College Prep Physics II – Video Lecture Notes – Chapter 19
Video Lecture #1
Introduction to Conventional Current and Direct Current & Example Problem

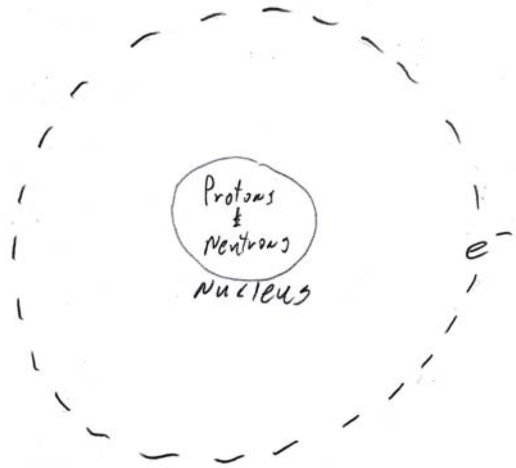
Current, I: The movement of charges. The rate at which charges pass by a point in a wire.

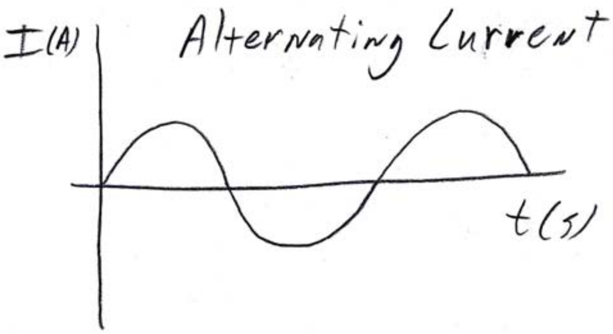
Bohr Model of the Atom: Protons and Neutrons in the nucleus with electrons in orbital shells. Electrons are much easier to remove from the atom; therefore it is generally electrons that flow in wires.

$$I = \frac{\Delta Q}{\Delta t} \Rightarrow \frac{Coulombs}{second} = \frac{C}{s} \Rightarrow Amperes, Amps, A$$

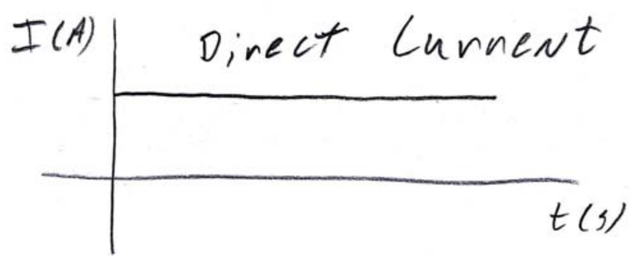
(Base SI Dimension)

Conventional Current: The direction that positive charges would flow. The reality is that negative charges flow in a negative direction.





Alternating Current, AC: Direction and magnitude of the current changes. Has a frequency like a sine or cosine wave. Less power loss over distance.



Direct Current, DC: Direction and magnitude of the current is constant. Large power loss over distance.

Many electronic devices have an AC/DC power converter to convert the alternating current that comes to your house to direct current. That is what the “brick” attached to your electronic devices is for.

Example Problem: A charge of 13.0 mC passes through a cross-section of wire in 4.5 seconds. (a) What is the current on the wire? (b) How many electrons pass through the wire in this time?

$\Delta Q = 13.0mC \times \frac{1C}{1000mC} = 0.013C$; $\Delta t = 4.5s$; a) I = ? b) # of electrons = ?

a) $I = \frac{\Delta Q}{\Delta t} = \frac{0.013}{4.5} = 0.002\overline{8}\,A \approx \boxed{0.0029A = 2.9mA}$

b) $Q = ne \Rightarrow n = \frac{Q}{e} = \frac{0.013}{1.6 \times 10^{-19}} = 8.125 \times 10^{16} e^{-} \approx \boxed{8.1 \times 10^{16} e^{-}}$

$n \approx 81 \times 10^{15} e^{-} = 81 Pe^{-} = 81{,}000{,}000{,}000{,}000{,}000\, e^{-}$ (That is a lot of electrons, eh?)

Flipping Physics Lecture Notes:

Resistivity

https://www.flippingphysics.com/resistivity.html

An open circuit does not contain a closed loop for current to flow and therefore current does not flow. A closed circuit does contain a closed loop for current to flow and therefore current does flow.

We tested various materials and discovered conductors such as aluminum, stainless-steel, and gold do allow current to flow. However, insulators such as plastic rubber and glass, do not allow current to flow. This is because conductors have electrons which are loosely bound to their atoms which allows current to flow. Whereas insulators have electrons which are tightly bound to their atoms which does not allow current to flow.

We have already learned about resistance, R, which is how an object limits current flow. Resistance is a physical property of an object.

$$\Delta V = IR \Rightarrow R = \frac{\Delta V}{I}$$

Today we learn about resistivity, ρ, which is a material property. Resistivity is a fundamental property of the material to limit electric current flow.

Because the difference between Resistance and Resistivity can be difficult for students to remember, I will repeat myself:

- Resistance is the property of an object.
- Resistivity is the property of a material.

Resistance and Resistivity are related by the following equation:

$$R = \frac{\rho L}{A}$$

- R = Resistance
- ρ = Resistivity. (I know. I am sorry. It's not density.)
- L = Length
- A = cross sectional area

The units for resistivity are: $\Omega \bullet m$

$$R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L} = \frac{\Omega \bullet m^2}{m} = \Omega \bullet m$$

Resistivities for some common materials at 20°C are:

Material	***ρ* @ 20°C (Ω·m)**		**Type**
Silver	1.6×10^{-8}	0.000000016	Conductor
Copper	1.7×10^{-8}	0.000000017	Conductor
Gold	2.4×10^{-8}	0.000000024	Conductor
Aluminum	2.8×10^{-8}	0.000000028	Conductor
Stainless Steel	6.9×10^{-7}	0.00000069	Conductor
Germanium	4.6×10^{-1}	0.46	Semiconductor
Silicon	6.4×10^{2}	620	Semiconductor
Glass	$10\times10^{10} – 10\times10^{14}$	100,000,000,000 – 1,000,000,000,000,000	Insulator
Hard Rubber	10×10^{13}	100,000,000,000,000	Insulator
Air	$1.3\times10^{16} – 3.3\times10^{16}$	13,000,000,000,000,000 – 33,000,000,000,000,000	Insulator

Resistivities compiled from electronics-notes.com[1] and sciencenotes.org[2].

Resistivity is temperature dependent.

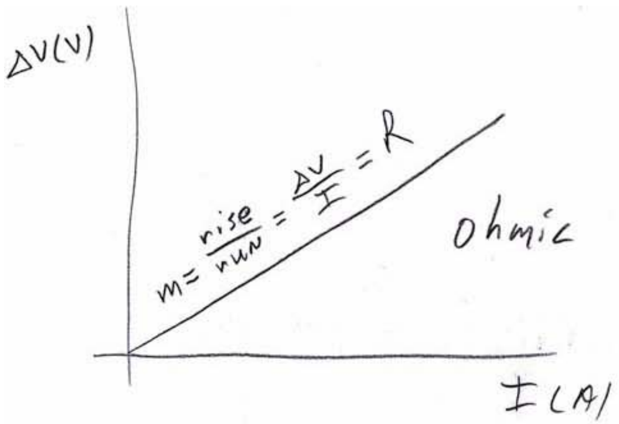
- Conductors: As temperature increases, resistivity increases.
- Semiconductors: As temperature increases, resistivity decreases.

