-direction and the tangential-direction, just like we did with the toy car and the curved hill. (http://www.flippingphysics.com/car-hill-force-normal.html) But, we don't have a numbers dependency, so let's stop here and not have any numbers in this lesson. ☺

[♥] For those of you taking AP Physics 1, be aware I have yet to see a problem on the AP Physics 1 exam that has the bucket in a position other than the top, bottom, or side.

Flipping Physics Lecture Notes:

Analyzing Water in a Bucket Revolving in a Vertical Circle

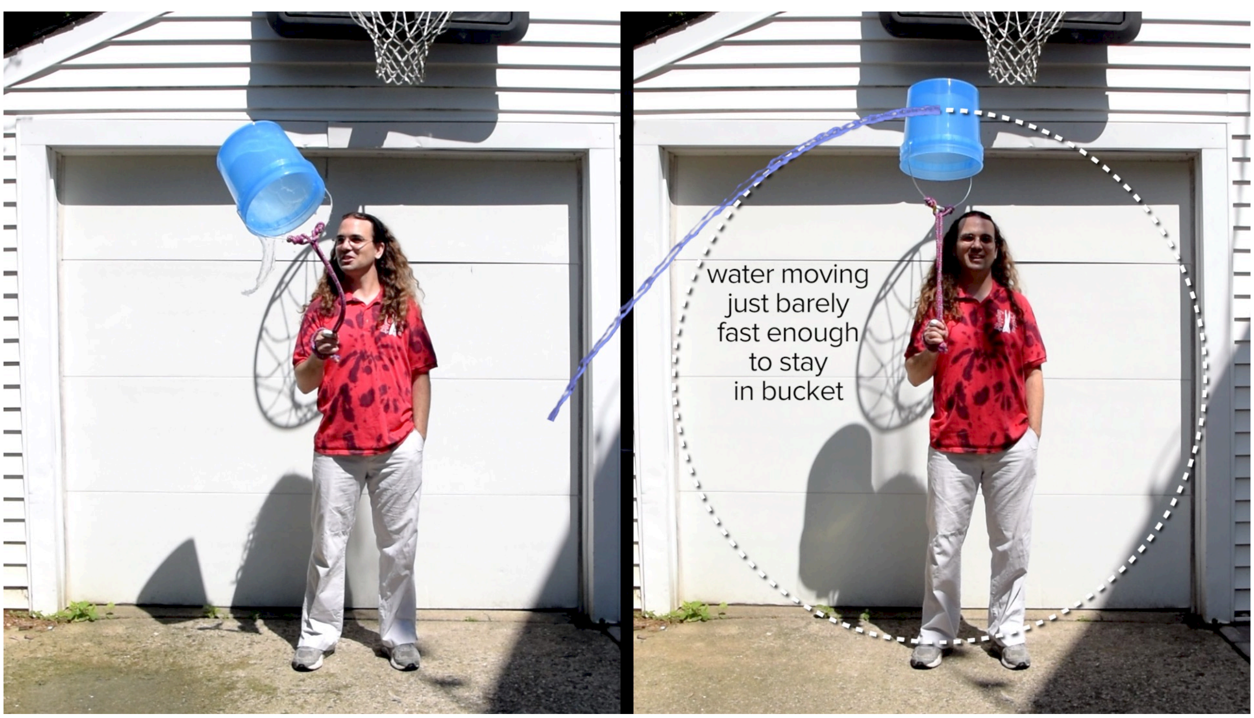
Example: What is the minimum angular speed necessary to keep water in a vertically revolving bucket? The rope radius is 0.77 m.



$$\sum F_{in} = F_g + F_T = ma_c \Rightarrow mg + 0 = mr\omega^2 \Rightarrow g = r\omega^2$$

At the "minimum angular speed necessary to keep water in a vertically revolving bucket" the tension in the rope is reduced to zero. At this angular speed the centripetal force is just the force of gravity.

$$\Rightarrow \omega^2 = \frac{g}{r} \Rightarrow \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81}{0.77}} = 3.5694 \approx \boxed{3.6\frac{rad}{s}}$$



Flipping Physics Lecture Notes:

The Right Hand Rule for Angular Velocity and Angular Displacement

Step #1) Don't be too cool for the Right Hand Rule.
Step #2) Limber Up! (Not kidding. You have to rotate your hips and shoulders sometimes when doing the right hand rule)
Step #3-1) Take the fingers of your right hand.
Step #3-2) Curl your fingers in the direction of the turning motion of the object.
Step #3-3) Stick out your thumb.
Step #3-4) Your thumb indicates the direction of the object's angular velocity and angular displacement.

Two examples:

Note: Clockwise and counterclockwise are observer dependent directions. The direction that results from the Right Hand Rule is not observer dependent. This is one of the reasons we use the Right Hand Rule in physics to determine the direction of angular displacement and angular velocity.

Also note: The direction of the angular displacement and angular velocity of a turning object is perpendicular to the plane in which the object is located.

Flipping Physics Lecture Notes:

A Tale of Three Accelerations
or
The Differences between Angular, Tangential, and Centripetal Accelerations

An object moving in a circle can have three different types of accelerations:

- Angular Acceleration: $\alpha = \frac{\Delta\omega}{\Delta t}$ in $\frac{rad}{s^2}$ is an *angular* quantity.
- Tangential Acceleration: $a_t = r\alpha$ in $\frac{m}{s^2}$ is a *linear* quantity.
- Centripetal Acceleration: $a_c = \frac{{v_t}^2}{r} = r\omega^2$ in $\frac{m}{s^2}$ is a *linear* quantity.

