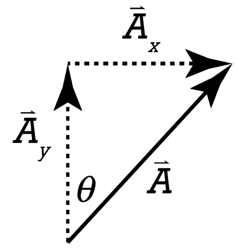
Flipping Physics Lecture Notes:
AP Physics 1 Review of *Kinematics*
https://www.flippingphysics.com/ap1-kinematics-review.html

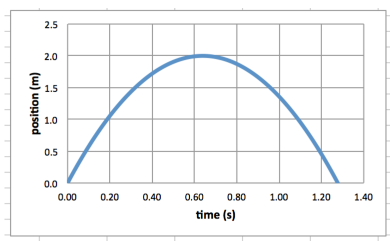
Introductory Concepts:

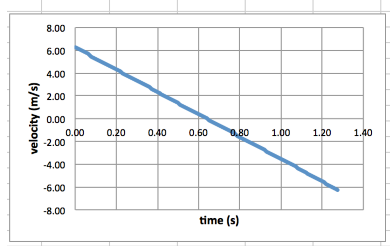


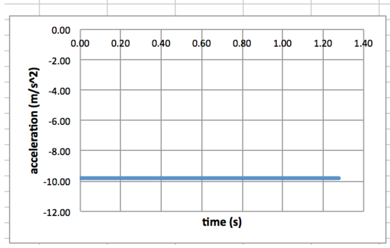
- Vector: Magnitude and Direction
  - Magnitude means the “amount” of the vector or the value of the vector without direction.
- Scalar: Magnitude only, no direction
- Component Vectors
  - Theta won’t always be with the horizontal, so the component in the x direction won’t always use cosine.
  - $\sin\theta = \frac{O}{H} = \frac{\vec{A}_x}{\vec{A}} \Rightarrow \vec{A}_x = \vec{A}\sin\theta$

Kinematics:

- Distance vs. Displacement
  - Distance is how far something moves and it includes the path travelled.
    - Distance is a scalar.
  - Displacement is the straight-line distance from where the object started to where it ended.
    - Displacement is a vector.
    - Displacement is the change in position of an object. $\Delta\vec{x} = x_f - x_i$
- $Speed = \frac{Distance}{Time}$, is a scalar.
- Velocity, $\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$, is a vector.
- Acceleration, $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$, is a vector.
- The slope of a position vs. time graph is velocity.
- The slope of a velocity vs. time graph is acceleration.
- On an acceleration vs. time graph, the area between the curve & the time axis is change in velocity.
- On a velocity vs. time graph, the area between the curve & the time axis is change in position which is also called displacement.
- In Free Fall, $a_y = -g = -9.81\frac{m}{s^2}$.
  - An object is in free fall if the only force acting on it is the force of gravity. In other words: the object is flying through the vacuum you can breathe♥ and not touching any other objects.







♥ Vacuum you can breathe = no air resistance.

- The Uniformly Accelerated Motion Equations (UAM Equations):

AP® Physics 1 Equation Sheet	*Flipping Physics®*
$v_x = v_{x0} + a_x t$	$v_f = v_i + a\Delta t$
$x = x_0 + v_{xo} t + \frac{1}{2} a_x t^2$	$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$
$v_x^2 = v_{x0}^2 + 2a_x \left(x - x_0\right)$	$v_f^2 = v_i^2 + 2a\Delta x$
	$\Delta x = \frac{1}{2}\left(v_f + v_i\right)\Delta t$

- The AP Physics 1 UAM Equations assume $t_i = 0$; $\Delta t = t_f - t_i = t_f - 0 = t$
- Projectile Motion: An object flying through the vacuum you can breathe in at least two dimensions.

x direction	*y direction*
$a_x = 0$	Free-Fall
Constant Velocity	$a_y = -g = -9.81 \frac{m}{s^2}$
$v_x = \frac{\Delta x}{\Delta t}$	Uniformly Accelerated Motion
Δt is the same in both directions because it is a *scalar* and has magnitude only (no direction).	

  - Remember to break your initial velocity into its components if it is not directly in the x direction and if the initial velocity is directly in the x direction, then the initial velocity in the y direction equals zero.
- Relative Motion is Vector Addition.
  - Draw vector diagrams.
  - Break vectors into components using SOH CAH TOA.
  - Make a right triangle.
  - Use SOH CAH TOA and the Pythagorean theorem to determine the magnitude and direction of the resultant vector.
- Center of mass.
  - Only need to know center of mass qualitatively, in other words, without numbers.
  - For the purposes of translational motion, which is essentially non-rotational motion, the whole object or system of objects can be considered to be located at its center of mass. For example, an object or group of objects in projectile motion is described by only analyzing the motion of the center of mass not each individual part of the object or system.

Flipping Physics Lecture Notes:
Projectile Motion - AP Physics 1: Kinematics Review Supplement
http://www.flippingphysics.com/ap1-kinematics-projectile-motion.html

First off, understand this video assumes you have already watched my video:
AP Physics 1: Kinematics Review (http://www.flippingphysics.com/ap1-kinematics-review.html)
So, if you haven't watched that video yet. Go do that now, eh!

Also, if you find this video (and lecture notes) helpful, please consider signing up for my AP Physics 1 Ultimate Review Packet at www.UltimateReviewpPacket.com! You'll find more of these supplemental videos there, along with practice multiple choice problems, free response questions, a practice AP Physics 1 exam, and solutions to all of those, of course! Anyway, let's get to the lecture notes for this video…

Kinematics serves as the backbone of the AP Physics 1 curriculum. It's where you get your first experience understanding motion graphs which leads to better understanding of more graphs in physics. It's where you first begin working with multiple variables and multiple equations. It's where you first begin breaking vectors into components. These are all skills you need to master for the entire course. So, please, take the time to understand Kinematics. It will help you understand the rest of the topics in AP Physics 1 much better.

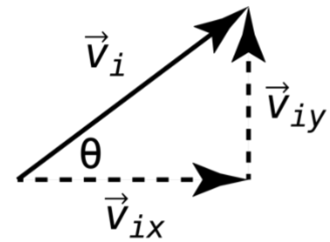
And now we review Kinematics through multiple-choice problems, starting with projectile motion problems:

1) A projectile is launched with speed v_i at an angle of θ above the horizontal. At its maximum height, the horizontal and vertical components of the projectile's velocity and acceleration are:

	Horizontal Velocity Component	Vertical Velocity Component	Horizontal Acceleration Component	Vertical Acceleration Component
(A)	$v_i \cos\theta$	0	0	-g
(B)	$v_i \sin\theta$	$v_i \cos\theta$	0	-g
(C)	$v_i \cos\theta$	$v_i \sin\theta$	-g	0
(D)	$v_i \sin\theta$	0	-g	0

*At the very top of its path, the velocity of a projectile changes from moving up to moving down, therefore, at the very top, it has **zero velocity in the y-direction**.*

*The **acceleration of a projectile in the x-direction is zero**, therefore, the velocity of a projectile in the x-direction is constant. We need to find the x-component of the initial velocity, which is the constant horizontal velocity of the projectile.*

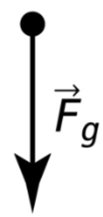


$$\cos\theta = \frac{A}{H} = \frac{v_{ix}}{v_i} \Rightarrow v_{ix} = v_i \cos\theta = v_x$$

*The acceleration of a projectile in the y-direction has a magnitude of g and is directed downward, therefore, **$a_y = -g$**.*

The correct answer is (A).

Once you learn about Newton's Second Law and Dynamics, you can understand all of this using a free body diagram and Newton's Second Law. The only force acting on the projectile is the force of gravity which acts down.



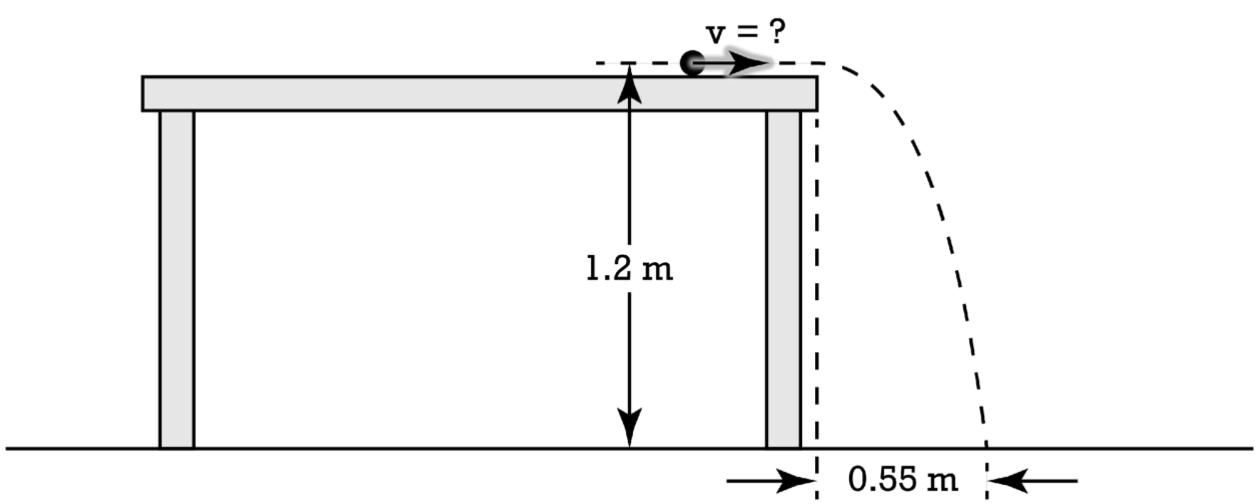
$$\sum F_y = -F_g = ma_y \Rightarrow -mg = ma_y \Rightarrow a_y = -g$$

There are no forces acting in the x-direction, therefore, the acceleration in the x-direction is zero.

$$\sum F_x = 0 = ma_x \Rightarrow a_x = 0$$

2) A small steel ball rolls off a horizontal table with a height of 1.2 m and lands a horizontal distance of 0.55 m from the edge of the table. What was the speed of the ball as it rolled on the table? (Friction is negligible)

(A) 0.27 m/s (B) 0.49 m/s (C) 0.89 m/s (D) 1.1 m/s



y-direction: $v_{iy} = 0; \Delta y = -1.2m; a_y = -g = -10\frac{m}{s^2}$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 \Rightarrow -1.2 = \frac{1}{2}(-10)\Delta t^2 = -5\Delta t^2$$

$$\Rightarrow \Delta t = \sqrt{\frac{1.2}{5}} = 0.489898s$$

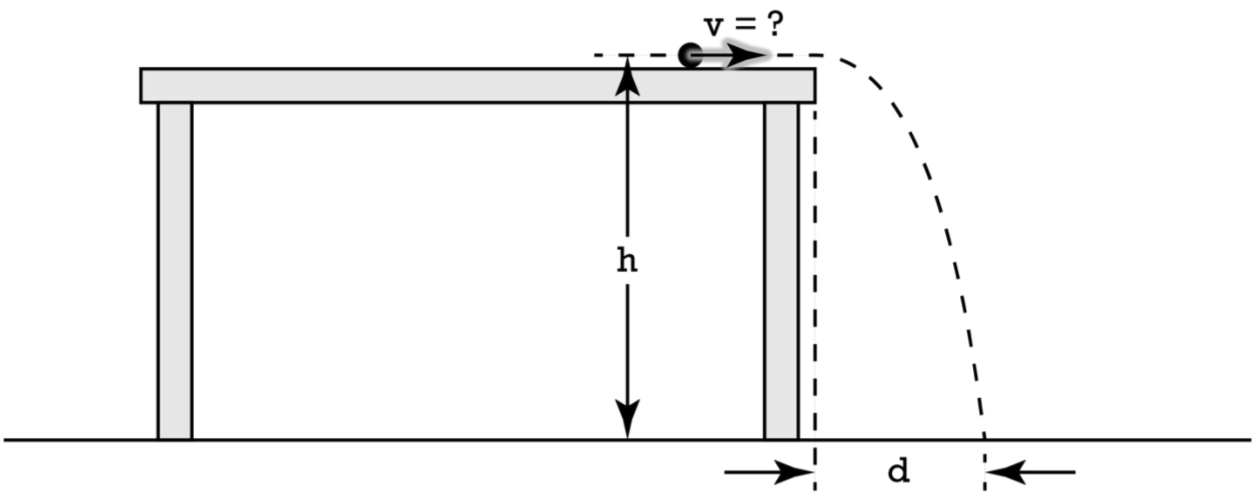
x-direction: $\Delta t = 0.489898s; \Delta x = 0.55m; v_x = \frac{\Delta x}{\Delta t} = \frac{0.55}{0.489898} \approx 1.1\frac{m}{s}$

The correct answer is (D).

