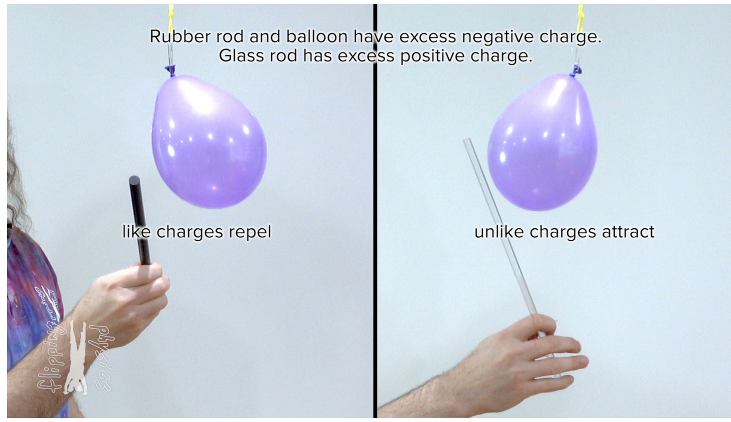
Flipping Physics Lecture Notes:

Electric Charge

https://www.flippingphysics.com/charge.html

The Law of Charges:

- Like charges repel &
- Unlike charges attract



When we rub fur against a rubber rod and then a rubber balloon, electrons transfer from the fur to the rubber objects leaving the rubber objects with a net negative charge and the fur with a net positive charge. When we rub silk against a glass rod, electrons transfer from the glass rod to the silk leaving the glass rod with a net positive charge and the silk with a net negative charge.

Electrons and Protons are very, very tiny particles with charge magnitude equal to e:

- e = 1.60 x 10-19 coulombs, C
- e = elementary charge
- e = smallest charge measured on an *isolated* particle.
- Coulombs, C = SI unit for charge
- Charge on electron = -e & Charge on proton = +e
- $m_{electron}=9.11\times10^{-31}kg$ & $m_{proton}=1.67\times10^{-27}kg$
- A proton is much more massive than an electron. $\dfrac{m_{proton}}{m_{electron}}=\dfrac{1.67\times10^{-27}kg}{9.11\times10^{-31}kg}\approx1830$
- electron = elementary particle
- proton is not an elementary particle because it is composed of quarks.

Quarks are elementary particles which make up protons and neutrons:

- $q_{up\,quark}=+\dfrac{2}{3}e$ & $q_{down\,quark}=-\dfrac{1}{3}e$
- $q_{proton}=2q_{up\ quark}+1q_{down\ quark}=2\left(+\dfrac{2}{3}e\right)+\left(-\dfrac{1}{3}e\right)=+\dfrac{4}{3}e-\dfrac{1}{3}e=+e$
- $q_{neutron}=1q_{up\ quark}+2q_{down\ quark}=\left(+\dfrac{2}{3}e\right)+2\left(-\dfrac{1}{3}e\right)=+\dfrac{2}{3}e-\dfrac{2}{3}e=0$

When you take a rubber balloon and rub it against fur, three things are possible.

1) The balloon will stay in your hair.
   a. Rub rubber balloon against hair and electrons transfer from hair to balloon.
   b. Balloon now has a net negative charge and hair now has a net positive charge.
   c. Law of Charges: unlike charges attract.
   d. Electric force pulls hair and balloon together.
2) Pull the balloon away from your hair and some of your hairs will stick out.
   a. Rub rubber balloon against hair and electrons transfer from hair to balloon.
   b. Hair now has a net positive charge.
   c. Law of Charges: like charges repel.
   d. • Electric force pushes hair apart.

3) The balloon will stick to a wall.
    a. This is polarization which we will learn about in a future lesson.
        i. https://www.flippingphysics.com/polarization.html

The elementary charge is very small: $e = 1.60 \times 10^{-19} C = 0.000000000000000000160 C$

Example: How many excess protons does it take to get a charge of 1 coulomb on an object?

For this we need a new equation: $q = ne$

- q = net charge on an object
- n = excess number of charge carriers
- e = elementary charge

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{1C}{1.60 \times 10^{-19} \frac{C}{proton}} = 6.25 \times 10^{18}\, protons$$

$$q = 6.25\ quintillion\ protons = 6.25\ million\ million\ million\ protons = 6{,}250{,}000{,}000{,}000{,}000{,}000\ protons$$

Example: Can an object have a net negative charge of 2.00 times 10 to the negative 19 coulombs?