[1] https://www.electronics-notes.com/articles/basic_concepts/resistance/electrical-resistivity-table-materials.php
[2] https://sciencenotes.org/table-of-electrical-resistivity-and-conductivity/

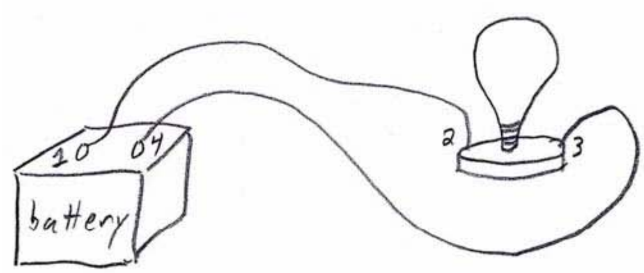
College Prep Physics II – Video Lecture Notes – Chapter 19
Video Lecture #2
Defining Resistance, Ohmic vs. Non-Ohmic, Electrical Power

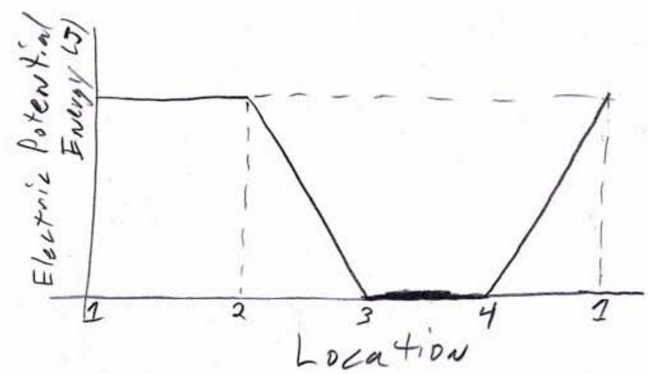
Resistance, R, is the resistance to current flow. $R = \frac{\Delta V}{I} \Rightarrow \boxed{\Delta V = IR}$ (Ohm's Law)

$R = \frac{\Delta V}{I} \Rightarrow \frac{Volts}{Amps} = \Omega$ or Ohms (Capital Omega, an upside down horse shoe, it's unlucky.)



Materials that follow Ohm's Law are called Ohmic. If they don't they are Non-Ohmic.
We will consider all resistors to be Ohmic, unless otherwise stated.





Electric Power: The rate at which electrical potential energy is being converted to heat, light and sound.

From 1-2 and 3-4 the charges are moving along the wire and we consider wires to have zero resistance unless otherwise stated.
From 2-3 the electric potential energy of the electrons is converted to heat, light and sound.
From 4-1 the electrons are being given electric potential energy by the battery.

Derivation of Electric Power Equation:

$$P = \frac{W}{t} = \frac{\Delta PE_{electric}}{t} \Rightarrow \frac{J}{s} = Watts \ \& \ \Delta V = \frac{\Delta PE_{ele}}{q} \Rightarrow \Delta PE_{ele} = q\Delta V$$

Therefore: $P = \frac{\Delta PE_{electric}}{t} = \frac{q\Delta V}{t} = \left(\frac{q}{t}\right)\Delta V = I\Delta V \ \& \ \Delta V = IR$

Gives: $P = I\Delta V = I(IR) = I^2 R \ \& \ \Delta V = IR \Rightarrow I = \frac{\Delta V}{R}$

Gives: $P = I^2 R = \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2 R}{R^2} = \frac{\Delta V^2}{R}$ Therefore: $\boxed{P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}}$

College Prep Physics II – Video Lecture Notes – Chapter 19
Video Lecture #3
Finding the cost to power light bulbs and Defining Kilowatt Hour

$0.12

Example Problem: 3 Light Bulbs; P_f = 15 watts; P_i = 60 watts; ΔV = 120 V; $\frac{\$0.12}{kW \cdot hr}$

$\frac{3hr}{day}$ (Bulbs are powered for this much time)
(yes, there are 24 hours in a day, however, the light bulbs are not on 24 hours each day.)

(A standard household circuit in the United States has a potential difference of 120 V.)

$$\Delta P = P_f - P_i = 15 - 65 = -50\,watts \Rightarrow \|\Delta P\| = 50\,watts$$

$$\Delta V = IR \Rightarrow R = \frac{\Delta V}{I} = \frac{120}{I} = ?????$$ (we don't know the current.)

$$P = \frac{\Delta V^2}{R} \Rightarrow R = \frac{\Delta V^2}{P} = \frac{120^2}{15} = \boxed{960\Omega}$$

Power saved by three bulbs: $P_{saved} = 3 \times 50 = 150\,watts \times \frac{1kW}{1000watts} = 0.15kW$

$$0.15kW \times \frac{3hr}{day} = \frac{0.45kW \cdot hr}{day} \Rightarrow \left(\frac{0.45kW \cdot hr}{day}\right)\left(\frac{\$0.12}{kW \cdot hr}\right)\left(\frac{365.242days}{1year}\right) = \frac{\$19.723}{year}$$

$$\$108 \times \frac{1year}{\$19.723} = 5.4758 \approx \boxed{5.5\,years}$$

What is a KiloWatt Hour?

$$(kW \cdot hr)\left(\frac{1000W}{1kW}\right)\left(\frac{3600s}{1hr}\right) = 3{,}600{,}000W \cdot s = 3{,}600{,}000\frac{J}{s} \cdot s = 3{,}600{,}000s = 3.6MJ$$

$1kW \cdot hr = 3.6MJ$ (you will not be given this as a conversion, you must derive it, every time.)

Flipping Physics Lecture Notes:

Graphing Resistivity

https://www.flippingphysics.com/graphing-resistivity.html

Given a length of nichrome wire and a variable power supply which displays both current and electric potential difference, what data would you need to collect and what would need to go on the axes of a graph such that the resistivity of nichrome would be the slope of the best-fit line of the data?

Let's start with Ohm's Law and solve for resistance and we also have an equation for resistance in terms of resistivity which we can set equal to one another.

$$\Delta V = IR \Rightarrow R = \frac{\Delta V}{I} = \frac{\rho L}{A}$$

Then we remove all variables from the denominators. We know have: $\Delta V A = \rho\left(IL\right)$

Comparing that to the slope intercept form equation for a line: $y = mx + b$

And you can see what variables go on the y and x axes:

$$\Rightarrow y = \Delta V A;\ m = slope = \rho;\ x = IL;\ b = 0$$

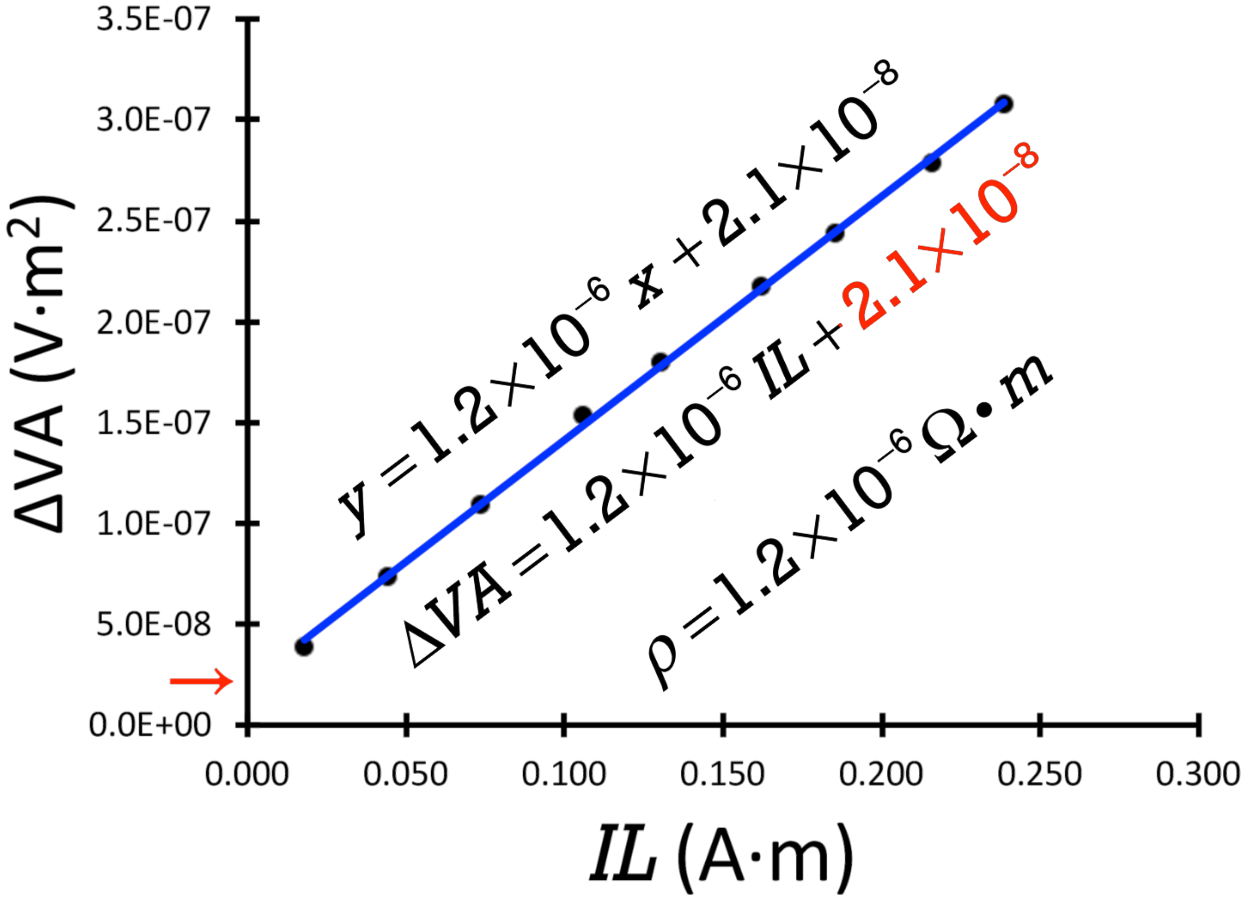
We will need the cross-sectional area of the wire. We know the wire is a 32-gauge wire. Which, according to American Wire Gauge standards, has a diameter of 0.202 mm, and a radius which is half that or 0.101 mm or 0.000101 m. Therefore, the cross-sectional area is the area of a circle or πr^2.