It is also plausible they would ask you questions like this without numbers. That would look like this:

2a) A small steel ball rolls off a horizontal table with a height of h and lands a horizontal distance of d from the edge of the table. What was the speed of the ball as it rolled on the table? (Friction is negligible)

(A) $\sqrt{\frac{2d^2h}{g}}$ (B) $\sqrt{\frac{2h}{g}}$ (C) $\sqrt{\frac{2h}{d^2g}}$ (D) $\sqrt{\frac{d^2g}{2h}}$



y-direction: $v_{iy} = 0; \Delta y = -h; a_y = -g$

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 \Rightarrow -h = \frac{1}{2}(-g)\Delta t^2 = -\frac{g\Delta t^2}{2} \Rightarrow \Delta t = \sqrt{\frac{2h}{g}}$$

x-direction: $\Delta t = \sqrt{\frac{2h}{g}}; \Delta x = d; v_x = \frac{\Delta x}{\Delta t} = \frac{d}{\sqrt{\frac{2h}{g}}} \Rightarrow v_x = \sqrt{\frac{d^2g}{2h}}$

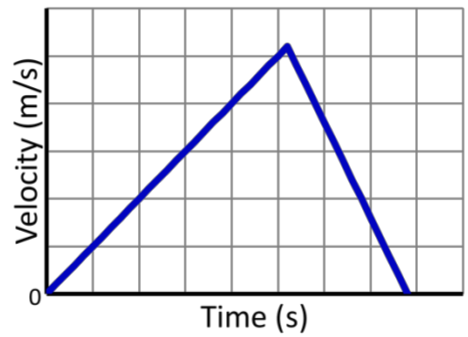
The correct answer is still (D).

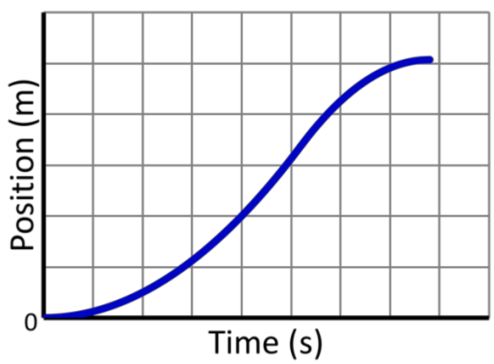
Flipping Physics Lecture Notes:
Motion Graphs - AP Physics 1: Kinematics Review Supplement
http://www.flippingphysics.com/ap1-kinematics-motion-graphs.html

This lesson is a part of my AP Physics 1 Ultimate Review Packet. Please consider signing up for access to the whole Review Packet at www.UltimateReviewpPacket.com!

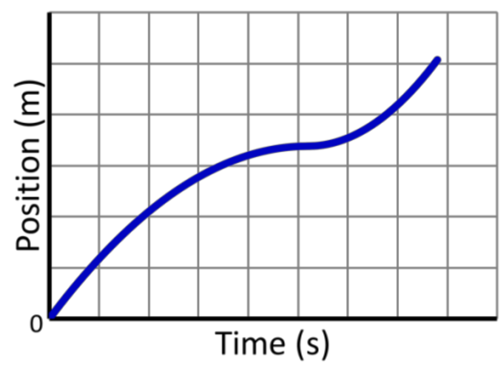
I can pretty much **guarantee** you will have problems where you have to interpret position, velocity, and acceleration as functions of time graphs. For example:

1) You slam your foot down on the accelerator pedal in your car causing it to speed up with a uniform acceleration. After a few seconds, you take your foot off the accelerator pedal and immediately slam it down on the brake pedal, causing your car to slow down with a uniform acceleration. Your velocity as a function of time graph is shown. Which graph could correctly show your position as a function of time?

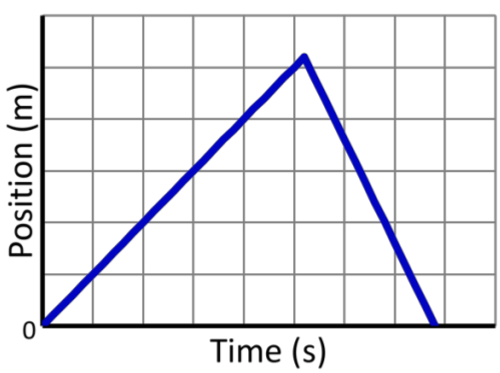




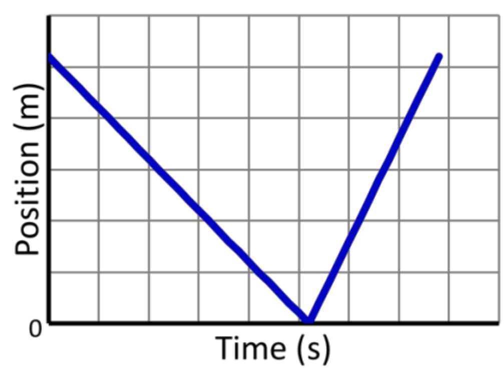
(A)



(B)



(C)

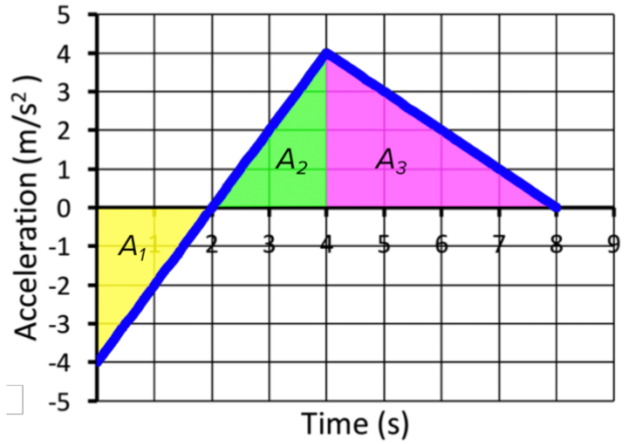


(D)

The key to remember here is that the slope of a position vs. time graph is velocity. The initial velocity is zero, therefore, the initial ***slope*** *of the position vs. time graph needs to be zero. The only graph which shows that is choice (A). The correct answer is (A).*

2) The graph shows the acceleration of a particle with respect to time. Assuming the velocity of the particle at t = 0 seconds is -10 m/s, which of the following is the velocity of the particle at t = 8 seconds?

(A) -2 m/s
(B) 8 m/s
(C) 6 m/s
(D) 18 m/s



The area "under" an acceleration as a function of time graph is change in velocity, however, remember that area under the time axis is negative and area above the time axis is positive.

The area "under" the curve from 0 to 2 seconds and 2 to 4 seconds are of equal magnitude, however, from 0 to 2 seconds the area is negative and from 2-4 seconds the area is positive. In other words, those two areas cancel one another out. Therefore, the only area "under" the curve we need to calculate is from 4 to 8 seconds.

$$A_1 + A_2 = 0$$

$$\text{Area "Under" Curve} = A_3 = \frac{1}{2}bh = \frac{1}{2}(8s - 4s)\left(4\frac{m}{s^2}\right) = 8\frac{m}{s}$$

$$\Rightarrow 8\frac{m}{s} = \Delta v = v_f - v_i = v_f - (-10) = v_f + 10 \Rightarrow v_f = 8 - 10 = -2\frac{m}{s}$$

Correct answer is (A).

Just so you know, the basic concept that:

- the slope of a position versus time graph is velocity,
- the slope of a velocity versus time graph is acceleration,
- the area[1] "under" a velocity versus time graph is change in position,
- the area "under" an acceleration versus time graph is change in velocity,

Will be combined with other topics later in the AP Physics 1 curriculum. 😊

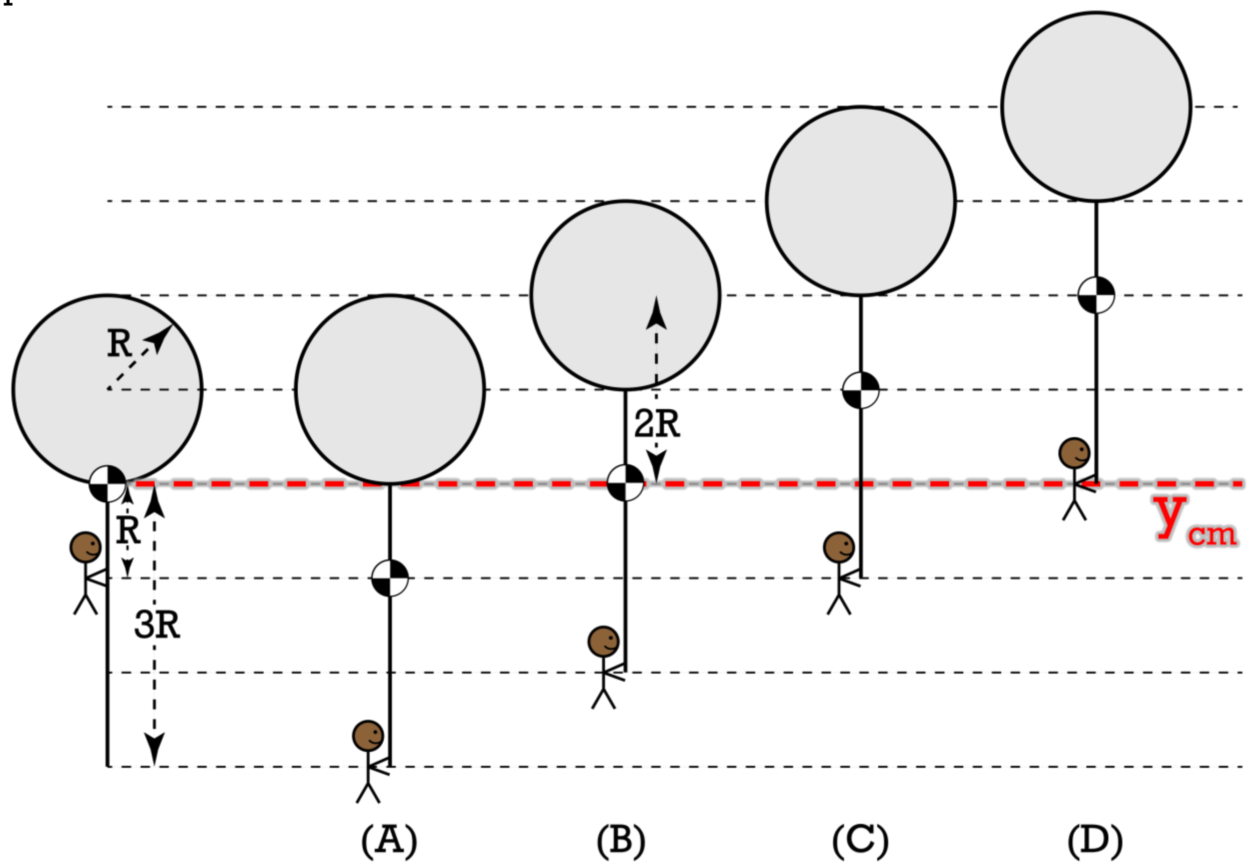
[1] Technically this is the *signed area* or the total area above the horizontal axis minus the total area below the horizontal axis.

Flipping Physics Lecture Notes:
Center of Mass - AP Physics 1: Kinematics Review Supplement
http://www.flippingphysics.com/ap1-kinematics-center-of-mass.html

This lesson is a part of my AP Physics 1 Ultimate Review Packet. Please consider signing up for access to the whole Review Packet at www.UltimateReviewpPacket.com!

This is what a typical center of mass problem could look like:

1) A giant, spherical helium balloon with mass M and radius R has a massless rope of length 3R hanging from it. A person with a mass of M is hanging on the rope in perfect equilibrium such that the balloon-person system does not move up or down. If the person starts a distance R from the bottom of the balloon and climbs down to the end of the rope, which figure best illustrates the final position of the balloon and person? Assume there is no wind.



Because the net external force on the balloon-person system is zero, the system will not accelerate, and the center of mass of the system will stay in the same location while the person climbs down to the bottom of the rope.

Because both objects have the same mass, the initial center of mass of the balloon-person system will be directly in the middle between the centers of mass of the two objects. In other words, because the initial distance between the two objects is 2R, the *initial center of mass of the balloon-person system will be half that distance or a distance R from the centers of mass of both objects.*

Again, because both objects have the same mass, the final center of mass of the balloon-person system will again be in the middle between the centers of mass of the two objects. In other words, because the final distance between the two objects is 4R, the final center of mass of the balloon-person system will be half that distance or a distance 2R from the centers of mass of both objects. Therefore, the correct answer is (B).

The center of mass equation is not required for AP Physics 1; however, I often find that the equation helps solidify understanding. So, here is the solution using the equation. Note, I have set the zero y-position to be at the center of mass of the balloon.

$$y_{cm} = \frac{m_b y_b + m_p y_p}{m_b + m_p} \Rightarrow y_{cm_i} = \frac{(M)(0) + (M)(2R)}{M + M} = \frac{2MR}{2M} = R$$

$$\& \ y_{cm_f} = \frac{(M)(0) + (M)(4R)}{M + M} = \frac{4MR}{2M} = 2R$$

Another typical center of mass question involves the standard “frozen, frictionless ice”.

2) While peacefully reading your physics textbook, you are sliding at 3 m/s East on a very large patch of frozen, frictionless ice. In frustration, you decided to throw your physics textbook and give it a speed of 8 m/s North. If your mass is 40 times larger than the mass of your physics textbook, what is the velocity of the center of mass of the you-textbook system after you throw the book?

(A) 0
(B) 3 m/s East
(C) 4 m/s @ 53° South of East
(D) 4 m/s @ 37° South of East

Considering the net external force of the you-textbook system is zero when you throw the book, the acceleration of the system is also zero. Therefore, the you-textbook system has no change in its momentum or velocity when you throw the book. So, the velocity of the center of mass of the you-textbook system is still 3 m/s East after you throw the book. The correct answer is (B).