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{-2.00 \times 10^{-19} C}{-1.60 \times 10^{-19} \frac{C}{electron}} = 1.25\, electrons$$

However, you cannot have a quarter of an electron because charge is *quantized*.

- Charge comes in discrete quantities in multiples of the elementary charge.
- Charge is caused by having more or fewer charged particles (protons or electrons).
- The charge on an object, q, must be an integer multiple of the elementary charge, e.
- In $q = ne$, n, the number of charge carriers, has to be an integer.
- Because you cannot cut protons and electrons into pieces.

So the answer is … No, you cannot have a net charge of $-2.00 \times 10^{-19} C$ on an object.

Flipping Physics Lecture Notes:
Introduction to Coulomb's Law or the Electric Force
https://www.flippingphysics.com/coulombs-law.html

We have already learned about the Law of Charges which governs the directions of the forces on pairs of charges.[1] Today we learn about the magnitude of that force.

The electric force is described by Coulomb's Law: $F_e = \dfrac{kq_1q_2}{r^2}$

- This is the electric force which exists between any two charged particles.
- It is sometimes called the Coulomb Force or Electrostatic Force. I will call it the Electric Force.
- k is the Coulomb Constant: $k = 8.99\times10^9 \dfrac{N\cdot m^2}{C^2}$
- q_1 and q_2 are the two electric charges.
- r is the distance between the centers of *charge* of the two charges.
- Note the similarities to Newton's Universal Law of Gravitation: $F_g = \dfrac{Gm_1m_2}{r^2}$
- k, the Coulomb Constant is much larger than, G, the Universal Gravitational Constant.

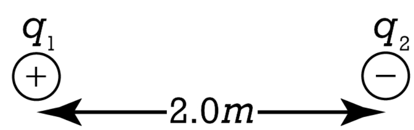
$$\frac{k}{G} = \frac{8.99\times10^9 \frac{N\cdot m^2}{C^2}}{6.67\times10^{-11} \frac{N\cdot m^2}{kg^2}} = 1.347826087\times10^{20} \approx 1.35\times10^{20} \frac{kg^2}{C^2}$$

A point charge is are just like a point mass, only the description has to do with charge rather than mass. In other words, a point charge is an object which has zero size and carries an electric charge. A point charge is an object whose mass is small enough that its mass is negligible when compared to its charge.

Three prefixes you should be familiar with when using coulombs:

- μ means micro or 1 millionth or $\times10^{-6}$ so 1 microcoulomb $= 1\mu C = 1\times10^{-6} C$
- n means nano or 1 billionth or $\times10^{-9}$ so 1 nanocoulomb $= 1nC = 1\times10^{-9} C$
- p means pico or 1 trillionth or $\times10^{-12}$ so 1 picoulomb $= 1pC = 1\times10^{-12} C$

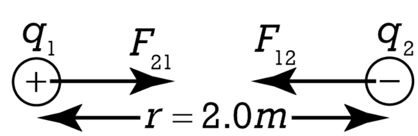
Example #1: Two equal magnitude point charges are located 2.0 meters apart. If the magnitudes of their charges are both 5.0 μC and one is positive and one is negative, what is the electric force acting on each charge caused by the other charge?



Knowns: $q_1 = +5.0\mu C \times \dfrac{1C}{1\times10^6 \mu C} = 5\times10^{-6} C;\ q_2 = -5.0\mu C = -5\times10^{-6} C;\ r = 2.0m;\ F_e = ?$

$$F_e = \frac{kq_1q_2}{r^2} = \frac{\left(8.99\times10^9\right)\left(5\times10^{-6}\right)\left(-5\times10^{-6}\right)}{2^2} = -0.0561875 \approx -0.056N$$

$$\boxed{F_{\text{each charge}} \approx 0.056N \text{ toward the other charge}}$$



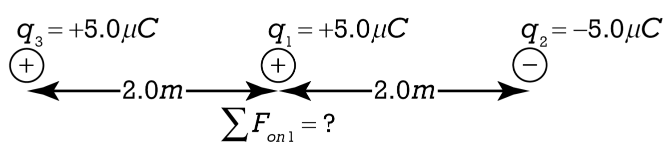
[1] See video "Electric Charge, Law of Charges, and Quantization of Charge" at https://www.flippingphysics.com/charge.html

But what does the negative on $F_e \approx -0.056N$ mean?