$$Diameter = 0.202mm \Rightarrow r = \frac{Dia}{2} = \frac{0.202mm}{2} = 0.101mm = 0.000101m$$

$$Area = \pi r^2 = \pi\left(0.000101\right)^2 = 3.20474 \times 10^{-8}\,m^2$$

And, if we adjust the length of the wire, then we can measure the electric potential difference across the wire and current through the wire.

ΔV (V)	Current (A)	Length (m)	ΔVA ($V \cdot m^2$)	IL (A·m)
1.2	0.36	0.050	3.8E-08	0.018
2.3	0.44	0.100	7.4E-08	0.044
3.4	0.49	0.150	1.1E-07	0.074
4.8	0.53	0.200	1.5E-07	0.11
5.6	0.52	0.250	1.8E-07	0.13
6.8	0.54	0.300	2.2E-07	0.16
7.6	0.53	0.350	2.4E-07	0.19
8.7	0.54	0.400	2.8E-07	0.22
9.6	0.53	0.450	3.1E-07	0.24



The resistivity we get from our experiment is roughly 1.2 x 10^{-6} Ω·m, which is right in the range we expect because the published value for the resistivity for nichrome at 20°C is in the range 1.0 – 1.5 x 10^{-6} Ω·m.[1]

Notice that b, the y-intercept, does not actually work out to be zero. It ends up being a small, positive number. That is because our solution assumes the wires we use in the experiment have zero resistance. The wires do have a small amount of resistance, which causes the y-intercept to have a small, positive number.

Another item to note is that I purposefully reduced the electric potential difference as the length of the wire was reduced. This is because resistivity of conductors increases with temperature. I did not want the nichrome wire to heat up too much during the experiment.

[1] https://hypertextbook.com/facts/2007/HarveyKwan.shtml

Flipping Physics Lecture Notes:

Electric Potential Difference and Circuit Basics
https://www.flippingphysics.com/electric-potential-difference.html

A mass, m, in a gravitational field can have gravitational potential energy, U_g.
Similarly, a charge, q, in an electric field can have electric potential energy, U_e.
We can determine the electric potential energy per unit charge, V, it is called electric potential:

$$V = \frac{U_e}{q}$$

However, typically we are interested in the *change* in electric potential energy per unit charge, which is called the electric potential difference, ΔV:

$$\Delta V = \frac{\Delta U_e}{q}$$

In other words, between any two points in an electric field there can exist an electric potential difference which represents the difference between the electric potential energies of those two locations per unit charge. This means you do not need a charge for that electric potential difference to be there. That electric potential difference is always there, essentially waiting for a charge to then provide that charge with a change in electric potential energy. You can determine the change in electric potential energy on a charge by multiplying the charge by the electric potential difference.

$$\Delta V = \frac{\Delta U_e}{q} \Rightarrow \Delta U_e = q\Delta V$$

It is not unusual for people to drop the "electric" from electric potential difference and just call it potential difference. I will do my best not to do that and always clearly identify ΔV as *electric* potential difference.

The units for electric potential difference are joules per coulomb:

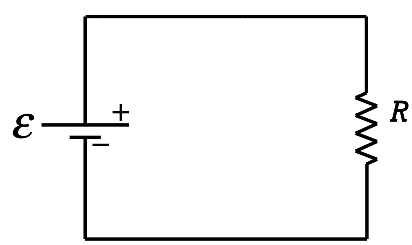
$$\Delta V = \frac{\Delta U_e}{q} \Rightarrow \frac{J}{C} = Volts, V$$

- Joules per coulomb are called Volts. The symbol for Volts is V.
- Electric potential difference is also commonly called the "voltage".
- Electric Potential Energy is a scalar; therefore, Electric Potential Difference is also a scalar.
- Please be careful to distinguish electric potential difference, ΔV, from the units for electric potential difference, V. I know they use the same symbol, which is irksome.
- Volts are named after the Italian physicist Alessandro Volta (1745–1827). He was a pioneer of electricity and power, and is credited with the invention of the electric battery.

Speaking of an electric battery, the maximum possible voltage a battery can provide between its terminals is called the electromotive force or emf. In a non-ideal battery, the emf differs from the battery's terminal voltage, ΔV_t, because the terminal voltage (the electric potential difference measured between the terminals of the battery) will be less than the emf of the battery because the internal resistance of the battery decreases the terminal voltage of the battery. Until further notice, however, all batteries are "ideal" and therefore, emf and ΔV_t are identical. In summary:

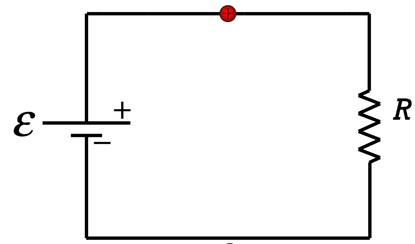
- emf (electromotive force) is the ideal, maximum voltage across a battery.
- ΔV_t (terminal voltage) is the voltage measured at the terminals of the battery.
- In a real battery, ΔV_t < emf due to internal resistance in the battery.
- However, for now, we assume all batteries are ideal and we are assuming ΔV_t = emf. ☺
- Oh, and emf has its own symbol, Ɛ.
- Also, I will point out that electromotive force is an "historical term" which, unfortunately, we still use even though it's a misnomer because it is *not* … a … force.

Let's take a look at a basic circuit with a battery, 2 connecting wires, and a resistor. The circuit diagram looks like this:

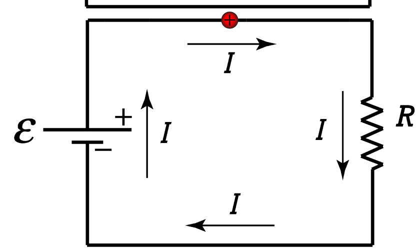


- The battery, which is indicated with the emf symbol, Ɛ, has a positive terminal which is indicated with a long line and a + and a negative terminal which is indicated with short line and a -.
- The resistor is labelled R for resistor.
- Please remember that, unless otherwise stated, all wires are considered to have zero resistance.

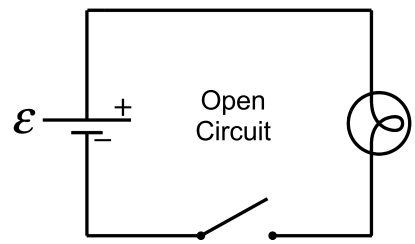
We need to determine which direction current will flow in the circuit. To do that we place a small, positive test charge at a location in the circuit (as shown in the diagram) and discuss, using the Law of Charges, which direction that small, positive test charge will experience an electric force.