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Dynamics*
https://www.flippingphysics.com/ap1-dynamics-review.html

- Inertial Mass vs. Gravitational Mass
  - Inertial mass: the measure of an object's inertia or a measure of its resistance to acceleration.
  - Gravitational Mass: used to determine the force of gravity or weight of an object. $\vec{F}_g = m\vec{g}$
  - Inertial Mass and Gravitational Mass are experimentally identical.
- Newton's First Law: "An object at rest will remain at rest and an object in motion will remain at a constant velocity unless acted upon by a net external force."
  - Common mistake: "an object in motion will remain in *motion*" is wrong. It will remain at a constant velocity which means it will have a constant speed and a constant direction.
  - Common mistake: "unless acted upon by an external force." Do **not** leave out the word "*net*". It is the *sum of all the forces* that needs to be zero for an object to remain at rest or at a constant velocity.
- Newton's Second Law: $\sum \vec{F} = m\vec{a}$
  - It is arranged differently on the equation sheet: $\vec{a} = \frac{\sum \vec{F}}{m}$, but it is the same equation.
  - When you use Newton's Second Law, you must identify object(s) and direction.
  - Free Body Diagrams: always draw them to use Newton's Second Law.
    - On the AP Test, do ***NOT*** break forces in to components in your initial Free Body Diagram.
- The Force of Gravity or Weight of an object is always down. $\vec{F}_g = m\vec{g}$
- The Force Normal is caused by a surface, is normal or perpendicular to the surface and always a push.
- Dimensions for Force are Newtons, N: $\sum \vec{F} = m\vec{a} \Rightarrow N = \frac{kg \cdot m}{s^2}$
- The Force of Friction is parallel to the surface, opposes motion and independent of the direction of the force applied. On equation sheet: $\left|\vec{F}_f\right| \le \mu \left|\vec{F}_n\right|$, which works out to be three equations because we have two types of friction.
  - Static or non-moving friction: the two surfaces do *not* slide relative to one another. $\vec{F}_{sf} \le \mu_s \vec{F}_n$ and $\vec{F}_{sf_{max}} = \mu_s \vec{F}_n$
  - Kinetic or moving friction: the two surfaces *do* slide relative to one another. $\vec{F}_{kf} = \mu_k \vec{F}_n$
  - For two surfaces, the coefficient of kinetic friction is always less that the coefficient of static friction. $\mu_k < \mu_s$
- Newton's Third Law: $\vec{F}_{12} = -\vec{F}_{21}$, For every force from object one on object two there is an equal but opposite force from object two on object one where both forces are vectors.
- Newton's Third Law Force Pairs or Action-Reaction Pairs:
  - Act on two different objects and act simultaneously.
- Inclines: Break the Force of Gravity in to its components that are parallel and perpendicular to the incline. $F_{g_{\parallel}} = mg\sin\theta$ & $F_{g_{\perp}} = mg\cos\theta$
- Translational Equilibrium: $\sum \vec{F} = 0 = m\vec{a} \Rightarrow \vec{a} = 0$
  - The object is either at rest or moving with a constant velocity.

Flipping Physics Lecture Notes:
Newton's Second Law - AP Physics 1: Dynamics Review Supplement
http://www.flippingphysics.com/ap1-dynamics-newtons-second-law.html

This lesson is a part of my AP Physics 1 Ultimate Review Packet. Please consider signing up for access to the whole Review Packet at www.UltimateReviewpPacket.com!

1) A rock is freely falling downward toward the Earth. The weight of the rock is W. Which of the following best describes the force from the rock on the Earth as the rock falls? The force the rock causes on the Earth as it falls is…

(A) Zero. (B) W upward (C) W downward
(D) Increasing in magnitude as the rock's speed increases.

This question is testing your understanding of Newton's Third Law: For every force object 1 applies on object 2, object 2 applies an equal but opposite force on object 1.

$$\vec{F}_{12} = -\vec{F}_{21}$$

In this case the two objects are the Earth and the rock, and the force the Earth applies on the rock is the weight of the rock. Therefore:

$$\vec{W} = \vec{F}_{\text{Earth rock}} = -\vec{F}_{\text{rock Earth}}$$

In other words, the force the rock causes on the Earth is equal to the downward weight of the rock only in the opposite direction or upwards. The correct answer is B.

2) A 300 N object is accelerating to the North at 1 m/s^2. A force, F_1, of 40 N acts due East on the object. If there is only one other force, F_2, acting on the object, what is the magnitude and direction of F_2?

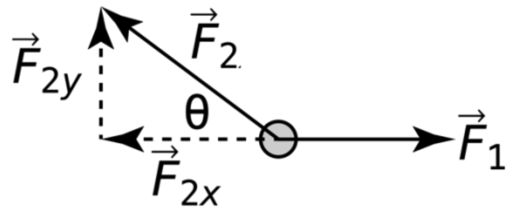
(A) 50 N @ 37° N of W (B) 50 N @ 53° N of W
(C) 35 N @ 37° N of W (D) 35 N @ 53° N of W

A "300 N" object has a weight of 300 newtons. So, let's use that to determine the mass of the object.

$$F_g = mg \Rightarrow m = \frac{F_g}{g} = \frac{300N}{10\frac{m}{s^2}} = 30kg$$

We know the acceleration of the object is completely North, let's call that the positive y-direction and identify East as the positive x-direction. Because F_1 is entirely in the positive x-direction, F_1 has no component in the y-direction. Therefore, the acceleration of the object is caused entirely by the y-component of F_2. That means we can find the y-component of F_2 by summing the forces in the y-direction.

$$\sum F_y = F_{2y} = ma_y = (30kg)\left(1\frac{m}{s^2}\right) = 30N$$



We also know the acceleration of the object in the x-direction is zero, therefore, the x-component of F_2 must be in the negative x-direction to counteract F_1. That means we can find the x-component of F_2 by summing the forces in the x-direction.

$$\sum F_x = -F_{2x} + F_1 = ma_x = m(0) = 0 \Rightarrow F_{2x} = F_1 = 40N$$

Now that we have the components of F_2 we can determine its magnitude and direction.

$$A^2 + B^2 = C^2 \Rightarrow {F_{2x}}^2 + {F_{2y}}^2 = {F_2}^2$$

$$\Rightarrow F_2 = \sqrt{{F_{2x}}^2 + {F_{2y}}^2} = \sqrt{40^2 + 30^2} = 50N$$

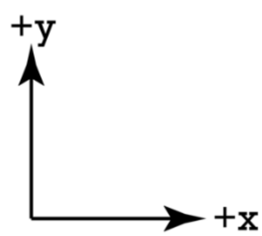
$$\tan\theta = \frac{O}{A} = \frac{F_{2y}}{F_{2x}} \Rightarrow \theta = \tan^{-1}\left(\frac{F_{2y}}{F_{2x}}\right) = \tan^{-1}\left(\frac{30}{40}\right) \approx 37°$$

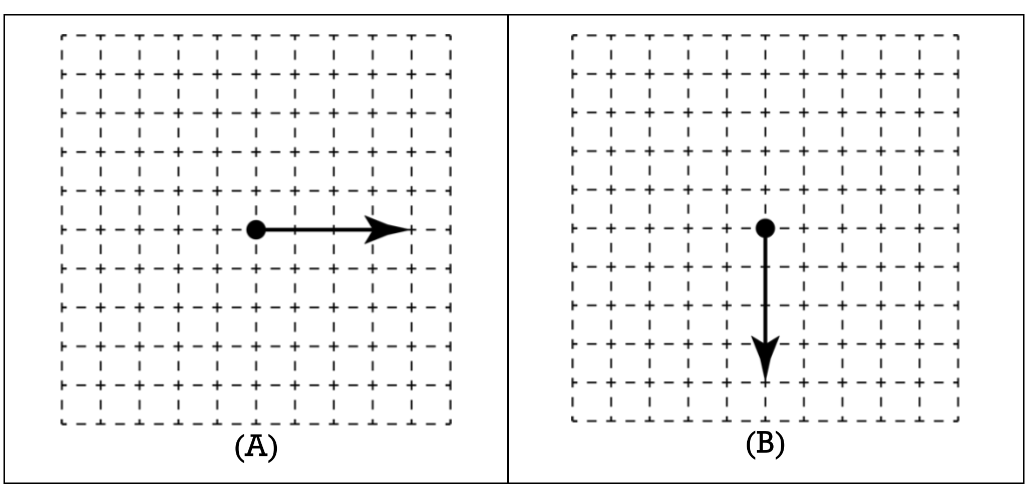
Correct answer is A.

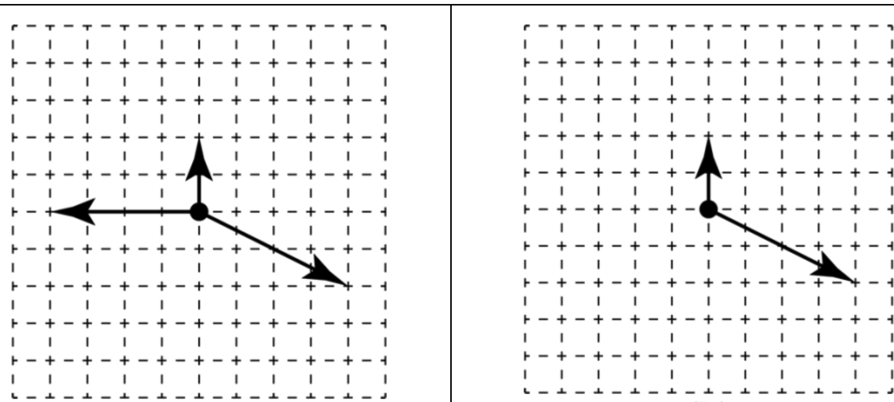
A few things to point out. One is that the College Board will often use common triangles like 3, 4, 5 triangles and 30, 60, 90 triangles, etc. It's just good to be aware of that.

Also, notice there must be a downward force of gravity in this problem, and therefore a counteracting upward force. That upward force is probably a normal force and the object is probably moving on a frictionless surface. All of that is ignored in the problem, which is too bad, however, you will likely see things like this on the AP Physics exam. Sorry!

3) Which of the following **two** free body diagrams could be for an object which has a constant velocity in the y-direction and a constant acceleration in the x-direction?







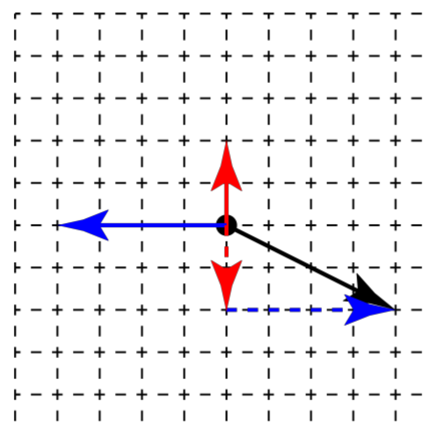
(C)	(D)

Because net force equals mass times acceleration, to have a constant velocity in the y-direction, the net force in the y-direction must be zero, and to have a constant acceleration in the x-direction, the net force in the x-direction must be nonzero.

Choice A has a net force in the y-direction equal to zero and a net force in the x-direction which is nonzero. So, choice A is a correct answer.

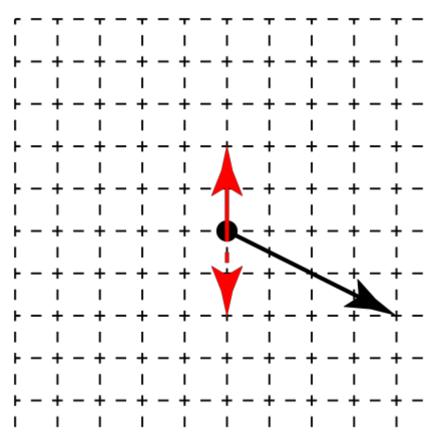
Choice B has a net force in the y-direction which is nonzero and a net force in the x-direction equal to zero. So, choice B is an incorrect answer.

Choice C has a net force in the y-direction equal to zero and a net force in the x-direction equal to zero. So, choice C is an incorrect answer.



To clarify, look at the grid. The y-component of the force at an angle (in red) is two grid spacings in length down which balances out the force vector which is two grid spacings in length up. And the x-component of the force at an angle (in blue) is four grid spacings to the right which balances out the force vector which is four grid spacings to the left. This is why the net force in all directions is equal to zero.

Choice D has a net force in the y-direction equal to zero and a net force in the x-direction which is nonzero. So, choice D is a correct answer.

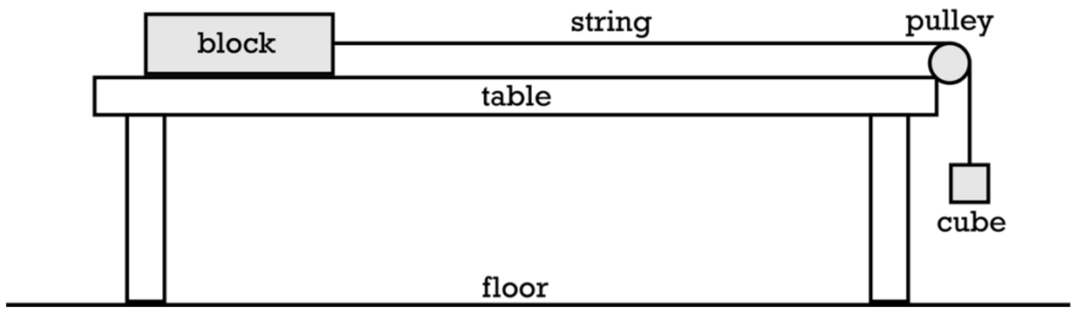


To clarify, again, look at the grid. The y-component of the force at an angle (in red) is two grid spacings in length down which balances out the force vector which is two grid spacings in length up. This is why the net force in the y-direction is zero, and the net force in the x-direction is nonzero.