Angular acceleration separates itself from the others:

1) Because it is an *angular* quantity, whereas the other two are linear quantities.
2) Because angular acceleration applies to the whole rigid object, however, tangential acceleration and centripetal acceleration are for a specific radius.

A major difference between tangential acceleration and centripetal acceleration is their direction.

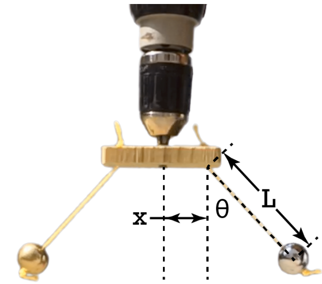
- Centripetal means “center seeking”. Centripetal acceleration is always directed inward.
- Tangential acceleration is always directed tangent to the circle.
  - By definition, tangential acceleration and centripetal acceleration are perpendicular to one another.

Another major difference between tangential acceleration and centripetal acceleration is that circular motion cannot exist without centripetal acceleration.

- No centripetal acceleration means the object is not moving in a circle.
  - Centripetal acceleration results from the change in direction of the tangential velocity. If the tangential velocity is not changing directions, then the object is not moving in a circle.
- Tangential acceleration results from the change in magnitude of the tangential velocity of an object. An object can move in a circle and not have any tangential acceleration. No tangential acceleration simply means the angular acceleration of the object is zero and the object is moving with a constant angular velocity.
  - $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0}{\Delta t} = 0 \Rightarrow a_t = r\alpha = r(0) = 0$

Flipping Physics Lecture Notes:

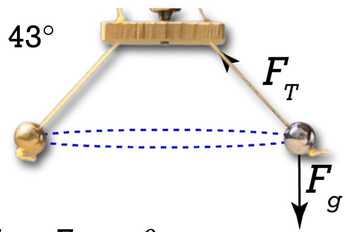
Conical Pendulum Demonstration and Problem

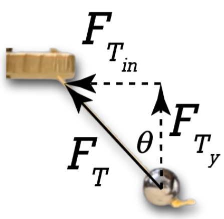


Example: Two spheres attached to a horizontal support are rotating with a constant angular velocity. Determine the angular velocity.
As shown in the figure: L = 9.7 cm, x = 3.4 cm and θ = 43°.

Knowns:

$$\omega = ?;\ x = 3.4cm\left(\frac{1m}{100cm}\right) = 0.034m;\ L = 9.7cm\left(\frac{1m}{100cm}\right) = 0.097m;\ \theta = 43^\circ$$

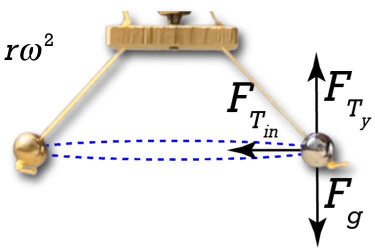




$$\sin\theta = \frac{O}{H} = \frac{F_{T_{in}}}{F_T} \Rightarrow F_{T_{in}} = F_T\sin\theta \ \&\ \cos\theta = \frac{A}{H} = \frac{F_{T_y}}{F_T} \Rightarrow F_{T_y} = F_T\cos\theta$$

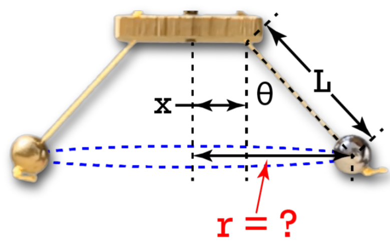
$$\sum F_y = F_{T_y} - F_g = ma_y \Rightarrow F_T\cos\theta - mg = m(0) = 0 \Rightarrow F_T\cos\theta = mg \Rightarrow F_T = \frac{mg}{\cos\theta}$$

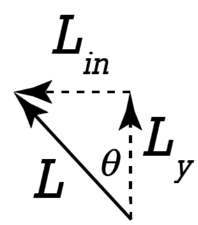
$$\sum F_{in} = F_{T_{in}} = ma_c \Rightarrow F_T\sin\theta = mr\omega^2 \Rightarrow \frac{mg}{\cos\theta}\sin\theta = mr\omega^2 \Rightarrow g\tan\theta = r\omega^2$$



$$\Rightarrow \omega^2 = \frac{g\tan\theta}{r} \Rightarrow \omega = \sqrt{\frac{g\tan\theta}{r}}$$

Need r: $r = L_{in} + x$ therefore we need L_{in}





$$\sin\theta = \frac{O}{H} = \frac{L_{in}}{L} \Rightarrow L_{in} = L\sin\theta \ \&\ \ r = L_{in} + x = L\sin\theta + x$$

$$\Rightarrow \omega = \sqrt{\frac{g\tan\theta}{L\sin\theta + x}} = \sqrt{\frac{(9.81)\tan(43)}{(0.097)\sin(43) + 0.034}} = \pm 9.55716 \approx \boxed{-9.6\frac{rad}{s}}$$

According to the right hand rule, the direction of the angular velocity and angular displacement of the spheres is down, which is negative.

$$\omega_{measured} = \frac{\Delta\theta}{\Delta t} = \frac{-2\pi rad}{0.66s} = -3\pi\frac{rad}{s} = -9.51998\frac{rad}{s}$$

Use the measured value as our observed value and the predicted value as our accepted value:

$$E_r = \frac{O - A}{A}\times 100 = \frac{-9.55716 - (-9.51998)}{-9.51998}\times 100 = -0.39052 \approx \boxed{-0.39\%}$$