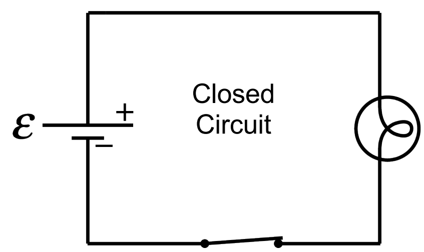
- We are using a positive test charge because conventional current is defined by the direction positive charges would flow. We do this even though we know negative charges (electrons) actually flow opposite the direction of conventional current. Yea!
- According to the Law of Charges, the positive charge is repelled away from the positive terminal of the battery (like charges repel) and the positive charge is attracted to the negative terminal of the battery (unlike charges attract). In other words…
- Current flows in a clockwise direction in this circuit.



Now let's add a switch to the circuit and replace the resistor with a light bulb so we can see evidence of current flow. As you can see in the video, with the switch open, there is no closed loop for the current to flow through, so current does not flow, the light bulb does not glow, and this is called an open circuit.

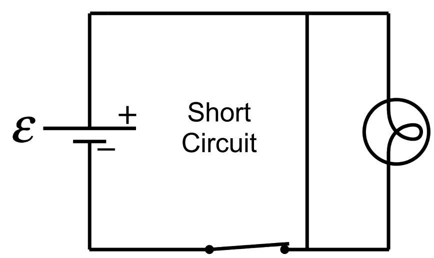


When we close the switch, there is now a closed loop for the current to flow through, current flows, the light bulb glows and this is called a closed circuit. The light bulb (or the resistor in the previous circuit) is called the "electrical load" of the circuit. The "electrical load" is the part of the circuit which is converting eclectic potential energy to heat, sound, and (in the case of the lightbulb) light. Because the switch and all the wires are "ideal" and considered to have zero resistance, those items are not a part of the electrical load. The battery is not a part of the electrical load because the battery is an electrical power source; the battery is a source of electric potential energy.



If we were to add a wire to this circuit which bypasses the load, this would be a short circuit. A short circuit is a circuit which has a very small resistance and therefore a very large current. Short circuits are usually the result of some sort of accident and should be avoided because, with a very small resistance and a constant electric potential difference, the electric power, or the rate at which electric potential energy is converted to heat, light, and sound, can be very large and dangerous.

$$P = \frac{\Delta V^2}{R}$$



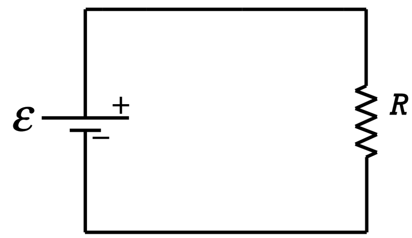
Flipping Physics Lecture Notes:

Kirchhoff's Rules of Electrical Circuits

https://www.flippingphysics.com/kirchhoff.html

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0$$

Kirchhoff's Two Rules for circuits are very basic rules which are used to understand circuits. Let's start with Kirchhoff's Loop Rule which states that the net electric potential difference around a closed loop equals zero.



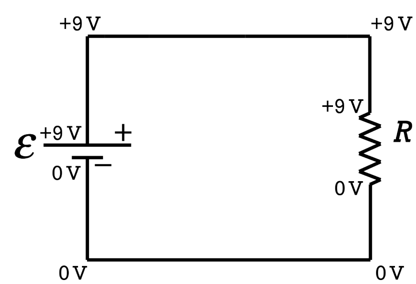
The Loop Rule is essentially conservation of electric potential energy in a circuit. Because electric potential difference equals change in electric potential energy per unit charge, the net change in electric potential energy in a closed loop then equals zero.

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0 \;\&\; \Delta V = \frac{\Delta U_e}{q} \Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \frac{\Delta U_e}{q} = 0 \Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \Delta U_e = 0$$

Using a gravitational potential energy analogy here, this is like saying, if you drop a mass off a wall, then pick up the mass and return it to its original location, the change in gravitational potential energy of that mass equals zero. We know this to be true because the mass returns back to the same height as where it started, so the mass will have the same gravitational potential energy at the end as it did at the beginning, no matter where we place the horizontal zero line.

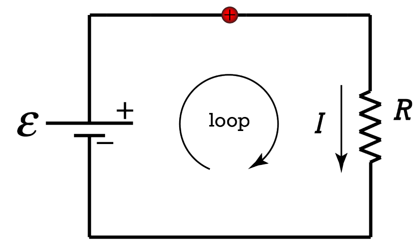
Going back to electric potential energy, this means, after a charge goes through one full, closed loop around a circuit, the electric potential energy of the charge will return back to its original value. But because we are using electric potential, we are really talking about the electric potential energy per unit charge at each location.

Let's say we have a 9-volt battery. That means we know the electric potential difference across the battery equals 9 volts. As we go from the negative to the positive terminals of the battery, the electric potential will go up. Technically we do not know the electric potential at any point, only the *difference* in the electric potential, however, it is customary to assume the minimum electric potential is zero. That means we are assuming the negative terminal of the battery is at zero volts and the positive terminal of the battery is at positive 9 volts.



Because ideal wires have zero resistance, that means the electric potential in the upper left corner must also be 9 volts, the electric potential in the upper right corner equals 9 volts, and the electric potential at the top of the resistor is 9 volts. Also, the electric potential in the lower left corner must be the same as the negative terminal of the battery, so electric potential in the lower left corner equals 0 volts. Therefore, electric potential in the lower right corner is 0 volts, and the electric potential at the bottom of the resistor equals 0 volts. This means the electric potential difference across the resistor also has a magnitude of 9 volts. In other words, in this circuit with two circuit elements, the two elements, the battery and the resistor, both have the same magnitude electric potential difference.

In a previous lesson we determined that a positive charge in the circuit would be repelled from the positive terminal of the battery and attracted to the negative terminal of the battery, therefore the current in this circuit is clockwise. This means the current is down through the resistor.



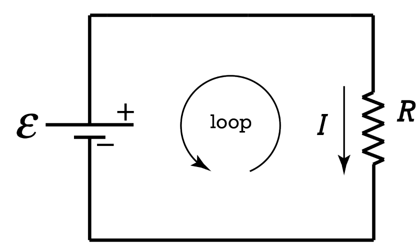
There is only one closed loop in our present circuit, so it might not seem obvious that we need to do this, however, we need to define a loop direction. Often the loop direction is the same as the direction which goes from the negative terminal to the positive terminal of the battery and through the battery, therefore, our loop direction for this circuit is clockwise.

This means as we go in the direction of the loop across the battery, the electric potential goes up because we go from the negative to the positive terminal of the battery. Therefore, when we sum the electric potential differences in our Kirchhoff's loop equation, the electric potential difference across the battery is positive. When we go in the direction of the loop across the resistor, as we illustrated before, the electric potential goes down. Therefore, in our loop equation, the electric potential difference across the resistor is negative. We know the electric potential difference across the battery equals the electromotive force or the emf of the battery. And the electric potential difference across the resistor equals current times resistance. Therefore, we can determine the current in the circuit in terms of the emf of the battery and the resistance of the resistor.

$$\Delta V_{Battery} = \varepsilon \ \& \ \Delta V_{Resistor} = IR$$

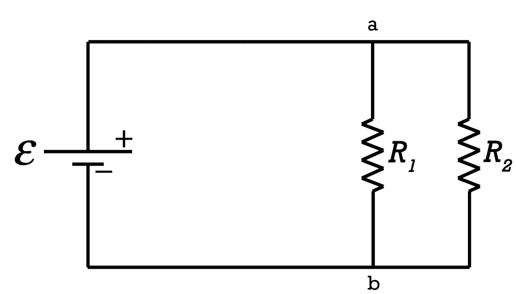
$$\Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = \Delta V_{battery} - \Delta V_{Resistor} = 0 = \varepsilon - IR \Rightarrow \varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R}$$

If we had chosen counterclockwise as the loop direction, all of our electric potential differences in Kirchhoff's Loop Rule would have been reversed. Because the loop direction goes from the positive to the negative terminals of the battery, the electric potential difference across the battery is negative, because the electric potential is going down. Because the loop direction through the resistor is opposite the direction of the current direction we defined through the resistor, the electric potential goes up through the resistor and the electric potential difference the across resistor is positive.