I have seen three different versions of Coulomb's Law:

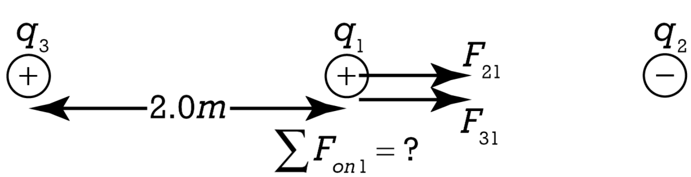
- The one we have been working with: $F_e = \dfrac{kq_1q_2}{r^2}$
- The magnitude of the electric force: $\left|\vec{F}_e\right| = k\left|\dfrac{q_1q_2}{r^2}\right|$
    - This ignores direction information, so we are not going to use it.
- And the unit vector version: $\vec{F}_{12} = \dfrac{kq_1q_2}{{r_{12}}^2}\hat{r}_{12}$
    - $\vec{F}_{12}$ is the electric force by charge 1 on charge 2
    - $\hat{r}_{12}$ is the unit vector directed from charge 1 toward charge 2
    - We have not worked with unit vectors in this algebra based class yet, so we are not going to use this verson of Coulomb's Law.
- Therefore we will use the first one: $F_e = \dfrac{kq_1q_2}{r^2}$
    - A negative force means an attractive force.
    - A positive force means a repulsive force.
- Note: $F_e = \dfrac{kq_1q_2}{r^2}$ can be attractive or repulsive however, $F_g = \dfrac{Gm_1m_2}{r^2}$ is always attractive.

Building on Example #1, Example #2 is .. If we place a third charge, q 3, with the same charge as the positively charged object, q 1, only now 2.0 meters on the opposite side from the other negative charge, q 2, what is the net force acting on q 1, the positive charge in the middle?



Adding the third charge does not affect the electric force of 0.056 newtons which is from q_2 on q_1 and acting to the right. We just need to add the electric force from q_3 on q_1. Because both q_1 and q_3 are positive, force 3-1 will be a positive, repulsive force, on q_1 that would be to the right.

$$F_{31} = \frac{kq_1q_3}{{r_{31}}^2} = \frac{\left(8.99\times10^9\right)\left(+5\times10^{-6}\right)\left(+5\times10^{-6}\right)}{2^2} = 0.0561875N \text{ to the right}$$



$$\sum F_{on\,1} = F_{21} + F_{31} = 0.0561875 + 0.0561875 = 0.112375 \approx \boxed{0.11\text{N to the right}}$$

Flipping Physics Lecture Notes:
Charging via Conduction and Induction
https://www.flippingphysics.com/conduction-and-induction.html

An electroscope is an instrument for demonstrating electric charge. This electroscope has a metal ball on top of a vertical metal rod with a hook on the end of it and two thin foils of aluminum hanging from the hook. The metal ball, rod, hook, and two aluminum foils are electrically insulated from the surroundings via the rubber stopper and glass flask.

Charge by Conduction:
- charge the balloon
- bring balloon close to electroscope and foils move apart
- touch balloon to electroscope
- electroscope is charged by conduction
- touch electroscope with hand and foils fall down to original positions

In order to charge the balloon, we rub fur on balloon. This causes electrons to move from the fur to the rubber balloon. The fur now has excess positive charge and the balloon has excess negative charge. This is called charging by friction. Before balloon is close to the electroscope the net charge on electroscope equals zero. The electroscope has an equal number of protons and electrons. The balloon has larger number of electrons then protons.

The negatively charged balloon is brought close to the electroscope, however, the electroscope and the balloon have not touched yet. The aluminum foils are pushed away from one another. The foils must have the same charge. The foils must have an electric force pushing them apart.

We can use the Law of Charges to determine the charge on the foils. We know protons do not easily move because they are in the nucleus of the atoms. That means the electrons are the ones that move. So, the electrons in the electroscope, which are negatively charged, are repelled by the negative charges in the balloon and flow down to the metal foils. That must mean, when the balloon is brought near the electroscope, the metal foils have an excess of electrons, or a net negative charge, and an electric force pushes the metal foils away from one another. That means the top of the electroscope has a net positive charge because there are fewer electrons than protons in the ball at the top of the electroscope. However, realize the electroscope still has a net neutral charge.

When the balloon touches the electroscope, electrons transfer from the balloon to the electroscope. The balloon and electroscope now both have excess electrons, however, total number of excess electrons in the system remains the same. This is called Conservation of Charge. Because the balloon and electroscope have excess electrons, the foils also have excess electrons and repel one another.