Flipping Physics Lecture Notes:
Friction - AP Physics 1: Dynamics Review Supplement
http://www.flippingphysics.com/ap1-dynamics-friction.html

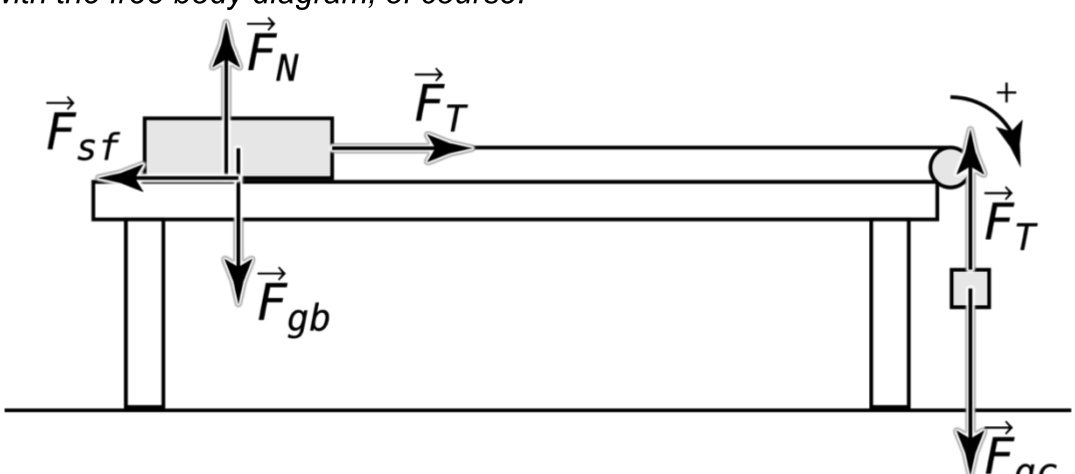
This lesson is a part of my AP Physics 1 Ultimate Review Packet. Please consider signing up for access to the whole Review Packet at www.UltimateReviewpPacket.com!

1) As shown, a block of mass m_b on a table is attached to a string which goes over an ideal pulley and is attached to a cube of mass m_c. The block and cube are currently at rest. If the coefficient of static friction between the block and the table is μ_s, the magnitude of the force preventing the block from accelerating is best described by:



(A) $\mu_s m_b g$ (B) $\mu_s m_c g$ (C) $(m_c + m_b)g$ (D) $m_c g$

Start with the free body diagram, of course!



You can see it is the force of static friction between the block and the table which prevents the block from accelerating. We need to solve for the magnitude of that force of static friction. Defining the positive direction as down and to the right, we can sum the forces on the entire block-cube system in that positive direction:

$$\sum F_+ = F_{gc} - F_T + F_T - F_{sf} = ma_+ = m(0) = 0 \Rightarrow F_{sf} = F_{gc} = m_c g$$

The correct answer is D.

Many of you may want to solve this problem this way…
Sum the forces on the block in the y-direction:

$$\sum F_y = F_N - F_{gb} = m_b a_y = m_b(0) = 0 \Rightarrow F_N = F_{gb} = m_b g$$

and then solve for the force of static friction like this:

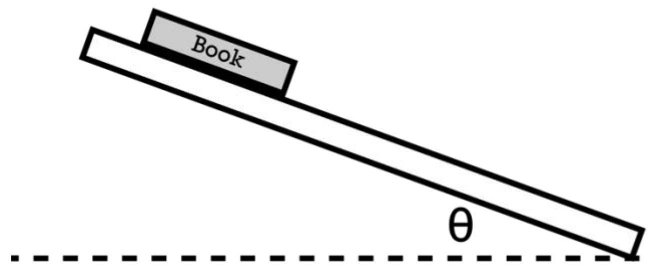
$$F_{sf} = \mu_s F_N = \mu_s m_b g$$

Which would make you think that choice A is correct. However, the equation for the force of static friction you used is not correct. The correct equation for the force of static friction is:

$$F_{sf} \le \mu_s F_N$$

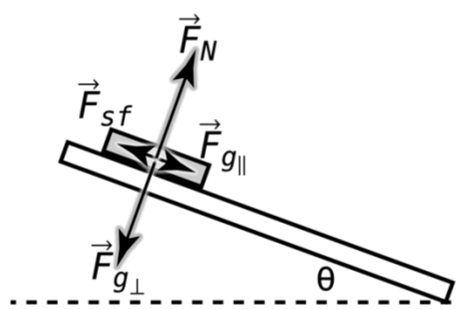
In other words, the force of static friction likely is not at its maximum value, which means you have to solve for the force of static friction the way I did in this problem.

2) A book with mass m is at rest on an incline of angle θ as shown. If the coefficient of static friction between the book and the incline is μ_s, and the coefficient of kinetic friction between the book and the incline is μ_k, which expression best represents the force of friction currently acting on the book?



(A) $\mu_s mg\cos\theta$ (B) $\mu_k mg\cos\theta$ (C) $mg\cos\theta$ (D) $mg\sin\theta$

Start with the free body diagram, of course!



$$\sum F_\parallel = F_{g_\parallel} - F_{sf} = ma_\parallel = m(0) = 0$$

$$F_{sf} = F_{g_\parallel} = mg\sin\theta$$

The correct answer is D.

Many of you may want to solve this problem this way…
Sum the forces on the book in the perpendicular direction:

$$\sum F_\perp = F_N - F_{g_\perp} = ma_\perp = m(0) = 0 \Rightarrow F_N = F_{g_\perp} = mg\cos\theta$$

and then solve for the force of static friction like this:

$$F_{sf} = \mu_s F_N = \mu_s mg\cos\theta$$

Which would make you think that choice A is correct. However, the equation for the force of static friction you used is not correct. The correct equation for the force of static friction is:

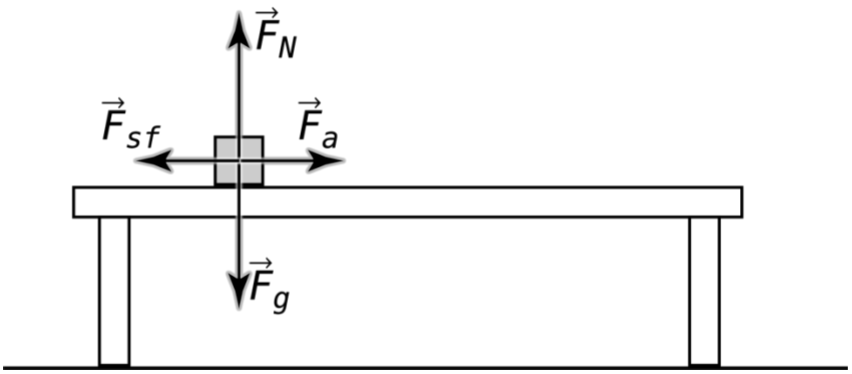
$$F_{sf} \le \mu_s F_N$$

In other words, the force of static friction likely is not at its maximum value, which means you have to solve for the force of static friction the way I did in this problem.
This should seem eerily familiar to you.

3) A 20 kg mass is at rest on a table. A horizontal force of 140 N is applied to a block, however, it is not enough to move the block. Which of the following best describes what we know about the coefficient of static friction between the block and the table?

(A) $\mu_s \geq 0.7$ (B) $\mu_s \geq 1.4$ (C) $\mu_s = 0.7$ (D) $\mu_s = 1.4$

Start with the free body diagram:



Then use Newton's Second Law, twice:

$$\sum F_y = F_N - F_g = ma_y = m(0) = 0 \Rightarrow F_N = F_g = mg$$

$$\sum F_x = F_a - F_{sf} = ma_x = m(0) = 0 \Rightarrow F_a = F_{sf} = \mu_s F_N = \mu_s mg$$

$$\Rightarrow \mu_s = \frac{F_a}{mg} = \frac{140}{(20)(10)} = 0.7$$

It is important to recognize that, because the force of static friction is less than or equal to the coefficient of static friction times force normal, $F_{sf} \leq \mu_s F_N$, what we have solved for here is the minimum coefficient of static friction to keep the block from moving. In other words, the coefficient of static friction could be equal to or greater than 0.7. The correct answer is A.

To better help you understand why the coefficient of static friction is equal to or greater than 0.7, let's solve for the force of static friction and force normal in the problem.

$$F_a = F_{sf} = 140N \;\&\; F_N = F_g = mg = (20)(10) = 200N$$

And then solve for the coefficient of static friction using the equation for the force of static friction:

$$F_{sf} \leq \mu_s F_N \Rightarrow 140 \leq \mu_s(200) \Rightarrow 0.7 \leq \mu_s \Rightarrow \mu_s \geq 0.7$$

And you can see the coefficient of static friction between the block and the table is greater than or equal to 0.7 in this problem.

For those of you who challenge this step: $0.7 \leq \mu_s \Rightarrow \mu_s \geq 0.7$

Here it is in several steps instead:

$$0.7 \leq \mu_s \Rightarrow 0.7 < \mu_s \text{ or } 0.7 = \mu_s \Rightarrow \mu_s > 0.7 \text{ or } \mu_s = 0.7 \Rightarrow \mu_s \geq 0.7$$

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Work, Energy and Power*
https://www.flippingphysics.com/ap1-work-review.html

- $\Delta E = W = F_{\parallel} d = Fd\cos\theta$: In terms of an object or a group of objects which we call the system, the change in energy of the system equals the work done on the system which is equal to force times displacement times the angle between the force and the displacement. Work causes a change in energy of the system.
  - $F_{\parallel} = F\cos\theta$: The force parallel to the displacement is the force times the cosine of the angle between the force and the displacement of the object.
  - Identify which force you are using in the work equation.
  - Use the magnitude of the force and the displacement.
  - Dimensions for Work are Joules or Newtons times meters:
- Three types of mechanical energy:
  - *Kinetic Energy:* $KE = K = \frac{1}{2}mv^2$ (can't be negative)
  - Dimensions for energy are also Joules:

    $$KE = \frac{1}{2}mv^2 \Rightarrow (kg)\left(\frac{m}{s}\right)^2 = \frac{kg \cdot m^2}{s^2} = \left(\frac{kg \cdot m}{s^2}\right)(m) = N \cdot m = J$$
  - *Elastic Potential Energy:* $PE_e = U_s = \frac{1}{2}kx^2$ (can't be negative)
  - *Gravitational Potential Energy:* $PE_g = mgh$ or $\Delta U_g = mg\Delta y$
    - PE_g **Can** be negative. If the object is below the horizontal zero line, then h, the vertical height above the zero, is line negative.
- Work and Energy are Scalars!
- Conservation of Mechanical Energy: $ME_i = ME_f$
  - Valid when there is no energy converted to heat, light or sound due to friction.
  - Identify the initial and final points. Identify the horizontal zero line.
  - Substitute in mechanical energies that are present.
- If there is friction & you need to use energy: $W_f = \Delta ME$ (Does not work when there is a force applied.)
- Power, the rate at which work is done or energy is transferred into or out of the system.
  - $P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \frac{Fd\cos\theta}{\Delta t} = Fv\cos\theta$
  - Dimensions for Power are Watts which are Joules per second:
    - $P = \frac{\Delta E}{\Delta t} \Rightarrow \frac{J}{s} = watts$ & $746\,watts = 1hp$
- Hooke's Law: $\left|\vec{F}_s\right| = k\left|\vec{x}\right|$: The force of a spring is linearly proportional to the displacement from equilibrium position.
  - The slope of a graph of Force of a Spring vs. displacement from equilibrium position is the spring constant. $slope = \frac{rise}{run} = \frac{F_s}{x} = k$
  - Typical dimensions for the spring constant are Newtons per meter: $\frac{F_s}{x} = k \Rightarrow \frac{N}{m}$

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Linear Momentum and Impulse*
https://www.flippingphysics.com/ap1-momentum-review.html

- Momentum: $\vec{p} = m\vec{v}$ (remember, momentum is a vector)
  - Dimensions for momentum have no special name: $\vec{p} = m\vec{v} \Rightarrow \frac{kg \cdot m}{s}$
- Conservation of momentum: $\sum \vec{p}_i = \sum \vec{p}_f$ (during all collisions and explosions)
  - Collisions in two dimensions: 2 different equations; $\sum \vec{p}_{xi} = \sum \vec{p}_{xf}$ & $\sum \vec{p}_{yi} = \sum \vec{p}_{yf}$
- Types of collisions:

Type of Collision	Is Momentum Conserved?	Is Kinetic Energy Conserved?
Elastic (bounce)	Yes	Yes
Perfectly Inelastic (stick)	Yes	No

  - Many collisions are in between Elastic and Perfectly Inelastic. They are called Inelastic collisions. During inelastic collisions the objects bounce off of one another, momentum is conserved however Kinetic Energy is not conserved. Elastic and Perfectly Inelastic collisions are the two ideal extremes.
- Rearranging Newton's Second Law in terms of momentum:
  - $\sum \vec{F} = m\vec{a} = m\left(\frac{\Delta \vec{v}}{\Delta t}\right) = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} \Rightarrow \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$
  - Gives us the equation for impulse: $\Delta \vec{p} = \sum \vec{F} \Delta t = \vec{J} = Impulse$
  - The Impulse Approximation gives us the equation on the equation sheet: $\sum \vec{F} \approx \vec{F}_{impact} \Rightarrow \Delta \vec{p} = \vec{F}_{impact} \Delta t = \vec{J} = Impulse$
  - On a Force of Impact vs. time graph, the area between the curve & the time axis is impulse.
  - Dimensions for impulse: $\Delta \vec{p} = \vec{F}_{impact} \Delta t \Rightarrow N \cdot s = \frac{kg \cdot m}{s}$