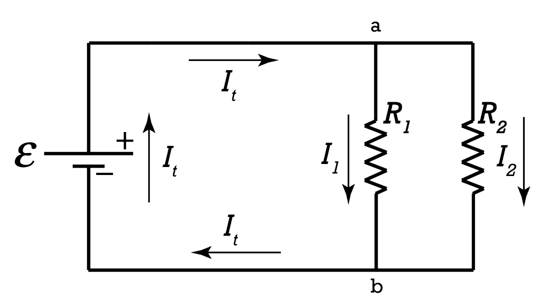
$$\Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = -\Delta V_{battery} + \Delta V_{Resistor} = 0 = -\varepsilon + IR \Rightarrow \varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R}$$

Realize, we get the same result for the current in the circuit regardless of which loop direction we choose. If we had chosen an incorrect direction for current, the current ends up being negative, which tells you that you chose the incorrect current direction.



Now let's add a resistor to the circuit and talk about Kirchhoff's Junction Rule which is the result of conservation of charge in the circuit. The rule is sum of the currents entering a junction must equal the sum of the currents leaving a junction, which is conservation of charge:

$$\sum I_{in} = \sum I_{out}$$



Junctions are locations in circuits where at least three circuit paths meet. That means in our circuit we have two junctions which are labelled *a* and *b*. Therefore, the current going into both of those junctions equals the current coming out of those junctions. This means we need to define current directions. We do this the same way we did before, we place a positive test charge in the circuit and see which direction the Law of Charges defines electric force direction on the charge. This means current will go to the right through the top wire, to the left through the bottom wire, and down through both resistors. Let's label those currents as current 1 through resistor 1, current 2 through resistor 2, and current t through the battery because it is the current through the terminals of the battery.

Kirchhoff's Junction Rule equations for this circuit are for:

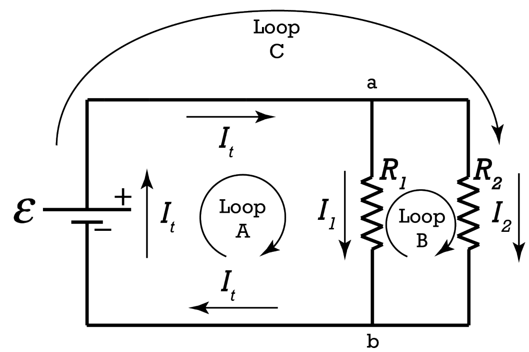
junction a: $I_t = I_1 + I_2$

junction b: $I_t = I_1 + I_2$

Yes, these two equations are actually the same.

But how do we know *a* and *b* are junctions and the four exterior "corners" of the circuit are not junctions? I know it may seem obvious because there are not at least three circuit paths at any of those locations, however, this is a simple circuit. Again, we return back to placing a positive test charge on the wire. Notice that a charge which approaches point a could go in the wire leading to resistor 1 or in the wire leading to resistor 2. Because junctions are defined as having three circuit paths, any time a charge comes to a fork in the wire, the charge could go down either wire, that makes it a junction. When a charge enters a corner, there is no other choice but to continue along the same wire, therefore none of the corners are junctions.

Let's identify the loops in this second circuit and determine their Kirchhoff's Loop Rule equations. We can define the first loop as the same as the previous circuit, but let's call it loop A with a clockwise direction. There is another loop that contains resistor 1 and resistor 2. Let's call that loop B and also have that be clockwise. Lastly there is a loop all the way around the outside; It includes the battery and resistor 2. Let's call that loop C and have it also be clockwise.



Kirchhoff's Loop Rule equations look like this:

$$\sum_{Loop\ A} \Delta V = \Delta V_t - \Delta V_{R_1} = \varepsilon - I_1 R_1 = 0 \Rightarrow \varepsilon = I_1 R_1 \Rightarrow I_1 = \frac{\varepsilon}{R_1}$$

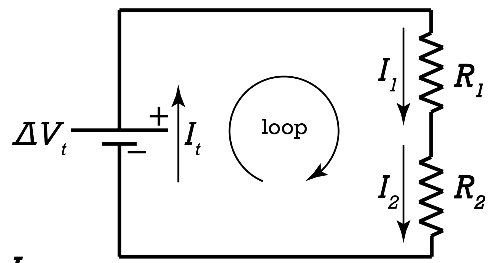
$$\sum_{Loop\ C} \Delta V = \Delta V_t - \Delta V_{R_2} = \varepsilon - I_2 R_2 = 0 \Rightarrow \varepsilon = I_2 R_2 \Rightarrow I_2 = \frac{\varepsilon}{R_2}$$

$$\sum_{Loop\ B} \Delta V = \Delta V_{R_1} - \Delta V_{R_2} = I_1 R_1 - I_2 R_2 = 0 \Rightarrow I_1 R_1 = I_2 R_2$$

But notice the third equation is actually just a combination of the previous two: $\varepsilon = I_1 R = I_2 R_2$

Flipping Physics Lecture Notes:

Resistors in Series and Parallel Equation Derivations
https://www.flippingphysics.com/series-parallel-resistors.html



We start with a circuit with a battery and two resisters in series. Because a positive charge would be repelled from the positive terminal of the battery and attracted to the negative terminal of the battery, the current in this circuit is clockwise or up through the battery and down through each resistor. Let's label those currents as the terminal current through the battery and current 1 and current 2 through their respective resistors. Hopefully you recognize that each charge on the wire has to go through all three of these circuit elements, therefore all of these currents are equal: $I_t = I_1 = I_2$

According to Kirchhoff's Loop Rule, a charge moving all the way around a loop in a circuit must end with the same electric potential energy it started with, therefore, the electric potential difference all the way around a loop is equal to zero. If we define the loop in a clockwise direction in our circuit, Kirchhoff's Loop Rule looks like this: $\Delta V_{loop} = 0 = \Delta V_t - \Delta V_1 + \Delta V_2 \Rightarrow \Delta V_t = \Delta V_1 + \Delta V$

Because electric potential difference equals current times resistance, we can substitute current times resistance for each of the electric potential differences. For the battery, the terminal voltage equals the current at the terminals of the battery times the equivalent resistance of the electrical load. The electrical load in this circuit is the two resistors in series.

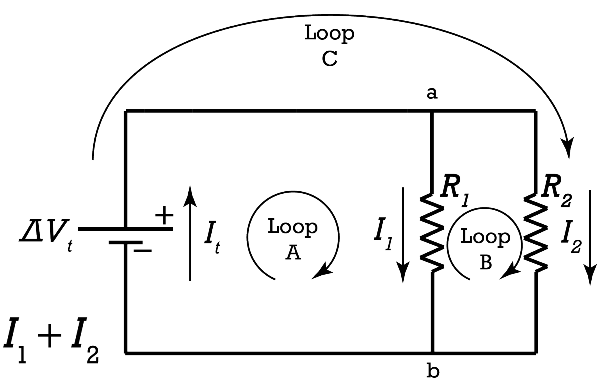
$$\Delta V = IR$$

$$\Rightarrow I_t R_{eq} = I_1 R_1 + I_2 R_2$$

All the currents are the same, so they cancel out and the equivalent resistance for two resistors in series equals the sum of those two resistors. We could perform this experiment with as many resistors in series as we wanted to, and the equivalent resistance would always be the sum of the resistances.

$$\Rightarrow R_{eq} = R_1 + R_2 \Rightarrow \boxed{R_{series} = R_1 + R_2 + R_3 + \ldots}$$

Note: In series circuit elements currents are the same and electric potential differences add.