Now I touch the electroscope with my finger and the foils fall to their original positions. This is because touching electroscope grounds the electroscope. That means the electroscope is no longer electrically isolated and electrons transfer from electroscope into ground. After grounding, the electroscope has neutral charge. An Ideal Ground is an infinite well of charge carriers. We call it a “ground” because electrical circuits are literally connected to the Earth or the “ground”. And the Earth has, relatively speaking, an infinite number of electrons which we can pull from it, or we can give to it, again relatively speaking, an infinite number of charges. If something goes wrong in a circuit, the “ground” will serve as a way to balance out the charges. In this example, when I touch the electroscope, the electroscope is no longer electrically isolated and the excess electrons on the electroscope flow out of the electroscope into the “ground”, into me, and the electroscope is now electrically neutral, which is why the foils are no longer repelled from one another.

Two items to remember about Charging by Conduction:
1. The two objects have to touch. In this case the two objects are the balloon and the electroscope.
2. The two objects end with the same sign of net charge. In this case they both end with an excess negative charge. (Before the electroscope is grounded and ends with a net neutral charge.)

Charging by Conduction in pictorial form:

And now we switch to Charging by Induction:

- charge the balloon
- bring balloon close to electroscope
- ground the electroscope
- remove the ground
- remove the balloon and electroscope is charged by induction
- ground the electroscope

This time when the electroscope is grounded the negatively charged balloon is held near the electroscope, therefore, electrons in the electroscope flow into the ground. They do that because, according to the Law of Charges, like charges repel one another. So, some of the electrons in the electroscope flow from the electroscope into the ground. The ground essentially provides an escape route for electrons to leave the electroscope. But there are still electrons in the electroscope, just fewer than before, and many are in the metal foils of the electroscope because they are repelled from the electrons in the balloon. That gives the metal foils a net neutral charge and the two foils are not repelled from one another.

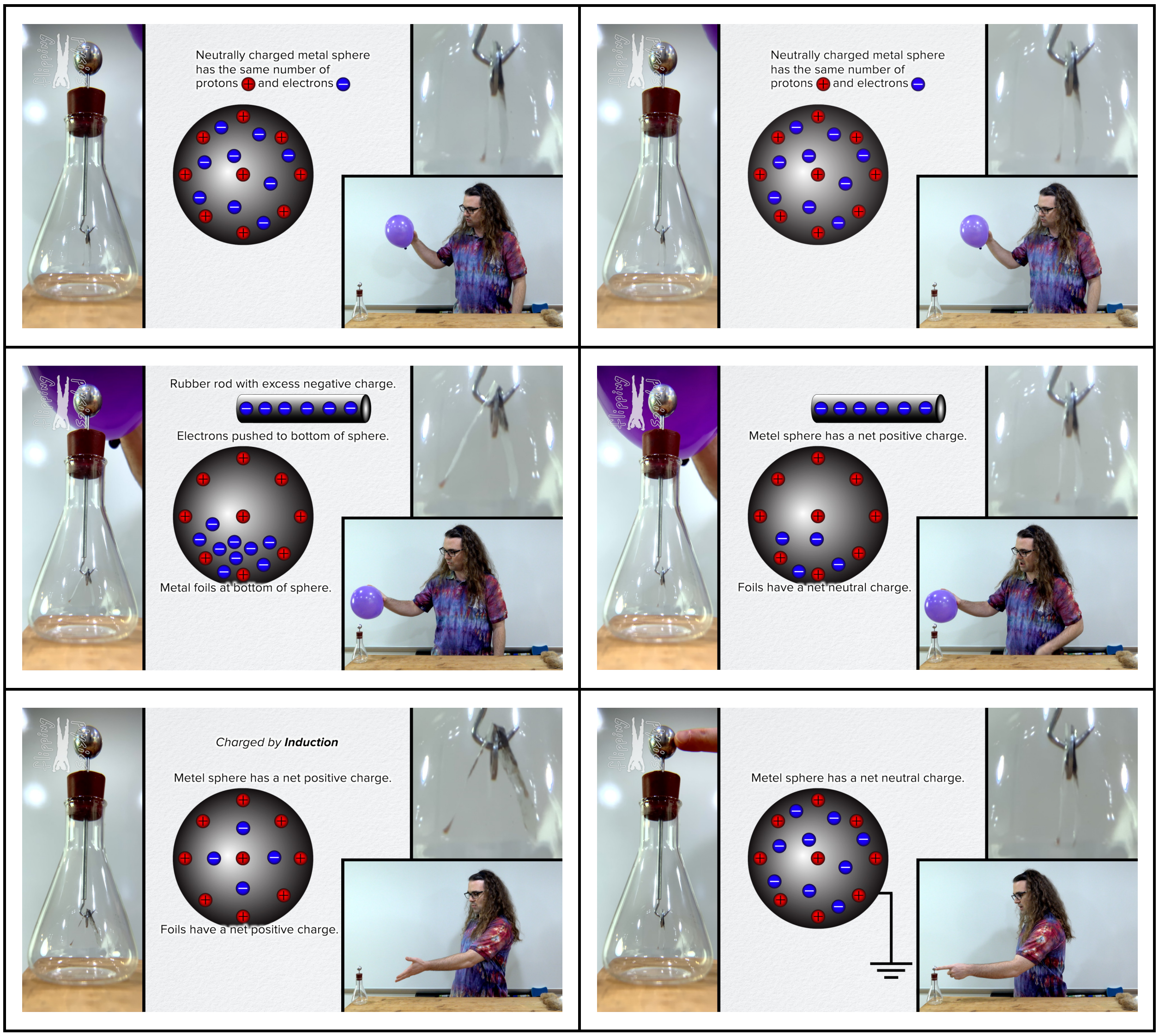
Then when the balloon is removed, the electrons in the electroscope are repelled from one another in the electroscope and get distributed throughout the electroscope. That leaves the electroscope with an excess of protons and a net positive charge. That is why the metal foils are repelled from one another, because the positive charges in the foils repel one another.