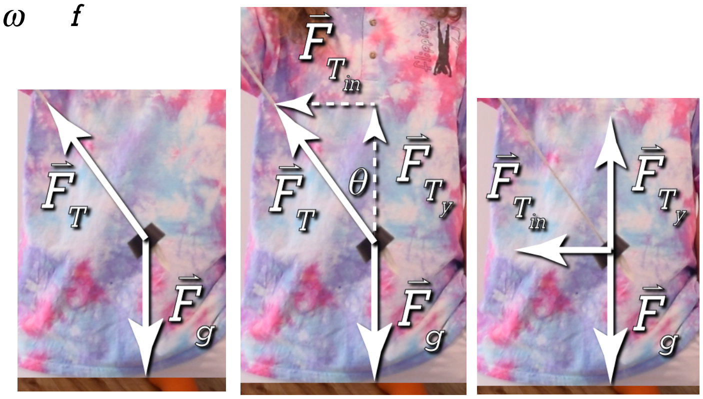
Flipping Physics Lecture Notes:
AP Physics 1 Review of *Rotational Kinematics*
https://www.flippingphysics.com/ap1-rotational-kinematics-review.html

- Angular Velocity: $\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t}\left(\frac{rad}{s}\ or\ \frac{rev}{min}\right)$
  - Remember for conversions: $1\ rev = 360° = 2\pi\ radians$
- Angular Acceleration: $\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}\left(\frac{rad}{s^2}\right)$
- Uniformly Angularly Accelerated Motion, UαM, is just like UAM, only it uses angular variables:
  - Equations are valid when $\vec{\alpha} = constant$

Uniformly Accelerated Motion, UAM	Uniformly Angularly Accelerated Motion, UαM
$v_x = v_{x0} + a_x t$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_{xo}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v_x^{\ 2} = v_{x0}^{\ 2} + 2a_x\left(x - x_0\right)$	$\omega_f^{\ 2} = \omega_i^{\ 2} + 2\alpha\Delta\theta$
$\Delta x = \frac{1}{2}\left(v_f + v_i\right)\Delta t$	$\Delta\theta = \frac{1}{2}\left(\omega_f + \omega_i\right)\Delta t$

- Tangential velocity is the linear velocity of an object moving along a circular path. $\vec{v}_t = r\vec{\omega}$
  - The direction of tangential velocity is tangent to the circle and normal to the radius.
  - Tangential velocity is a linear velocity so it has the same dimensions as linear velocity: $\frac{m}{s}$
- Centripetal Force and Centripetal Acceleration:
  - Centripetal means "Center Seeking"
  - Centripetal force is the net force in the in direction or the "center seeking" force which causes the acceleration of the object in toward the center of the circle which is the centripetal or "center seeking" acceleration.
  - Centripetal Force, $\sum \vec{F}_{in} = m\vec{a}_c$:
    - Not a new force.
    - Never in a Free Body Diagram.
    - The direction "in" is positive and the direction "out" is negative.
  - Centripetal Acceleration, $a_c = \frac{v_t^{\ 2}}{r} = r\omega^2$
- The Period, T, is the time for one full cycle or revolution.
  - Dimensions for period: seconds or seconds per cycle.
- The Frequency, f, is the number of cycles or revolutions per second.
  - Dimensions for frequency are cycles per second which are called Hertz, Hz: $f \Rightarrow \frac{cyc}{sec} = Hz$
  - Frequency and Period are inversely related: $T = \frac{1}{f}$

- We can use the equation for angular acceleration to derive an equation on the equation sheet:
    - $\vec{\omega} = \frac{\Delta\vec{\theta}}{\Delta t} = \frac{2\pi \ rad}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{1}{f}$
- The Conical Pendulum Example:



$$\sin\theta = \frac{O}{H} = \frac{\vec{F}_{T_{in}}}{\vec{F}_T} \Rightarrow \vec{F}_{T_{in}} = \vec{F}_T \sin\theta$$

$$\cos\theta = \frac{A}{H} = \frac{\vec{F}_{T_y}}{\vec{F}_T} \Rightarrow \vec{F}_{T_y} = \vec{F}_T \cos\theta$$

$$\sum F_y = F_{T_y} - F_g = ma_y = m(0)$$

$$\Rightarrow F_{T_y} = F_T \cos\theta = mg$$

$$\sum F_{in} = F_{T_{in}} = \vec{F}_T \sin\theta = ma_c = m\left(\frac{v_t^2}{r}\right)$$

$$v_t = r\omega \text{ or } v_t = \frac{\Delta x}{\Delta t} = \frac{C}{T} = \frac{2\pi r}{T}$$

We could even substitute further:

$$F_T \cos\theta = mg \Rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\vec{F}_T \sin\theta = m\left(\frac{v_t^2}{r}\right) \Rightarrow \left(\frac{mg}{\cos\theta}\right)\sin\theta = m\left(\frac{\left(\frac{2\pi r}{T}\right)^2}{r}\right) \Rightarrow g\tan\theta = \frac{4\pi^2 r^2}{T^2 r} = \frac{4\pi^2 r}{T^2}$$

And solve for the radius in terms of the length of the string.

$$\sin\theta = \frac{O}{H} = \frac{r}{L} \Rightarrow r = L\sin\theta$$

$$g\tan\theta = \frac{4\pi^2 r}{T^2} \Rightarrow g\frac{\sin\theta}{\cos\theta} = \frac{4\pi^2 L\sin\theta}{T^2} \Rightarrow \frac{g}{\cos\theta} = \frac{4\pi^2 L}{T^2} \Rightarrow \boxed{T^2 = \frac{4\pi^2 L\cos\theta}{g}}$$

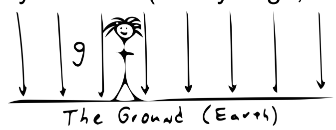
And we end with an expression for the period of the circular motion.

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Rotational Dynamics*
https://www.flippingphysics.com/ap1-rotational-dynamics-review.html

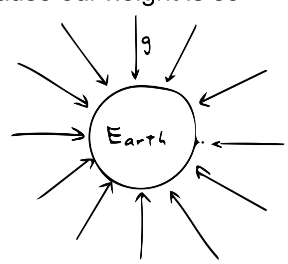
- Torque, the ability to cause an angular acceleration of an object: $\vec{\tau} = \vec{r}_{\perp}\vec{F} = \vec{r}\vec{F}\sin\theta$
  - The moment arm or lever arm is: $\vec{r}_{\perp} = \vec{r}\sin\theta$
  - A larger moment arm will cause a larger torque.
    - Maximize torque by maximizing r, the distance from axis of rotation to the force.
    - Maximize torque by using an angle of 90° because $(\sin\theta)_{max} = \sin(90°) = 1$
  - Dimensions for Torque are Newtons meters, N·m, not to be confused with Joules for energy:
  - Torque is a vector.
    - For direction use clockwise and counterclockwise. (sadly, not the right hand rule)
- Rotational form of Newton's Second Law: $\sum \vec{\tau} = I\vec{\alpha}$
- Moment of Inertia or Rotational Mass:
  - For a system of particles: $I = \sum_i m_i r_i^2$
  - Dimensions for Moment of Inertia: $I = \sum_i m_i r_i^2 \Rightarrow kg \cdot m^2$
  - For a rigid object with shape the value or the equation will be given to you. For example: $I_{solid\ cylinder} = \frac{1}{2}MR^2$; $I_{thin\ hoop} = MR^2$; $I_{solid\ sphere} = \frac{2}{5}MR^2$; $I_{thin\ spherical\ shell} = \frac{2}{3}MR^2$; $I_{rod} = \frac{1}{12}ML^2$; $I_{rod\ about\ end} = \frac{1}{3}ML^2$
    - With the exception of $I_{rod\ about\ end}$, these are all about the center of mass of the object.
- Rotational Kinetic Energy: $KE_{rot} = \frac{1}{2}I\omega^2$
  - KE_{rot}, like translational energy, is in Joules, J.
  - Rolling without slipping: When an object rolls down a hill, it will gain not only translational kinetic energy but also rotational kinetic energy. Which means, the higher the moment of inertia, the higher the rotational kinetic energy of the object and therefore the lower amount of energy that will be left over for translational kinetic energy and therefore a lower final linear velocity.
    - Using Conservation of Mechanical Energy: $ME_i = ME_f \Rightarrow PE_{gi} = KE_{rot\ f} + KE_{t\ f}$
    - Also need the equation for the velocity of the center of mass of a rigid object rolling without slipping: $v_{cm} = R\omega$
- Angular Momentum: $\vec{L} = I\vec{\omega}$
  - Dimensions for Angular Momentum: $\vec{L} = I\vec{\omega} \Rightarrow (kg \cdot m^2)\left(\frac{rad}{s}\right) = \frac{kg \cdot m^2}{s}$
- Angular Impulse: $\Delta\vec{L} = \vec{\tau}_{impact}\Delta t = Angular\ Impulse$
  - Dimensions for Angular Impulse: $\Delta\vec{L} = \vec{\tau}_{impact}\Delta t \Rightarrow N \cdot m \cdot s$

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Universal Gravitation*
https://www.flippingphysics.com/ap1-gravitation-review.html

- Newton's Universal Law of Gravitation: $F_g = \frac{Gm_1m_2}{r^2}$
  - Universal Gravitational Constant: $G = 6.67\times10^{-11}\frac{N\cdot m^2}{kg^2}$
  - r is not defined as the radius, it is defined as the distance between the centers of mass of the two objects which can be confusing because sometimes it does work out to be the radius. ☺
  - $\vec{F}_g = m\vec{g}$ is planet specific.
  - $F_g = \frac{Gm_1m_2}{r^2}$ is universally true.
  - We can combine the two to solve for the acceleration due to gravity on Earth (or any large, celestial body): $F_g = m_o g = \frac{Gm_om_E}{\left(R_E + alt\right)^2} \Rightarrow g = \frac{Gm_E}{\left(R_E + alt\right)^2}$

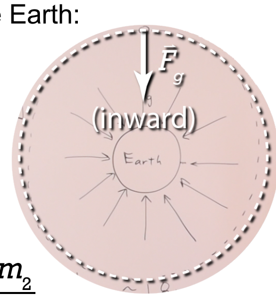


- The gravitational field is approximately constant on the surface of the Earth because our height is so small compared to the radius of the Earth. $h_{mr.p} \approx 1.8$ m, $R_E \approx 6{,}370{,}000$ m
- The gravitational field is not constant from a global perspective and decreases as altitude increases, this can be shown using a vector field diagram.



- Solving for the speed of the satellite in orbit around the Earth:
  - $\sum F_{in} = F_g = m_s a_c = \frac{Gm_sm_E}{r^2} = m_s\frac{v_t^{\,2}}{r}$

    $\Rightarrow v_t = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{Gm_E}{\left(R_E + alt\right)}}$

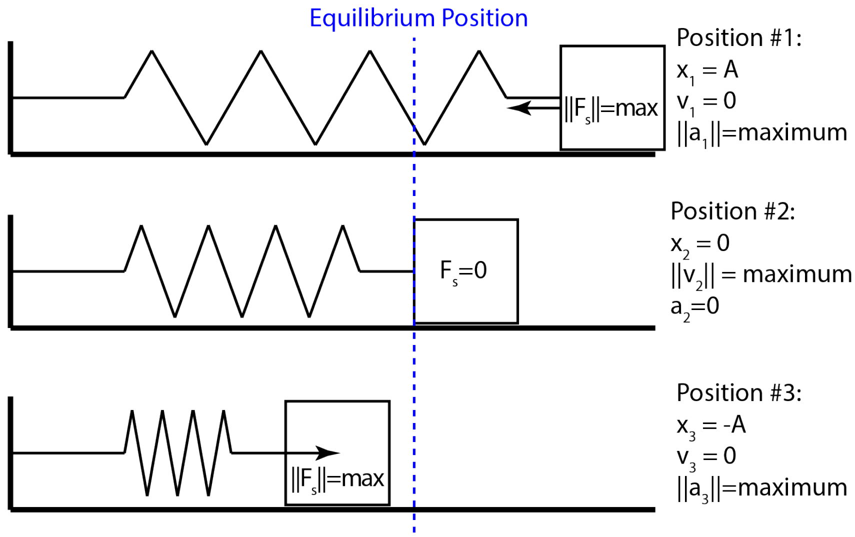


- Universal Gravitational Potential Energy: $U_g = -\frac{Gm_1m_2}{r}$
  - The equation used to find gravitational potential energy in a non-uniform gravitational field.
  - $U_g \le 0$: The zero line is infinitely far away. $U_{g_\infty} = -\frac{Gm_1m_2}{\infty} \approx 0$
  - A single object can *not* have Universal Gravitational Potential Energy. Universal Gravitational Potential Energy is defined as the Gravitational Potential Energy that exists between *two* objects.
    - Technically Gravitational Potential Energy in a constant gravitational field: $PE_g = mgh$, is the gravitational potential energy that exists between the object and the Earth. So even PE_g requires two objects.