Now let's do a circuit with a battery and two resistors in parallel. Again, the current directions are up through the battery and down through each of the resistors. There are two junctions in the circuit; junction a and junction b. Using Kirchhoff's Junction Rule, which is a result of conservation of charge, the fact that every charge that goes into the junction must come out of the junction, for junction a we get:

$$\sum I_{in} = \sum I_{out} \Rightarrow I_t = I_1 + I_2$$

We can define three loops for Kirchhoff's Loop Rule as shown in the figure. Remembering that electric potential goes up as you go from the negative to the positive terminals of the battery and, as you go in the direction of current across a resistor, the electric potential goes down; these are the equations for loop A and loop C:

$$\Delta V_{Loop\ A} = 0 = \Delta V_t - \Delta V_1 \Rightarrow \Delta V_t = \Delta V_1$$

$$\Delta V_{Loop\ C} = 0 = \Delta V_t - \Delta V_2 \Rightarrow \Delta V_t = \Delta V_2$$

Notice then that all of the electric potential differences in this circuit are the same. And because electric potential difference equals current times resistance, current equals electric potential difference divided by

$$\Rightarrow \Delta V_t = \Delta V_1 = \Delta V_2 \ \& \ \Delta V = IR \Rightarrow I = \frac{\Delta V}{R}$$

resistance. Therefore, we can combine these equations to solve for the equivalent resistance of the two resistors in parallel:

$$I_t = I_1 + I_2 \Rightarrow \frac{\Delta V_t}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

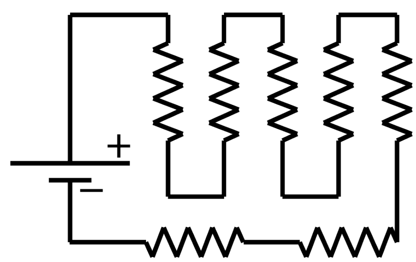
We could perform this experiment with as many resistors in parallel as we want, and the equivalent resistance will always be equal to the inverse of the sum of the inverses of all the resistors in parallel.

$$R_{\|} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\right)^{-1}$$

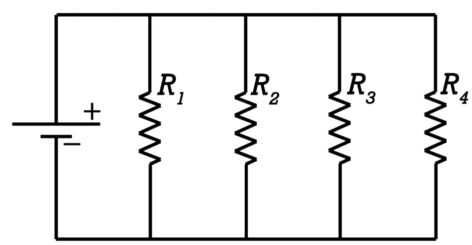
Note: In parallel circuit elements currents add and electric potential differences are the same.

So, notice that adding resistors in series increases the net resistance of the resistors and adding resistors in parallel decreases the net resistance of the resistors. Think of it this way, by adding a resistor in series, you are adding resistance to the path the charges to go through which, no matter how small that resistance is, still increases the resistance. When you add a resistor in parallel, you are adding an additional path for the charges have to go through and therefore, no matter how large the resistor is which you are adding in parallel, the addition of another pathway for the charges to travel decreases the overall resistance.

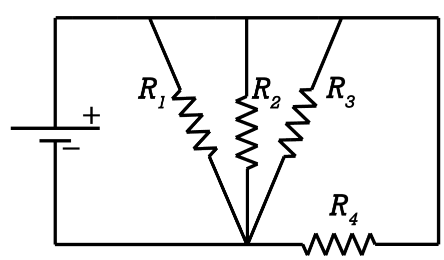
It may seem pretty obvious in simple circuits like the ones we just went through; however, it is important to identify when circuit elements are in series, parallel, or neither. Let's start with series. If every charge that goes through one element also has to go through the other element, those two circuit elements are in series. For example, all of the resistors in the following circuit are in series. This is because every charge in the circuit has to pass through every one of the resistors in the circuit.



Circuit elements which are in parallel all have the same electric potential difference. For example, all the resistors in the following circuit have the same potential at the top and bottom of the resistor, so their electric potential differences are the same. Another way to look at this is that if the charges are split between resistors and then all the charges come back together again, the resistors are in parallel.

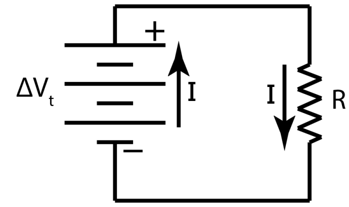


If you look at the next circuit, it appears to be different, however, the top of each resistor (or right side in the case of resistor 4) are all at the same electric potential and the bottom of each resistor (or left side in the case of resistor 4) are at the same electric potential, therefore, all four of these resistors are still in parallel. In fact, this is the same circuit as before, it has simply been drawn slightly differently.

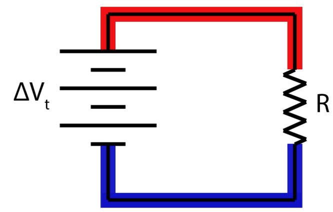


Flipping Physics Lecture Notes:

Basic Series and Parallel Resistor Circuit Demos and Animations
https://www.flippingphysics.com/series-parallel-resistors-basic.html



Before we analyze resistors in series and in parallel, let's get our bearings using a circuit with a battery and one resistor. First off, realize the current will go through the battery from the negative to the positive terminals of the battery. The current will therefore be up through the battery and down through the resistor. Because there is only one loop in the circuit, there is only one current, which we will simply label I.



Because there are only two elements in the circuit, both elements have the same magnitude electric potential difference equal to the terminal voltage, ΔV_t. You can see this by using the electric potential color-coding technique. Starting at the positive terminal of the power supply we draw red on the wire until we come to another circuit element. Everything in red is at the same electric potential. Starting at the negative terminal of the power supply we draw blue on the wire until we come to another circuit element. Everything in blue is at the same electric potential. Both the power supply and the resistor have an electric potential difference which is between red and blue, so both the power supply and the resistor have the same magnitude electric potential difference.

Now we can solve for the current in the circuit: $$\Delta V = IR \Rightarrow I = \frac{\Delta V_t}{R} = \frac{5.0V}{5.0\Omega} = 1.0A = 1.0\frac{C}{s}$$

With a real example of a 5.0 volt power supply and a 5.0 Ω resistor, we should expect to observe 1.0 amps of current through the circuit or 6.2 million million million electrons every second.

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{1C}{1.6\times10^{-19}\frac{C}{electron}} = 6.25\times10^{18} \approx 6.2\times10^{18}\,electrons$$

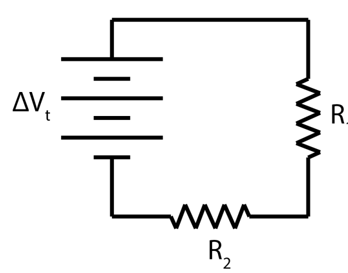
Unfortunately, we only get 0.97 Amps through the power supply. Measuring the electric potential difference across the resistor shows that the resistor actually has 4.9 volts across it. Likely this means two things. One is that we have real wires which have resistance instead of ideal wires which do not have resistance. And the power supply is actually delivering slightly less than what it is displaying.

We can also determine the rate at which the resistor is converting electric potential energy to heat:

$$P = I\Delta V = (1)(5) = 5.0\,watts$$

This means our resistor is converting 5.0 joules of energy to heat every second, which is why the resistor is getting HOT!

Please watch the video to see the animations relating charged particle location and electric potential energy. I could try to describe it here, but it's an animation. You should watch it instead.

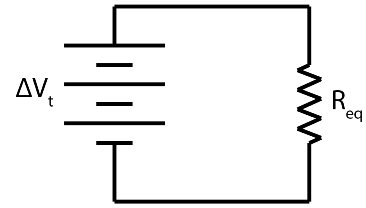


Next, let's analyze two resistors in series with a 5.0 V power supply. Both resistors have a resistance of 5.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.

First, we know these are two resistors in series because there are no other paths for charges to follow. Every charge which goes through resistor one must eventually go through resistor two. Because the two resistors are in series, their resistances add to get their equivalent resistance:

In Series: $R_{eq} = R_1 + R_2 = R + R = 2R = (2)(5) = 10\Omega$



Now we can replace the two resistors in our circuit with one equivalent resistor, R_{eq}. Notice how this is the same setup as our original circuit, one resistor and one power supply, therefore:

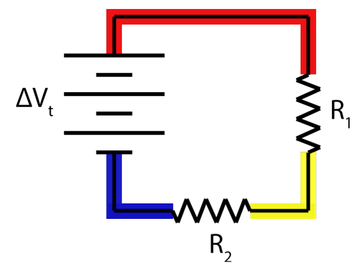
$$\Delta V_t = IR_{eq} \Rightarrow I = \frac{\Delta V_t}{R_{eq}} = \frac{5}{10} = 0.50A$$

The observed value for the current in the circuit is 0.49 A, which is to be expected based on our observed value for the first circuit. We have answered part (a), the accepted current through all circuit elements equals 0.50 A.

Because we know the current through and resistance of each resistor, we can now determine (b), the electric potential difference across each resistor, 2.5 V.

$$\Delta V_1 = I_1 R_1 = (0.5)(5) = 2.5V \ \& \ \Delta V_2 = I_2 R_2 = (0.5)(5) = 2.5V$$

Because both resistors have the same resistance and current through them, the electric potential differences across each resistor are equal.