And then the electroscope is grounded. In this case, that means electrons are pulled from the ground into the electroscope to balance out the excess protons in the electroscope and leave the electroscope with an equal number of protons and electrons and a net neutral charge.

Two items to remember about Charging by Induction:

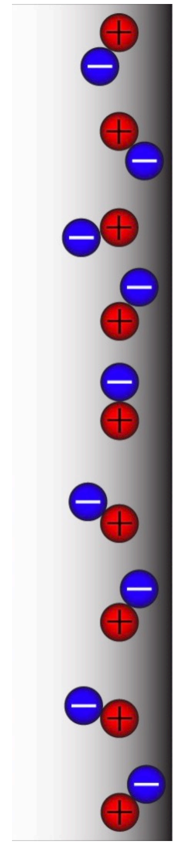
1. The two objects do not have to touch.
2. The two objects end with opposite sign of net charge. In this case the electroscope ends with an excess of positive charge and the balloon ends with an excess negative charge.
   (Before the electroscope is grounded and ends with a net neutral charge.)

Charging by Induction in pictorial form:

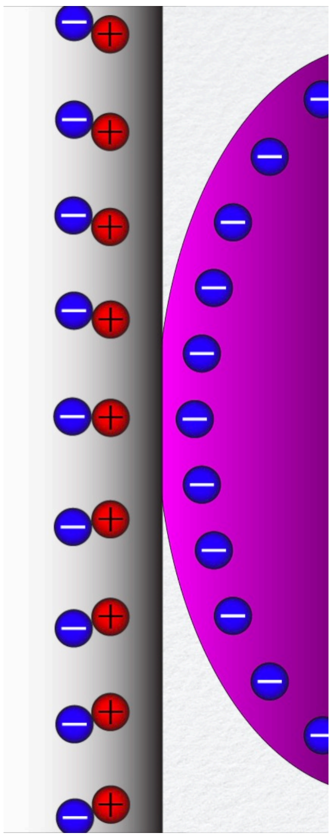


Flipping Physics Lecture Notes:
Polarization of Charge
https://www.flippingphysics.com/polartization-of-charge.html

A charged balloon is attracted to a wall because the molecules in the wall become polarized. Polarization does not mean the wall becomes charged, it simply means the molecules in the wall have aligned themselves such that there will be a net attractive force between the wall and the charged balloon.



To the right is an illustration of electrons and protons randomly oriented in the wall before the charged balloon is brought close to the wall.



When the balloon is brought near the wall (left illustration), the electrons in the wall move away from the electrons in the balloon because, according to the Law of Charges, the electrons in the wall will be repelled from electrons in the balloon because they both have negative charges.