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Simple Harmonic Motion*

https://www.flippingphysics.com/ap1-shm-review.html

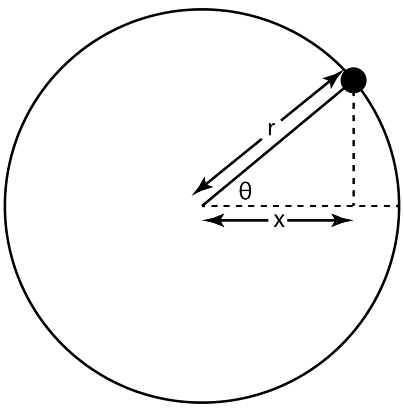
The mass-spring system shown at right is in simple harmonic motion. The mass moves through the following positions: 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, etc.



Simple Harmonic Motion (SHM) is caused by a Restoring Force:

- A Restoring Force is always:
  - Towards the equilibrium position.
  - Magnitude is proportional to distance from equilibrium position.

To derive the equation for position in SHM, we start by comparing simple harmonic motion to circular motion.



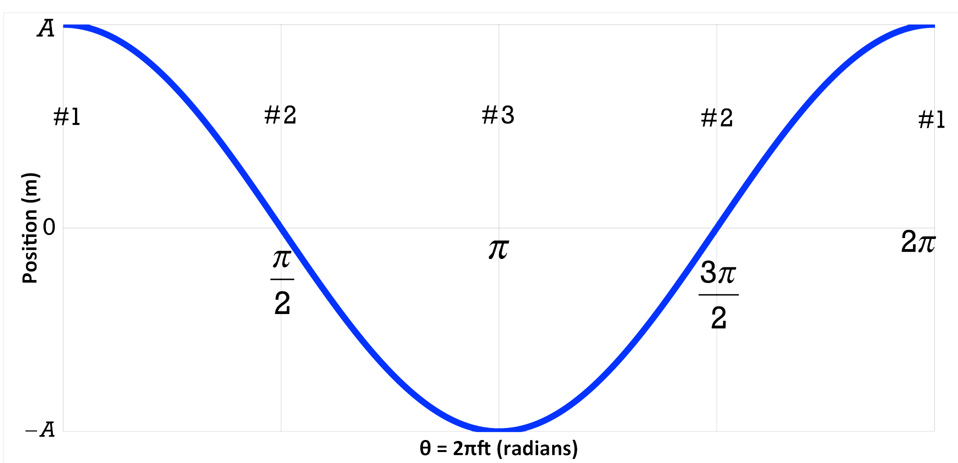
$$\cos\theta = \frac{A}{H} = \frac{x}{r} \Rightarrow x = r\cos\theta \;\;\&\;\; T = \frac{2\pi}{\omega} = \frac{1}{f} \Rightarrow \omega = 2\pi f$$

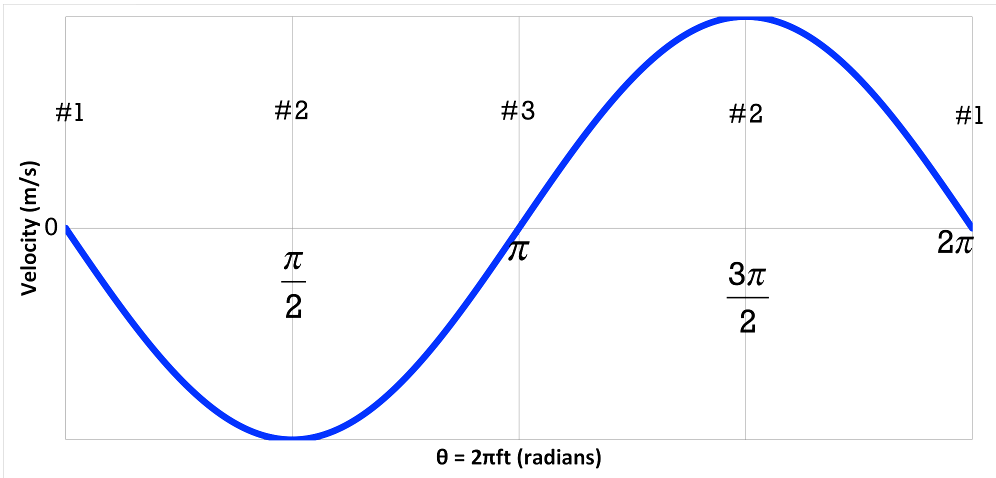
$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\theta_f - 0}{t_f - 0} = \frac{\theta}{t} \Rightarrow \theta = \omega t$$

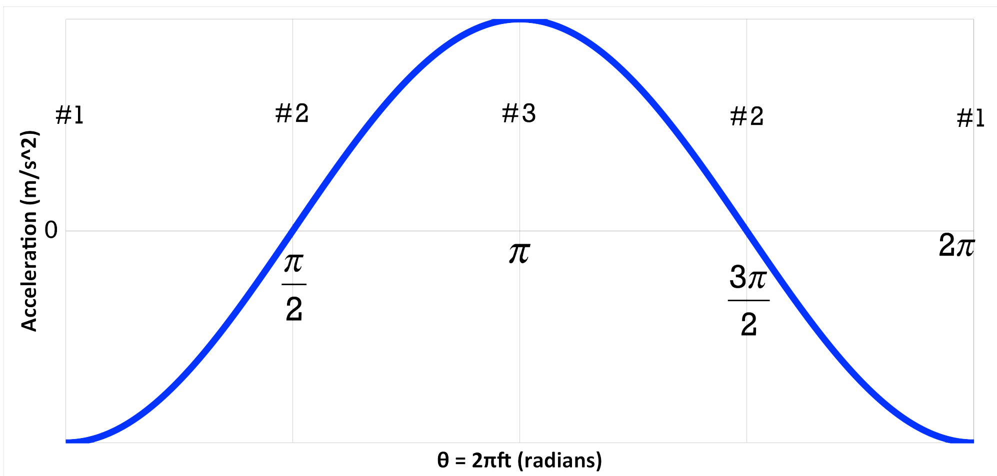
$$x = r\cos\theta = r\cos(\omega t) = r\cos\left[(2\pi f)(t)\right] = A\cos\left[(2\pi f)(t)\right]$$

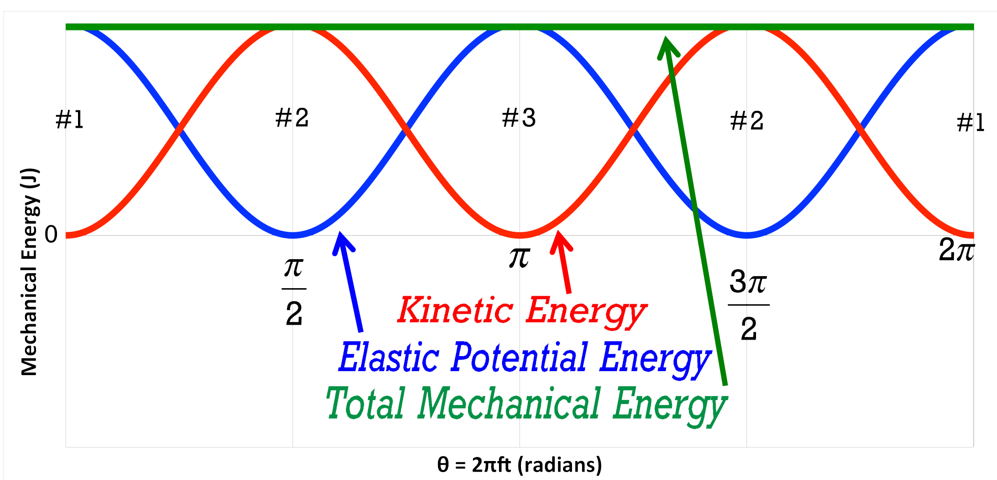
(letting r = A)

Looking at the graphs …





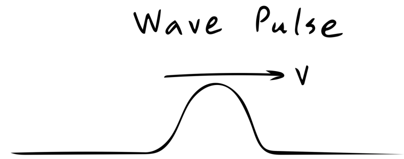




The period of a mass-spring system: $T_s = 2\pi\sqrt{\frac{m}{k}}$ Is independent of amplitude and acceleration due to gravity.

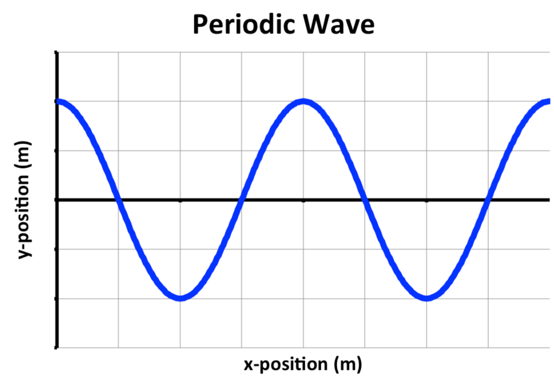
The period of a pendulum: $T_p = 2\pi\sqrt{\frac{L}{g}}$ Is independent of amplitude and mass.

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Waves*
https://www.flippingphysics.com/ap1-waves-review.html



A wave is the motion of a disturbance traveling through a medium not the motion of the medium itself.

- The disturbance of the medium is energy traveling through a medium.
- Wave Pulse: A single wave traveling trough a medium.
- Periodic Wave: Many wave pulses at specific, periodic time intervals.
- The energy moves through the medium as the wave pulse, however, the overall displacement of the medium is zero. $\Delta\vec{x}_{medium} = 0$



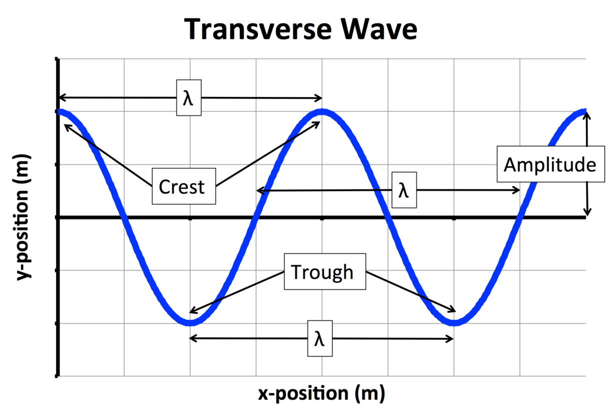
Transverse wave: the disturbance of the medium is perpendicular to the direction of wave propagation. (shown at right)

- Waves in rope, Ripples on a Pond

Longitudinal wave: the disturbance of the medium is parallel to the direction of wave propagation.

- Density on y-axis instead of y-position.
- Sound in Air, Seismic waves in the Earth

Electromagnetic waves are transverse waves that do not need a medium to travel through. They are the only waves we know of that do not need a medium.



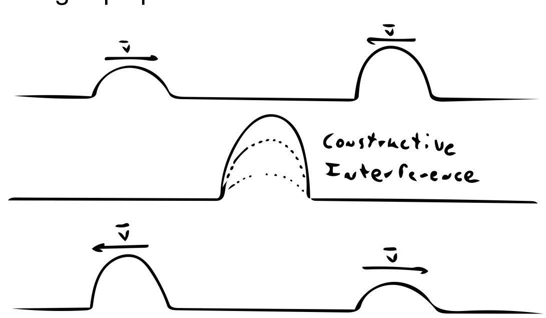
- The distance between two successive crests is called the wavelength, λ.
- The time it takes for one full cycle or for one wavelength to pass a point is called the period, T.

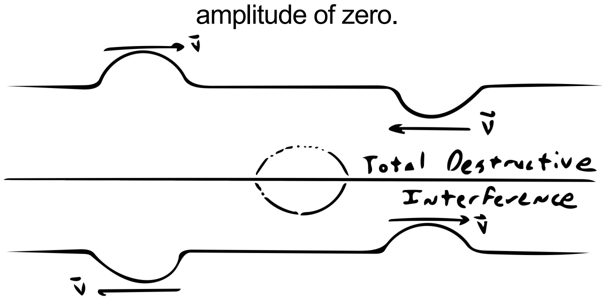
Using the equation for velocity, we can determine the equation for the velocity of a wave:

- $\vec{v} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\lambda}{T}$ & $f = \frac{1}{T} \Rightarrow v = f\lambda$
- Simple Harmonic Motion, SHM, does *not* have a wavelength, so you can *not* use $v = f\lambda$ with SHM.

Waves are not physical objects; they are energy traveling through a medium. Therefore, waves *can* occupy the same space at the same time. We determine their combined amplitude using superposition.

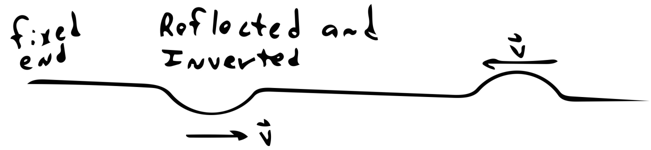
- Constructive Interference
  - Waves add together to create a larger amplitude.
- Total Destructive Interference
  - Waves cancel one another out to create an amplitude of zero.