Also, I want to use the electric potential color-coding technique to show that the electric potential differences across the two resistors add up to the terminal voltage. The highest electric potential wire is in red, the lowest electric potential wire is in blue, and the wire with a middle value electric potential is in yellow. Therefore, the terminal voltage is from blue to red, and across the resistors the electric potential goes from red to yellow plus yellow to blue. This means that:

$$\Delta V_t = \Delta V_1 + \Delta V_2 \Rightarrow 2.5 + 2.5 = 5.0V$$

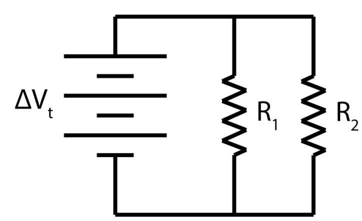
Again, we showed this because 2.5 volts plus 2.5 volts equals 5 volts.

For part (c), the power dissipated by each resistor is:

$$P_1 = P_2 = \frac{\Delta V_1^2}{R_1} = \frac{\Delta V_2^2}{R_2} = \frac{(2.5)^2}{5} = 1.25 \approx 1.2 watts$$

Therefore, the total power dissipated by both resistors is 2.5 watts; which is the same as the power delivered by the power supply:

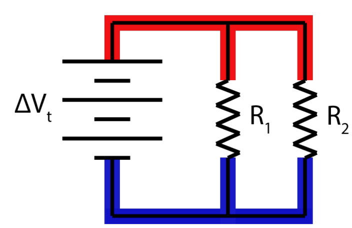
$$P_{both\ resistors} = P_1 + P_2 = 1.25 + 1.25 = 2.5 watts \ \& \ P_{power\ source} = I\Delta V_t = (0.5)(5) = 2.5 watts$$



Next, let's analyze two resistors in parallel with a 5.0 V power supply. Both resistors have a resistance of 10.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.

Let's start by color-coding the circuit to show that the two resistors are in fact in parallel. You can see all electric potential differences are the same, therefore the resistors are in parallel.



Part (b) $\Delta V_t = \Delta V_1 = \Delta V_2 = 5.0V$

We can determine the equivalent resistance of the two resistors by using the equation for resistors in parallel:

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = \left(\frac{2}{10}\right)^{-1} = \left(\frac{1}{5}\right)^{-1} = 5.0\Omega$$

The equivalent resistance for two 10.0 Ω resistors in parallel is 5.0 Ω. In other words, we can replace the two 10.0 Ω resistors with one 5.0 Ω resistor and the current through the circuit should remain unchanged.

But notice, this is exactly the same as the circuit we started with. Therefore, the current delivered by the power supply is 1.0 A and the power delivered by the power supply is 5.0 watts.

We know the electric potential across each circuit element is the same, so we can determine the current through each resistor:

$$\Delta V = IR \Rightarrow I_1 = \frac{\Delta V_1}{R_1} = I_2 = \frac{\Delta V_2}{R_2} = \frac{5}{10} = 0.50A$$

Part (a)

And we know the currents in parallel add: $I_t = I_1 + I_2 = 0.5 + 0.5 = 1.0A$

Which confirms our previous calculation for the current thorugh the battery.

And now we can calculate the power dissipated by each resistor:

$$P = I^2R \Rightarrow P_1 = P_2 = I_1^{\,2}R_1 = I_2^{\,2}R_2 = (0.5)^2(10) = 2.5watts$$

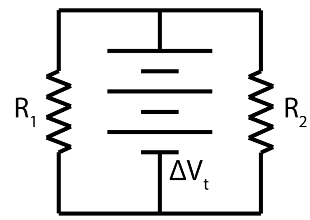
Part (c)

This makes sense because the total power delivered by the power supply should equal the total power dissipated by the resistors:

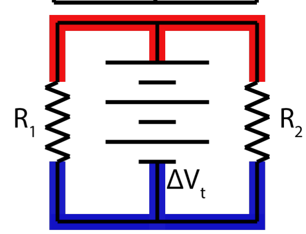
$$P_{both\ resistors} = P_1 + P_2 = 2.5 + 2.5 = 5.0watts\ \&\ P_{power\ source} = I\Delta V_t = (1)(5) = 5.0watts$$

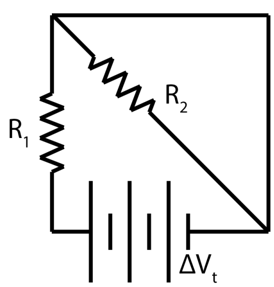
Next, let's analyze two resistors in this circuit with a 5.0 V power supply. Both resistors have a resistance of 10.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.



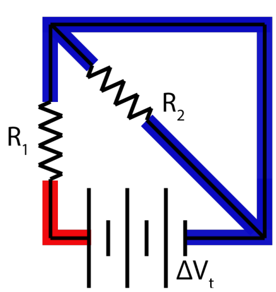
Let's start by color-coding the electric potential of the wires to determine if the resistors are in series or parallel. Hopefully you now recognize, because the electric potential color-coding is exactly the same, that this is the same circuit we just did, only drawn slightly differently. All the answers are the same.





Next, analyze two resistors in this circuit with a 5.0 V power supply. Both resistors have a resistance of 5.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.

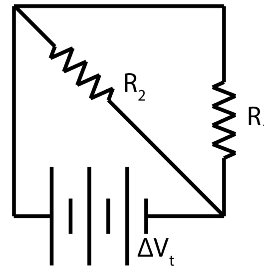


Let's start by color-coding the electric potential of the wires to determine if the resistors are in series or parallel. Notice the electric potential difference across resistor 2 is zero, this means no current will flow across resistor 2. This is because the wire in the upper right corner completely shorts resistor 2 out of the circuit. Because that wire has zero resistance, all charges will flow along that wire and none will flow through resistor 2. In other words, this circuit behaves the same as our very first circuit, other than resistor 2 having no current, no electric potential difference, and therefore no power dissipated. Therefore, our answers are:

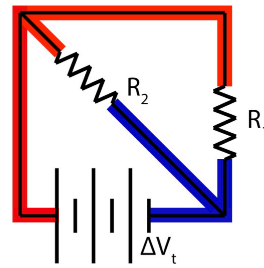
a) I_1 = 1.0 A and I_2 = 0
b) ΔV_1 = 5.0 V and ΔV_2 = 0
c) P_1 = 5.0 Watts and P_2 = 0

Okay, one last example. Analyze two resistors in this circuit with a 5.0 V power supply. Both resistors have a resistance of 10.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.



Again, we start by color-coding the electrical potential differences of the circuit.



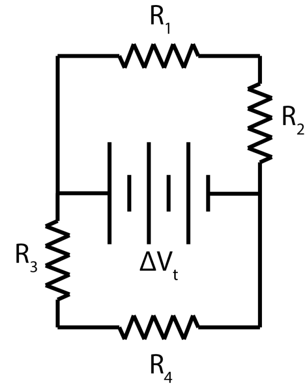
Both resistors have the same electric potential difference. This is two resistors in parallel with a power supply. This is actually the same as two circuits we already analyzed.

Please, be careful to look at circuits carefully before throwing equations at them. You need to first determine which circuit elements are in series and parallel, then you can start using equations.

Flipping Physics Lecture Notes:

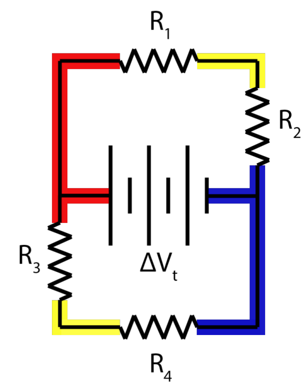
Intermediate Series and Parallel Resistor Circuit

https://www.flippingphysics.com/series-parallel-resistors-intermediate.html



The circuit shown has four 5.0 Ω resistors and a 5.0 V power source. Determine …

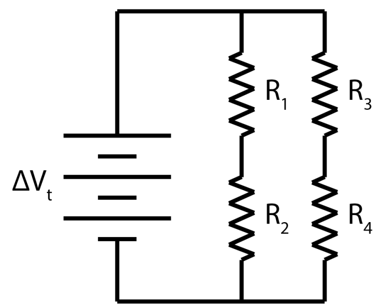
a) The equivalent resistance of all four resistors.
b) Current through each circuit element.
c) Electric potential difference across each resistor.
d) Power dissipated by each resistor.