We know like charges repel and unlike charges attract, however, it is important to notice that all the like charges are farther apart than all of the unlike charges. And because the electric force between the charges will be determined by Coulomb's Law: $F_e = \frac{kq_1q_2}{r^2}$, we know the closer the charged objects are to one another, the smaller the "r" value in Coulomb's Law and therefore the larger the electric force.

In other words, because the opposite charges are closer than the like charges, the attractive electric force is larger than the repulsive electric force and the net electric force between the balloon and the wall is an attractive force. This is how a charged object can be attracted to a neutrally charged object.

Two other examples of attracting by polarization are that the charged balloon can pick up little pieces of paper and cause an aluminum can to roll. In both cases, the charged balloon polarizes the other objects, the little pieces of paper and aluminum can, and therefore the objects are attracted to one another.

Please realize the electric forces in the polarization demonstrations are quite small. The masses of the balloon and little pieces of paper are small, so only a small electric force is required to hold them up and it only requires a small force to roll the aluminum can.

The electric force caused by polarization is typically larger for a conductor than an insulator, Because, in insulators, electrons are just pushed to the opposite side of the atom, however, in conductors, the electrons are free to move about more and actually end up farther away. That will produce a larger difference in attractive vs repulsive force and a larger net electric force in a conductor than an insulator.

Flipping Physics Lecture Notes:

Conservation of Charge Example Problems
https://www.flippingphysics.com/conservation-of-charge.html

Conservation of Charge: The total electric charge of an isolated system never changes.

What is an isolated system? We could start with the universe. In other words, the net electric charge of the universe never changes. Add up all the positive charges and subtract all the negative charges and you will always get the same number.

Or we could have a smaller isolated system, like the conductive metal pieces of an electroscope which are electrically isolated from the rest of the universe by the rubber and glass insulators. In other words, the net electric charge of the electroscope will remain constant, as long as it remains isolated.

Example Problem #1: Two charged, conducting objects collide and separate. Before colliding, the charges on the two objects are +3e and -6e. Which of the following are possible values for the final charges on the two objects? Choose all possible answers.

(a) +4e, -7e (b) +2e, -2e (c) -1.5e, -1.5e (d) -3.5e, +2.5e (e) +e, -4e

$q_{1i} = +3e; q_{2i} = -6e; q_{1f} = ?; q_{2f} = ?$ & $q_{total\ i} = q_{1i} + q_{2i} = +3e + (-6e) = -3e = q_{total\ f}$

a) $q_{1f} + q_{2f} = +4e + (-7e) = -3e = q_{total\ f}$ ✓ **it works**

b) $q_{1f} + q_{2f} = +2e + (-2e) = 0 \neq -3e = q_{total\ f}$ **does not work**

c) $q_{1f} + q_{2f} = -1.5e + (-1.5e) = -3e = q_{total\ f}$

But (c) **does not work** because you cannot have half an electron because charge is quantized!!

d) $q_{1f} + q_{2f} = -3.5e + 2.5e = -e = q_{total\ f}$ **does not work**

But (d) also does not work because you cannot have half an electron because charge is quantized!!

e) $q_{1f} + q_{2f} = +e + (-4e) = -3e = q_{total\ f}$ ✓ **it works**

Correct answers are (a) and (e) because those are the only two options which have a total final charge equal to the total initial charge and are integer multiples of the fundamental charge e.

Example Problem #2: Two identical, conducting spheres are held using insulating gloves a distance x apart. Initially the charges on each sphere are +3.0 pC and +6.0 pC. The two spheres are touched together and returned to the same distance x apart. You may assume x is the distance between their centers of charge.

(a) What is the final charge on each sphere?
(b) Is the final electric force between the two spheres increased, decreased, or the same when compared to the initial electric force?

.

$$q_{1i} = +3.0pC;\ q_{2i} = +6.0pC;\ r_i = r_f = x;\ \text{Part(a)}:\ q_{1f} = ?;\ q_{2f} = ?;\ \text{Part(b)}:\ F_{ef}\ ?\ F_{ei}$$

$$q_{total\ f} = q_{total\ i} = q_{1i} + q_{2i} = +3pC + 6pC = +9pC$$

Because the two spheres are identical, after touching, the spheres will have equal charge.

$$q_{1f} = q_{2f} = q_f \Rightarrow q_{total\ f} = q_{1f} + q_{2f} = 2q_f = 9pc \Rightarrow q_f = 4.5pC$$

(a) Both charges end with 4.5 pC of charge.