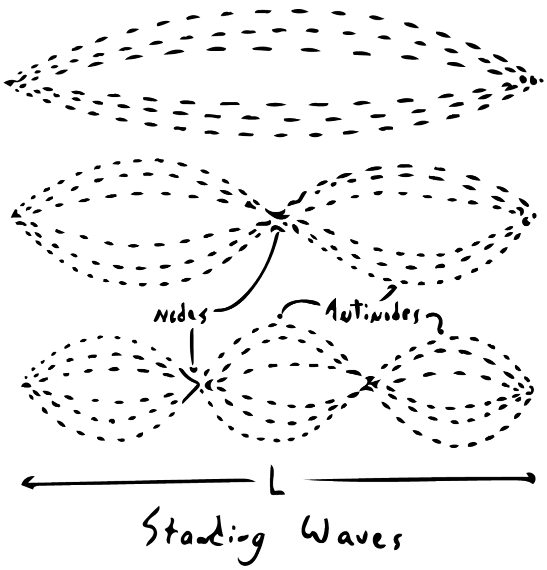




Standing waves: Periodic waves are reflected and inverted and interfere with one another creating standing waves.

- Nodes: Locations of total destructive interference.
- Antinode: Locations of constructive interference.





- n is called the Harmonic Number. n = 1 is the fundamental frequency and the 1st harmonic. n = 2 is the 2nd harmonic, etc.
- Pitch is our brain's interpretation of frequency. 440 Hz is typically concert pitch and is the A above middle C.
- Open pipe instrument (like the flute) is open at both ends and has the same equation for frequency as a stringed instrument.
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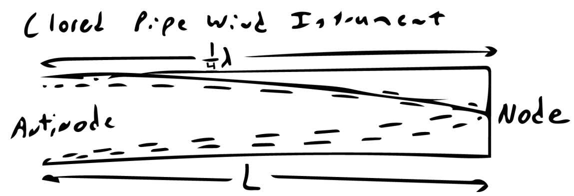


Standing waves will only occur at specific wavelengths and the wavelengths are determined by the length of the string or air column in the wind instrument!

An open end of a wind instruments creates an antinode and a closed end creates a node.

Stringed Instrument and Open Pipe:

- Fundamental Frequency or 1st harmonic: $\frac{1}{2}\lambda = L \Rightarrow \lambda = 2L$ & $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{v}{2L} = 1\left(\frac{v}{2L}\right)$
- 2nd harmonic: $\lambda = L$ & $f = \frac{v}{\lambda} = \frac{v}{L} = 2\left(\frac{v}{2L}\right)$
- 3rd harmonic: $\frac{3}{2}\lambda = L \Rightarrow \lambda = \frac{2L}{3}$ & $f = \frac{v}{\lambda} = \frac{v}{\left(\frac{2L}{3}\right)} = \frac{3v}{2L} = 3\left(\frac{v}{2L}\right)$
- $f = n\left(\frac{v}{2L}\right);\ n = 1, 2, 3, \ldots$

Closed Pipe Instrument:

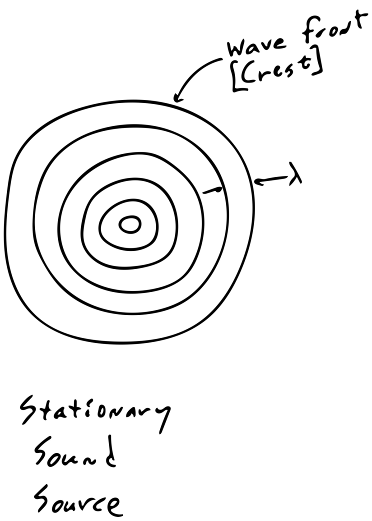
- Fundamental Frequency or 1st harmonic: $\frac{1}{4}\lambda = L \Rightarrow \lambda = 4L$ & $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{v}{4L} = 1\left(\frac{v}{4L}\right)$
- 3rd harmonic: $\frac{3}{4}\lambda = L \Rightarrow \lambda = \frac{4L}{3}$ & $f = \frac{v}{\lambda} = \frac{v}{\left(\frac{4L}{3}\right)} = 3\left(\frac{v}{4L}\right)$
- 5th harmonic: $\frac{5}{4}\lambda = L \Rightarrow \lambda = \frac{5L}{3}$ & $f = \frac{v}{\lambda} = \frac{v}{\left(\frac{5L}{3}\right)} = 5\left(\frac{v}{4L}\right)$
- $f = m\left(\frac{v}{4L}\right); m = 1,3,5,\ldots$

Beat Frequency: $f_{beat} = \left|f_1 - f_2\right|$

- When two notes are played that have frequencies that are close to one another, the constructive and destructive interference pattern creates "beats" in the sound.
- For example, $f_{beat} = \left|440 - 441\right| = 1hz$, will sound with 1 "beat" per second.

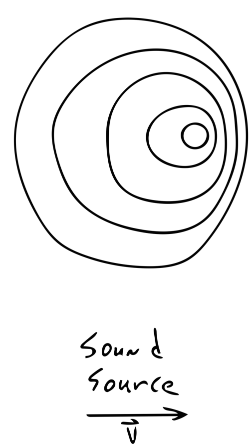
Doppler Effect: The change in the wavelength and therefore frequency and therefore pitch we hear of a moving sound source. (The observer can also move to cause the same effect.) Pictures!

- As the sound source moves towards the observer the crests are closer to one another and therefore the wavelength is decreased. $v = f\lambda$, therefore the frequency is increased and we hear a higher pitch.
- As the sound source moves away from the observer the crests are farther apart and therefore the wavelength is increased. $v = f\lambda$, therefore the frequency is decreased and we hear a lower pitch.



$$v = f\lambda \Rightarrow f = \frac{v}{\lambda}$$

Away from Observer:
$\lambda \uparrow$ & $f \downarrow$



Toward Observer:
$\lambda \downarrow$ & $f \uparrow$

Flipping Physics Lecture Notes:
AP Physics 1 Review of *Electrostatics*
https://www.flippingphysics.com/ap1-electrostatics-review.html

Elementary Charge: The smallest charge of an isolated particle. $e = 1.6 \times 10^{-19} C$

- Two examples: $q_{electron} = -e = -1.6 \times 10^{-19} C$ & $q_{proton} = +e = +1.6 \times 10^{-19} C$

The electron is a fundamental particle, however, the proton is not a fundamental particle.

Protons and neutrons are composed of "up" and "down" quarks: $q_{up\ quark} = +\frac{2}{3}e$ & $q_{down\ quark} = -\frac{1}{3}e$

- Proton is composed of 2 "up" quarks and 1 "down" quark.
  - $q_{proton} = 2q_{up\ quark} + 1q_{down\ quark} = 2\left(+\frac{2}{3}e\right) + \left(-\frac{1}{3}e\right) = +\frac{4}{3}e - \frac{1}{3}e = +e$
- Neutron is composed of 1 "up" quark and 2 "down" quarks.
  - $q_{neutron} = 1q_{up\ quark} + 2q_{down\ quark} = \left(+\frac{2}{3}e\right) + 2\left(-\frac{1}{3}e\right) = +\frac{2}{3}e - \frac{2}{3}e = 0$
- A quark can have a charge less than the Elementary Charge because a single quark has never been isolated; quarks are always found in groups like they are in the proton and neutron.

The Law of Charges: Unlike charges attract and like charges repel. For example:

- Two positive charges repel one another & two negative charges repel one another.
- A positive and a negative charge attract one another.

The force they repel or attract one another with is determine using Coulomb's Law: $F_e = \frac{kq_1q_2}{r^2}$

- This is called the Electrostatic Force. (Also sometimes called a Coulomb Force)
- Coulomb's Constant, $k = 8.99 \times 10^9 \frac{N \cdot m^2}{C^2}$
- q_1 & q_2 are the charges on the two charged particles.
- r is not the radius, it is the distance between the centers of charge of the two charges. (Sometimes r actually is the radius, however, that is not its definition.)
- Note the similarity to Newton's Universal Law of Gravitation: $F_g = \frac{Gm_1m_2}{r^2}$
  - However, comparing Coulomb's Constant to $G = 6.77 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$ shows that Coulomb's Constant is about 10^{20} times greater than the Gravitational Constant. In general, the electrostatic force is much, much, much greater than the gravitational force.

Conservation of Charge: In an isolated system the total charge stays constant. For example, if we start with two electrically isolated spheres, $q_{1i} = +4C$ & $q_{2i} = -2C$, we touch them together and pull them apart:

$$q_t = q_{1i} + q_{2i} = +4C + (-2C) = +2C \ \& \ q_{1f} = q_{2f} = q_f \Rightarrow q_t = q_{1f} + q_{2f} = q_f + q_f = 2q_f \Rightarrow q_f = \frac{q_t}{2} = \frac{+2C}{2} = +1C$$

Each sphere ends up with 6.24 x 10^{18} *excess* protons on it:

$$q_1 = n_1 e \Rightarrow n_1 = \frac{q_1}{e} = \frac{+1C}{1.6022 \times 10^{-19} C/proton} \approx 6.24 \times 10^{18}\ protons$$

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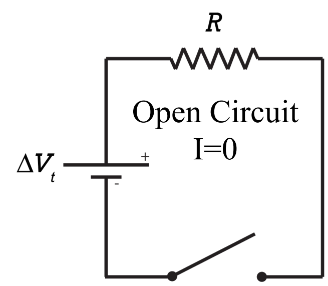
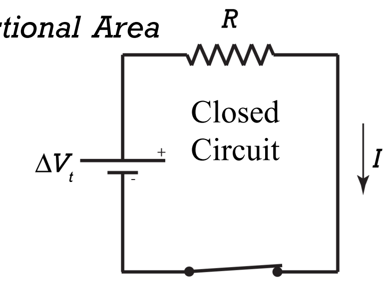
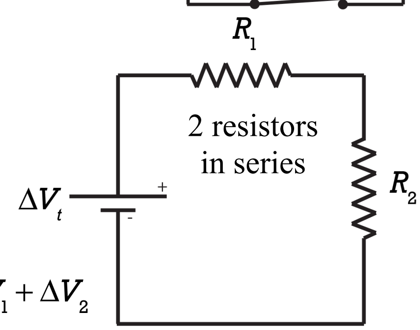
AP Physics 1 Review of *Electricity*
https://www.flippingphysics.com/ap1-electricity-review.html



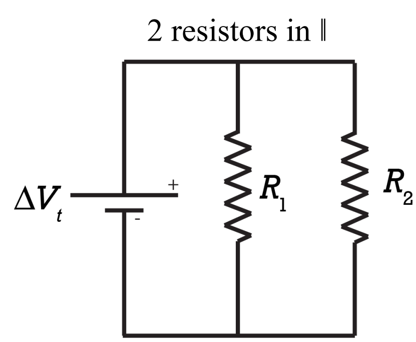
- Electric Current: The rate at which charges move.
    - $I = \frac{\Delta q}{\Delta t} \Rightarrow \frac{C}{s} = Amperes,\ Amps,\ A$
- Conventional current is the direction that positive charges "would" flow.
    - Even though it is usually negative charges flowing in the negative direction.
- Resistance, R: A resistor restricts the flow of charges.
    - $R = \frac{\rho \ell}{A};\ \rho = resistivity;\ \ell = length\ of\ wire;\ A = Cross\ Sectional\ Area$
    - Resistivity is a material property.



- Electric Potential Difference, $\Delta V = \frac{\Delta PE_{electrical}}{q}$
    - $\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow R = \frac{\Delta V}{I} \Rightarrow \frac{V}{A} = Ohm,\ \Omega$



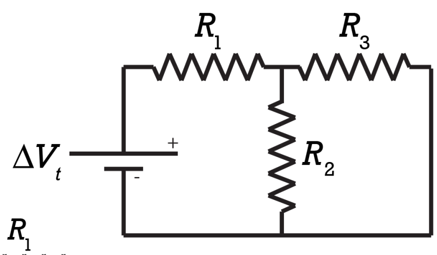
- Two resistors in series:
    - Using Kirchhoff's Loop Rule: $\Delta V_{loop} = 0$
    - $I_t = I_1 = I_2\ \&\ \Delta V_{loop} = 0 = \Delta V_t - \Delta V_1 - \Delta V_2 \Rightarrow \Delta V_t = \Delta V_1 + \Delta V_2$
      $\Rightarrow I_t R_{eq} = I_1 R_1 + I_2 R_2 \Rightarrow R_{eq} = R_1 + R_2 \Rightarrow R_{series} = R_1 + R_2 + R_3 + \ldots$



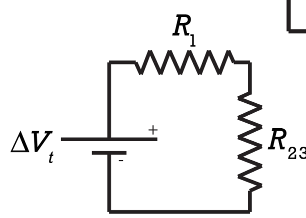
- Two resistors in parallel:
    - Using Kirchhoff's Junction Rule: $\sum I_{in} = \sum I_{out}$
    - $\Delta V_{loop} = 0 = \Delta V_t - \Delta V_1 \Rightarrow \Delta V_t = \Delta V_1$
    - $\Delta V_{loop} = 0 = \Delta V_t - \Delta V_2 \Rightarrow \Delta V_t = \Delta V_2$
    - $\Rightarrow \Delta V_t = \Delta V_1 = \Delta V_2\ \&\ \sum I_{in} = \sum I_{out} \Rightarrow I_t = I_1 + I_2$
    - $\Rightarrow \frac{\Delta V_t}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\right)^{-1}$

- Electric Power is the rate at which electric potential energy is being converted to heat and light. Also sometimes called the rate at which energy is dissipated in the circuit element.
    - $P = I\Delta V$ (the only equation for electric power on the equation sheet)
    - $P = I\Delta V = I(IR) = I^2 R = \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2}{R} \Rightarrow P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$
- Example Problem: Find the power dissipated in resistor #2.