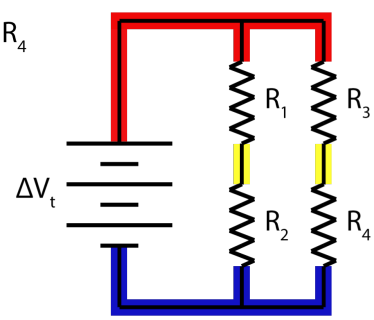
Start by color-coding the electric potential along all the wires of the circuit. From the color-coded circuit diagram, you can now see several things:

1) R_1 and R_2 are in series.
2) R_3 and R_4 are in series.
3) Those two sets of series resistors are in parallel.

In other words, we can redraw the circuit diagram like this:



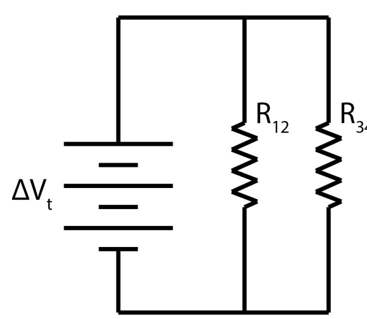
In fact, it is even easier to see they are the same circuit when we color-code the electric potential of the circuit:



I do want to point out that the only reason we know the wires between R_1 and R_2, and R_3 and R_4 are all at the same electric potential, and therefore the same color yellow, is because all four resistors have equal resistance. If they did not have the same number of ohms, then the electric potential difference across each resistor would be different and the electric potential between the resistors would not be the same.

Now we can begin solving the problem. Let's start by determining the equivalent resistance of the resistor pairs.

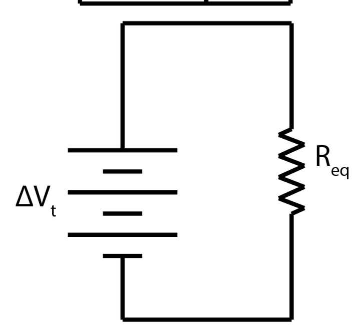
$$R_{12} = R_1 + R_2 = 5 + 5 = 10\Omega \quad \& \quad R_{34} = R_3 + R_4 = 5 + 5 = 10\Omega$$



We can replace R_1 and R_2 with equivalent resistor R_{12}.
We can replace R_3 and R_4 with equivalent resistor R_{34}.

And now we have two resistors in parallel and can deterimen the equivalent resistance of those two resistors:

$$R_{eq} = \left(\frac{1}{R_{12}} + \frac{1}{R_{34}}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = \left(\frac{2}{10}\right)^{-1} = \left(\frac{1}{5}\right)^{-1} = 5.0\Omega$$



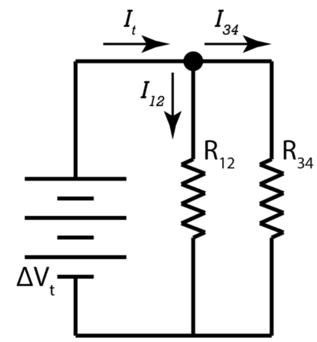
Part (a): We can replace all four resistors with one equivalent resistance of, R, or 5.0 Ω.

Now we can determine the current delivered by the power source to the equivalent resistor:

$$\Delta V = IR \Rightarrow I_t = \frac{\Delta V_t}{R_{eq}} = \frac{5}{5} = \boxed{1.0A}$$

So, the power source is delivering 1 coulomb of charge every second to the circuit. And we know the electric potential difference across the power source is the same magnitude as the electric potential difference across the two resistors R_{12} and R_{34}.

$$\Delta V_t = \Delta V_{12} = \Delta V_{34} = 5.0V$$



Therefore, we can determine the current through R_{12} and R_{34}.

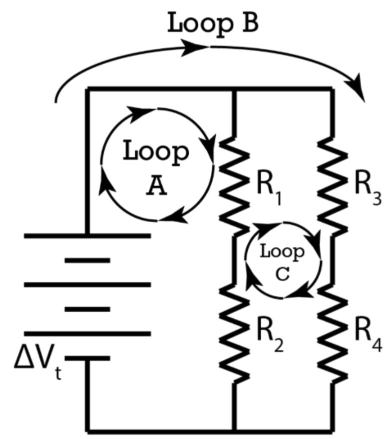
(b) $$I_{12} = \frac{\Delta V_{12}}{R_{12}} = \frac{5}{10} = \boxed{0.50A = I_1 = I_2} \;\&\; I_{34} = \frac{\Delta V_{34}}{R_{34}} = \frac{5}{10} = \boxed{0.50A = I_3 = I_4}$$

These currents make sense because we know Kirchhoff's Junction Rule states that the current going into a junction equals the current going out of a junction:

$$I_t = I_{12} + I_{34} = 0.5A + 0.5A = 1A$$

Now, we can determine the electric potential difference across each resistor:

(c) $$\boxed{\Delta V_1 = I_1 R_1 = (0.5)(5) = 2.5V = \Delta V_2 = \Delta V_3 = \Delta V_4}$$



This makes sense because we know Kirchhoff's Loop Rule states that the electric potential difference around any loop equals zero:

$$\Delta V_{loop\ A} = 0 = \Delta V_t - \Delta V_1 - \Delta V_2 = 5 - 2.5 - 2.5 = 0$$

$$\Delta V_{loop\ B} = 0 = \Delta V_t - \Delta V_3 - \Delta V_4 = 5 - 2.5 - 2.5 = 0$$

$$\Delta V_{loop\ c} = 0 = \Delta V_1 + \Delta V_2 - \Delta V_3 - \Delta V_4 = 2.5 + 2.5 - 2.5 - 2.5 = 0$$

And lastly, we can determine the rate at which energy is dissipated in each circuit element:

$$\boxed{\begin{aligned} P_t &= I_t \Delta V_t = (1)(5) = 5.0watts \\ P_1 &= I_1^2 R_1 = (0.5)^2(5) = 1.25watts \approx 1.2watts = P_2 = P_3 = P_4 \end{aligned}}$$

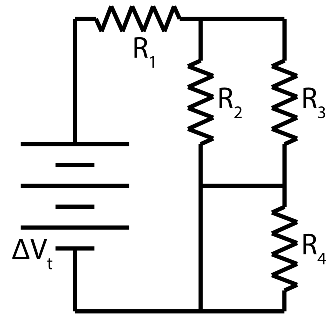
Which makes sense because the power added to the circuit from the power source needs to equal the power dissipated by the circuit:

$$P_t = P_1 + P_2 + P_3 + P_4 = 1.25 + 1.25 + 1.25 + 1.25 = 5.0watts$$

Flipping Physics Lecture Notes:

Resistor Circuit Example
http://www.flippingphysics.com/resistor-circuit-example.html

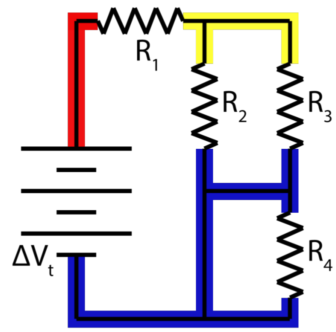
The circuit shown has four identical 5.0 Ω resistors and a 5.0 V battery. What is the current through, electric potential difference across, and power dissipated by resistor 4?



We know $\Delta V_4 = I_4 R_4$. Because we know the resistance of resistor 4, if we know either the current through or electric potential difference across resistor 4, we can determine the other unknown.

We also know $P_4 = I_4 \Delta V_4 = I_4^2 R_4 = \frac{\Delta V_4^2}{R_4}$. So again, if we know either current through or electric potential diference across resistor 4, then we can determine the power dissipated by resistor 4.

Let's do the electric potential difference color coding technique to help us determine the electric potential difference across resistor 4.



From this we can see that the electric potential difference across resistor 4 is zero, therefore the current through resistor 4 is also zero, and the power dissipated by resistor 4 is zero.

$$\Delta V_4 = 0$$

$$\Delta V_4 = I_4 R_4 \Rightarrow I_4 = \frac{\Delta V_4}{R_4} = \frac{0}{R_4} = 0$$

$$P_4 = I_4^2 R_4 = (0)^2 (0) = 0$$

Resistor 4 is short circuited in this circuit.

Also, please enjoy the animation of the charges moving through the circuit in the video. It really helps with understanding how the charges are moving in the circuit.