This is 4.5 picocoulombs of charge or 4.5 x 10^{-9} C which an object is physically able to have. Because then it will have:

$$q_f = n_f e \Rightarrow n_f = \frac{q_f}{e} = \frac{4.5\times10^{-9}C}{1.60\times10^{-19}\frac{C}{\text{charge carrier}}} = 2.8125\times10^{10} \approx 2.8\times10^{10}\text{excess protons}$$

Imagine that. 28 billion more protons than electrons on each sphere. Each sphere will have a heck of a lot more total protons and electrons, however, it has a deficit of 28 billion electrons and therefore has a net charge of 4.5 pC.

And now part (b): The two spheres have like charges, so they are repelled from one another with an electric force with a magnitude of: $F_e = \frac{kq_1q_2}{r^2}$ Therefore …

$$F_{ei} = \frac{kq_{1i}q_{2i}}{r_i^2} = \frac{k\left(3\times10^{-9}\right)\left(6\times10^{-9}\right)}{x^2} = 1.8\times10^{-17}\frac{k}{x^2}$$

$$F_{ef} = \frac{kq_{1f}q_{2f}}{r_f^2} = \frac{k\left(4.5\times10^{-9}\right)\left(4.5\times10^{-9}\right)}{x^2} = 2.025\times10^{-17}\frac{k}{x^2} \approx 2.0\times10^{-17}\frac{k}{x^2}$$

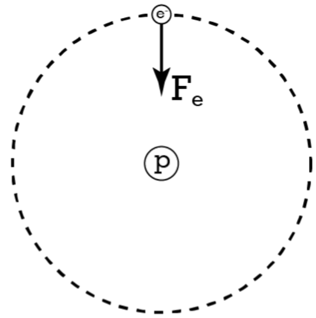
k is the Coulomb Constant and has a constant value: $k = 8.99\times10^{9}\frac{N\bullet m^2}{C^2}$

x is also a constant. Therefore: $F_{ef} \approx 2.0\times10^{-17}\frac{k}{x^2} > 1.8\times10^{-17}\frac{k}{x^2} = F_{ei}$

The final electric force is *greater than* the initial electric force.

Flipping Physics Lecture Notes:

Determining the Speed of the Electron
in the Bohr Model of the Hydrogen Atom
https://www.flippingphysics.com/electron-speed-bohr.html



Assuming a circular orbit of the electron about the nuclear proton in the Bohr model of the hydrogen atom, determine the speed of the electron. The electron orbits at a radius of 5.29 x 10^{-11} m.

Draw free body diagram of the electron.
Only one force, the electric force, inward toward the proton.

$$r_{orbit} = 5.29\times10^{-11}m;\ v_t = ?\left(\text{magnitude}\right)$$

$$\sum F_{in} = F_e = ma_c \Rightarrow \frac{kq_1q_2}{r^2} = m\left(\frac{v_t^{\,2}}{r}\right) \Rightarrow \frac{kq_1q_2}{r} = mv_t^{\,2} \Rightarrow v_t = \sqrt{\frac{kq_1q_2}{mr}}$$

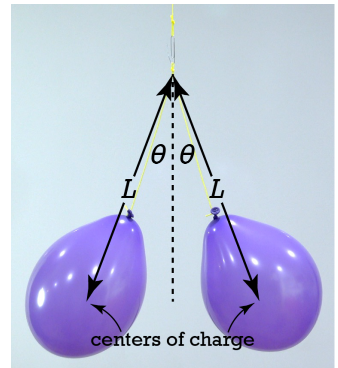
$$\Rightarrow v_t = \sqrt{\frac{\left(8.99\times10^{9}\right)\left(1.60\times10^{-19}\right)^2}{\left(9.11\times10^{-31}\right)\left(5.29\times10^{-11}\right)}} = 2.1853\times10^{6}\frac{m}{s} \approx \boxed{2.19\times10^{6}\frac{m}{s}}$$

$$v_t = 2.1853\times10^{6}\frac{m}{s}\left(\frac{3600sec}{1hour}\right)\left(\frac{1mile}{1609m}\right) \approx 4.89\times10^{6}\frac{mi}{hr}$$

Flipping Physics Lecture Notes:

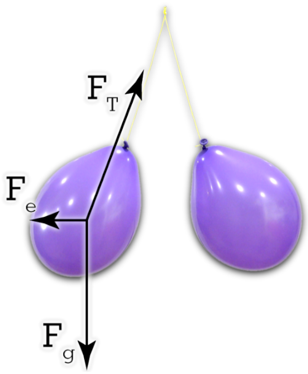
Balloon Excess Charges Experiment

https://www.flippingphysics.com/balloon-charges.html

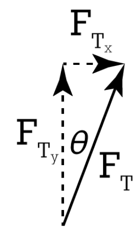


Two 0.0018 kg balloons each have approximately equal magnitude excess charges and hang as shown. If θ = 21° and L = 0.39 m, what is the average number of excess charges on each balloon?