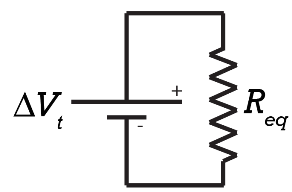
  $R_1 = 1.0\Omega,\ R_2 = 2.0\Omega,\ R_3 = 3.0\Omega,\ \Delta V_t = 6.0V,\ P_2 = ?$



- Resistors 2 and 3 are in parallel:
    - $R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{2} + \frac{1}{3}\right)^{-1} = 1.2\Omega$



- Resistors 1 and equivalent resistor 23 are in series:
    - $R_{eq} = R_1 + R_{23} = 1 + 1.2 = 2.2\Omega$



- We can find the current through the battery, which is the same as the current through resistor 1:
    - $\Delta V_t = I_t R_{eq} \Rightarrow I_t = \frac{\Delta V_t}{R_{eq}} = \frac{6}{2.2} = 2.\overline{72}A = I_1$
- We can now find the electric potential difference across resistor 1:
    - $\Delta V_1 = I_1 R_1 = (2.\overline{72})1 = 2.\overline{72}V$
- Now we can find the electric potential difference across equivalent resistor 23, which is the same as the electric potential difference cross resistor 2:
    - $\Delta V_t = \Delta V_1 + \Delta V_{23} \Rightarrow \Delta V_{23} = \Delta V_t - \Delta V_1 = 6 - 2.\overline{72} = 3.\overline{27}V = \Delta V_2$
- We have what we need to find the electric power in resistor 2:
    - $P_2 = \frac{(\Delta V_2)^2}{R_2} = \frac{(3.\overline{27})^2}{2} = 5.35537 \approx 5.4\ \frac{J}{s} \approx \boxed{5.4 watts}$

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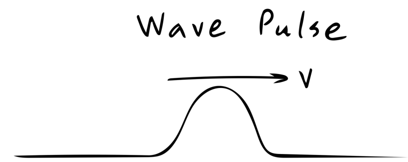

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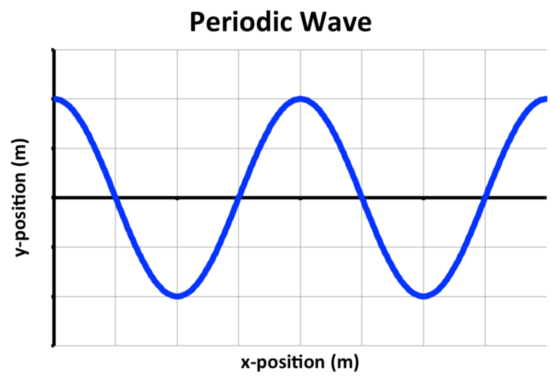

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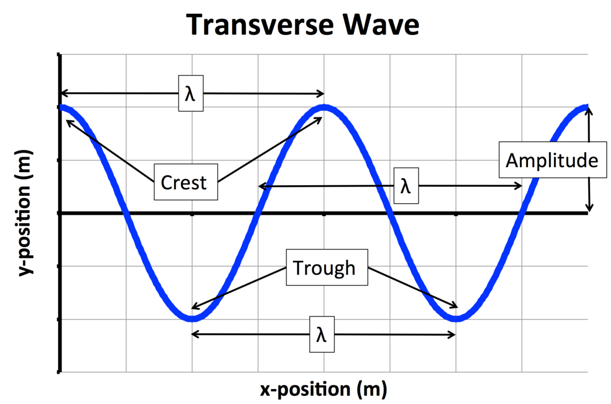

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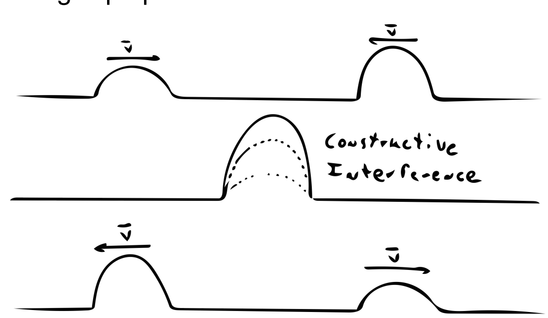

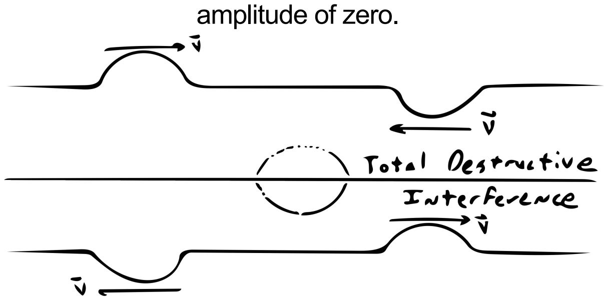

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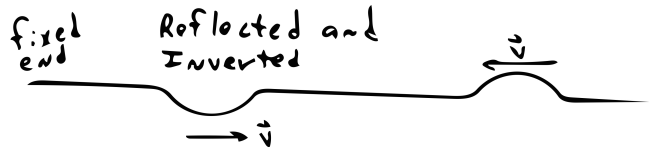

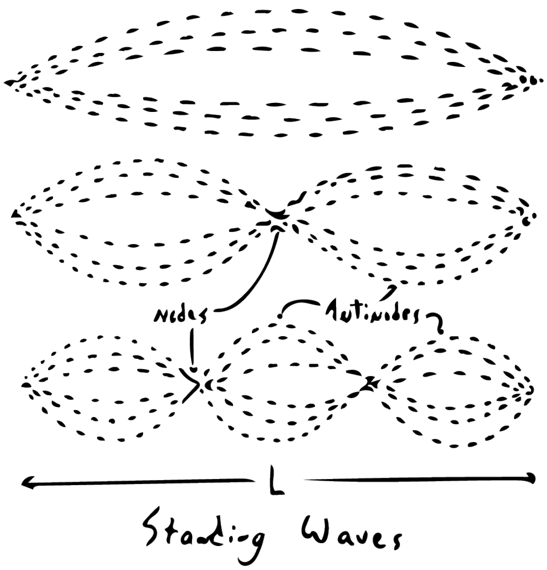

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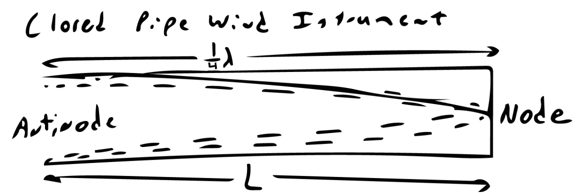

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- 2nd harmonic: $\lambda = L$ & $f = \frac{v}{\lambda} = \frac{v}{L} = 2\left(\frac{v}{2L}\right)$
- 3rd harmonic: $\frac{3}{2}\lambda = L \Rightarrow \lambda = \frac{2L}{3}$ & $f = \frac{v}{\lambda} = \frac{v}{\left(\frac{2L}{3}\right)} = \frac{3v}{2L} = 3\left(\frac{v}{2L}\right)$
- $f = n\left(\frac{v}{2L}\right);\ n = 1, 2, 3, \ldots$

Closed Pipe Instrument:

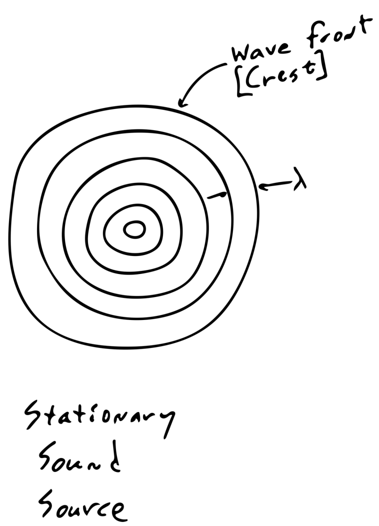
- Fundamental Frequency or 1st harmonic: $\frac{1}{4}\lambda = L \Rightarrow \lambda = 4L$ & $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{v}{4L} = 1\left(\frac{v}{4L}\right)$
- 3rd harmonic: $\frac{3}{4}\lambda = L \Rightarrow \lambda = \frac{4L}{3}$ & $f = \frac{v}{\lambda} = \frac{v}{\left(\frac{4L}{3}\right)} = 3\left(\frac{v}{4L}\right)$
- 5th harmonic: $\frac{5}{4}\lambda = L \Rightarrow \lambda = \frac{5L}{3}$ & $f = \frac{v}{\lambda} = \frac{v}{\left(\frac{5L}{3}\right)} = 5\left(\frac{v}{4L}\right)$
- $f = m\left(\frac{v}{4L}\right); m = 1,3,5,\ldots$

Beat Frequency: $f_{beat} = \left|f_1 - f_2\right|$

- When two notes are played that have frequencies that are close to one another, the constructive and destructive interference pattern creates "beats" in the sound.
- For example, $f_{beat} = \left|440 - 441\right| = 1hz$, will sound with 1 "beat" per second.

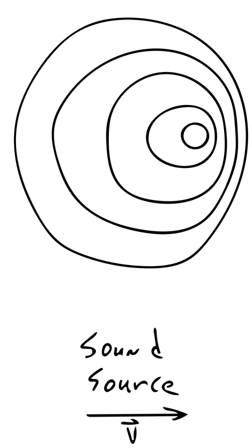
Doppler Effect: The change in the wavelength and therefore frequency and therefore pitch we hear of a moving sound source. (The observer can also move to cause the same effect.) Pictures!

- As the sound source moves towards the observer the crests are closer to one another and therefore the wavelength is decreased. $v = f\lambda$, therefore the frequency is increased and we hear a higher pitch.
- As the sound source moves away from the observer the crests are farther apart and therefore the wavelength is increased. $v = f\lambda$, therefore the frequency is decreased and we hear a lower pitch.



$$v = f\lambda \Rightarrow f = \frac{v}{\lambda}$$

Away from Observer:
$\lambda \uparrow$ & $f \downarrow$



Toward Observer:
$\lambda \downarrow$ & $f \uparrow$

Flipping Physics Lecture Notes:
AP Physics 1: *Equations to Memorize*
https://www.flippingphysics.com/ap1-equation-review.html

Let me be clear about what I mean by "memorize": I mean you should have the equation memorized, know what it means and know when you can use it. This is a lot more than just being able to write down the equation.

The following equations are *not* on the Equation Sheet provided by the AP College Board for the AP Physics 1 exam:

- $speed = \frac{distance}{time}$; $\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$; $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$
  - Please make sure you understand the differences between vectors and scalars, please.♥
- $\Delta x = \frac{1}{2}\left(v_f + v_i\right)\Delta t$
  - This is another Uniformly Accelerated Motion (UAM) equation you should know.
- $F_{g_\parallel} = mg\sin\theta$ & $F_{g_\perp} = mg\cos\theta$
  - When an object is on an incline, we often need to sum the forces in the parallel and perpendicular directions, which necessitates resolving the force of gravity into its components in the parallel and perpendicular directions.
  - Note: theta in this equation is the incline angle.
- Equations having to do with Mechanical Energy:
  - $ME_i = ME_f$: Conservation of Mechanical Energy can be used when there is no work done by the force of friction or the force applied.
  - $W_f = \Delta ME$: Can be used when there is no work done by the force applied.
  - $W_{net} = \Delta KE$: Is always true.
- $P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = \frac{Fd\cos\theta}{\Delta t} = Fv\cos\theta$
  - This is useful because you have power in terms of velocity.
- $\sum \vec{p}_i = \sum \vec{p}_f$
  - Conservation of linear momentum is valid when the net force acting on the system is zero, which is true during all collisions and explosions.
- $\vec{\omega} = \frac{\Delta \vec{\theta}}{\Delta t}$ & $\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$
  - Angular velocity and angular acceleration were, sadly, left off the equation sheet.

♥ Yes, that's a double please.

- $\omega_f{}^2 = \omega_i{}^2 + 2\alpha\Delta\theta$ & $\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t$
    - These two Uniformly Angularly Accelerated Motion (UαM) equations were also, sadly, left off the equation sheet.
- $\vec{v}_t = r\vec{\omega}$
    - The tangential velocity of an object.
- $v_{cm} = R\omega$
    - The velocity of the center of mass of an object rolling without slipping.
- $\sum \vec{F}_{in} = m\vec{a}_c$
    - The equation for the centripetal force acting on an object to keep it moving in a circle.
- $I = \sum_i m_i r_i^2$
    - The moment of inertia or "rotational mass" of a system of particles.
- $\sum \vec{L}_i = \sum \vec{L}_f$
    - Conservation of Angular Momentum, valid when the net external torque acting on the system is zero. $\sum \vec{\tau}_{external} = 0$
- $f_{beat} = |f_1 - f_2|$
    - The beat frequency heard due to the interference of two similar single frequency sounds.
- $q = ne$
    - The net change on an object equals the number of excess charges times the elementary charge.
- $\Delta V = \frac{\Delta PE_{electrical}}{q}$
    - The electric potential difference equals the change in electrical potential energy divided by charge.
- $P = I\Delta V$ is the only equation for electric power on the equation sheet.
    - However, using $\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow R = \frac{\Delta V}{I}$ we can find two more.
    - $P = I\Delta V = I(IR) = I^2R = \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2}{R}$
    - $P = I\Delta V = I^2R = \frac{\Delta V^2}{R}$ (This is what you should memorize for electric power.)