Knowns: $m = 0.0018kg;\ \theta = 21^\circ;\ L = 0.39m;\ q_{avg} = ?$



Draw Free Body Diagram on left balloon.

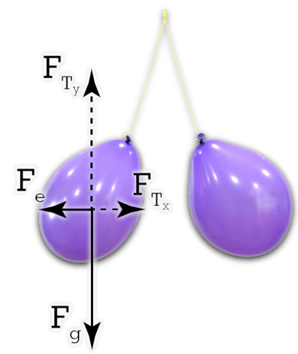


Break Force of Tension into its components.

$$\sin\theta = \frac{O}{H} = \frac{F_{T_x}}{F_T} \Rightarrow F_{T_x} = F_T \sin\theta$$

$$\cos\theta = \frac{A}{H} = \frac{F_{T_y}}{F_T} \Rightarrow F_{T_y} = F_T \cos\theta$$

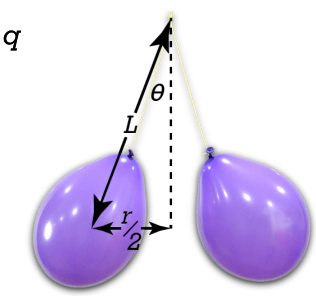
Redraw the Free Body Diagram.



$$\sum F_y = F_{T_y} - F_g = ma_y = m(0) = 0 \Rightarrow F_{T_y} = F_g \Rightarrow F_T \cos\theta = mg \Rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_{T_x} - F_e = ma_x = m(0) = 0 \Rightarrow F_{T_x} = F_e \Rightarrow F_T \sin\theta = \frac{kq_1q_2}{r^2}\ \&\ q_1 \approx q_2 \approx q_{avg} \approx q$$

$$\Rightarrow \left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{kqq}{r^2} \Rightarrow mg\tan\theta = \frac{kq^2}{r^2} \Rightarrow q = \sqrt{\frac{mgr^2\tan\theta}{k}} = r\sqrt{\frac{mg\tan\theta}{k}}$$



$$\sin\theta = \frac{O}{H} = \frac{r/2}{L} \Rightarrow r = 2L\sin\theta$$

$$\Rightarrow q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}} = 2(0.39)\sin(21)\sqrt{\frac{(0.0018)(9.81)\tan(21)}{8.99\times10^9}} = 2.42719\times10^{-7} \approx 2\times10^{-7}C$$

$$\Rightarrow q_{avg} \approx 2\times10^{-7}C\left(\frac{1\times10^9 nC}{1C}\right) \approx 200nC$$

Is the force of gravity which exists between the two balloons truly negligible?

$$F_e = \frac{kq_1q_2}{r^2} = \frac{kq^2}{4L^2\sin^2\theta} = \frac{\left(8.99\times10^9\right)\left(2.42719\times10^{-7}\right)^2}{\left(4\right)\left(0.39\right)^2\sin^2\left(21\right)} = 0.00677827 \approx 7\times10^{-3}N$$

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{Gm^2}{4L^2\sin^2\theta} = \frac{\left(6.67\times10^{-11}\right)\left(0.0018\right)^2}{\left(4\right)\left(0.39\right)^2\sin^2\left(21\right)} = 2.76582\times10^{-15} \approx 3\times10^{-15}N$$

$$\frac{F_e}{F_g} = \frac{0.00677827}{2.76582\times10^{-15}} = 2.45073\times10^{12} \Rightarrow F_e \approx 2\times10^{12}F_g$$

Given that the electric force is roughly 2 million million times larger than the force of gravity, I would say it is completely reasonable to assume the force of gravity which exists between the two balloons is negligible.

Now we actually answer the question, “what is the average number of excess charges on each balloon?”

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{2.42719\times10^{-7}C}{1.60\times10^{-19}\dfrac{C}{\text{charge carrier}}} = 1.51699\times10^{12} \approx \boxed{2\times10^{12}\text{ excess charge carriers}}$$