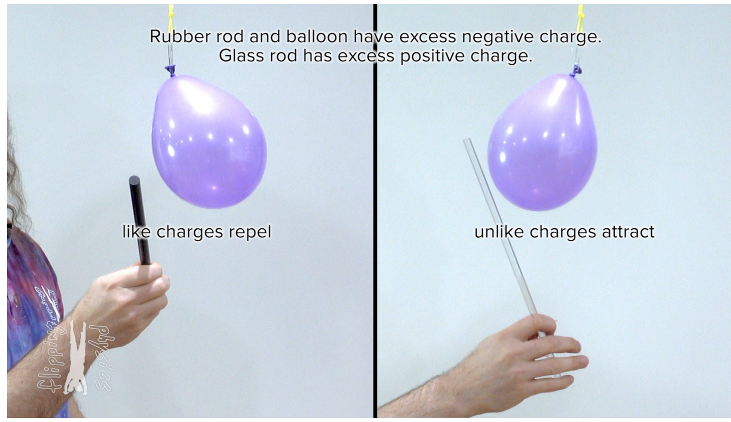
Flipping Physics Lecture Notes:

Electric Charge

https://www.flippingphysics.com/charge.html



The Law of Charges:

- Like charges repel &
- Unlike charges attract

When we rub fur against a rubber rod and then a rubber balloon, electrons transfer from the fur to the rubber objects leaving the rubber objects with a net negative charge and the fur with a net positive charge. When we rub silk against a glass rod, electrons transfer from the glass rod to the silk leaving the glass rod with a net positive charge and the silk with a net negative charge.

Electrons and Protons are very, very tiny particles with charge magnitude equal to e:

- e = 1.60 x 10-19 coulombs, C
- e = elementary charge
- e = smallest charge measured on an *isolated* particle.
- Coulombs, C = SI unit for charge
- Charge on electron = -e & Charge on proton = +e
- $m_{electron} = 9.11 \times 10^{-31} kg \ \& \ m_{proton} = 1.67 \times 10^{-27} kg$
- A proton is much more massive than an electron. $\frac{m_{proton}}{m_{electron}} = \frac{1.67 \times 10^{-27} kg}{9.11 \times 10^{-31} kg} \approx 1830$
- electron = elementary particle
- proton is not an elementary particle because it is composed of quarks.

Quarks are elementary particles which make up protons and neutrons:

- $q_{up quark} = +\frac{2}{3}e \ \& \ q_{down quark} = -\frac{1}{3}e$
- $q_{proton} = 2q_{up\ quark} + 1q_{down\ quark} = 2\left(+\frac{2}{3}e\right) + \left(-\frac{1}{3}e\right) = +\frac{4}{3}e - \frac{1}{3}e = +e$
- $q_{neutron} = 1q_{up\ quark} + 2q_{down\ quark} = \left(+\frac{2}{3}e\right) + 2\left(-\frac{1}{3}e\right) = +\frac{2}{3}e - \frac{2}{3}e = 0$

When you take a rubber balloon and rub it against fur, three things are possible.

1) The balloon will stay in your hair.
   a. Rub rubber balloon against hair and electrons transfer from hair to balloon.
   b. Balloon now has a net negative charge and hair now has a net positive charge.
   c. Law of Charges: unlike charges attract.
   d. Electric force pulls hair and balloon together.
2) Pull the balloon away from your hair and some of your hairs will stick out.
   a. Rub rubber balloon against hair and electrons transfer from hair to balloon.
   b. Hair now has a net positive charge.
   c. Law of Charges: like charges repel.
   d. • Electric force pushes hair apart.

3) The balloon will stick to a wall.
    a. This is polarization which we will learn about in a future lesson.
        i. https://www.flippingphysics.com/polarization.html

The elementary charge is very small: $e = 1.60 \times 10^{-19} C = 0.000000000000000000160 C$

Example: How many excess protons does it take to get a charge of 1 coulomb on an object?

For this we need a new equation: $q = ne$

- q = net charge on an object
- n = excess number of charge carriers
- e = elementary charge

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{1C}{1.60 \times 10^{-19} \frac{C}{proton}} = 6.25 \times 10^{18}\, protons$$

$$q = 6.25\ quintillion\ protons = 6.25\ million\ million\ million\ protons = 6{,}250{,}000{,}000{,}000{,}000{,}000\ protons$$

Example: Can an object have a net negative charge of 2.00 times 10 to the negative 19 coulombs?

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{-2.00 \times 10^{-19} C}{-1.60 \times 10^{-19} \frac{C}{electron}} = 1.25\, electrons$$

However, you cannot have a quarter of an electron because charge is *quantized*.

- Charge comes in discrete quantities in multiples of the elementary charge.
- Charge is caused by having more or fewer charged particles (protons or electrons).
- The charge on an object, q, must be an integer multiple of the elementary charge, e.
- In $q = ne$, n, the number of charge carriers, has to be an integer.
- Because you cannot cut protons and electrons into pieces.

So the answer is … No, you cannot have a net charge of $-2.00 \times 10^{-19} C$ on an object.

Flipping Physics Lecture Notes:
Introduction to Coulomb's Law or the Electric Force
https://www.flippingphysics.com/coulombs-law.html

We have already learned about the Law of Charges which governs the directions of the forces on pairs of charges.[1] Today we learn about the magnitude of that force.

The electric force is described by Coulomb's Law: $F_e = \frac{kq_1q_2}{r^2}$

- This is the electric force which exists between any two charged particles.
- It is sometimes called the Coulomb Force or Electrostatic Force. I will call it the Electric Force.
- k is the Coulomb Constant: $k = 8.99\times10^9 \frac{N\cdot m^2}{C^2}$
- q_1 and q_2 are the two electric charges.
- r is the distance between the centers of *charge* of the two charges.
- Note the similarities to Newton's Universal Law of Gravitation: $F_g = \frac{Gm_1m_2}{r^2}$
- k, the Coulomb Constant is much larger than, G, the Universal Gravitational Constant.

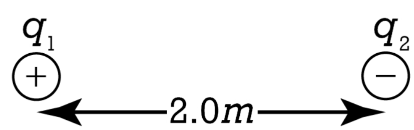
$$\frac{k}{G} = \frac{8.99\times10^9 \frac{N\cdot m^2}{C^2}}{6.67\times10^{-11} \frac{N\cdot m^2}{kg^2}} = 1.347826087\times10^{20} \approx 1.35\times10^{20}\frac{kg^2}{C^2}$$

A point charge is are just like a point mass, only the description has to do with charge rather than mass. In other words, a point charge is an object which has zero size and carries an electric charge. A point charge is an object whose mass is small enough that its mass is negligible when compared to its charge.

Three prefixes you should be familiar with when using coulombs:

- μ means micro or 1 millionth or $\times10^{-6}$ *so* 1 microcoulomb $= 1\mu C = 1\times10^{-6}C$
- n means nano or 1 billionth or $\times10^{-9}$ *so* 1 nanocoulomb $= 1nC = 1\times10^{-9}C$
- p means pico or 1 trillionth or $\times10^{-12}$ *so* 1 picoulomb $= 1pC = 1\times10^{-12}C$

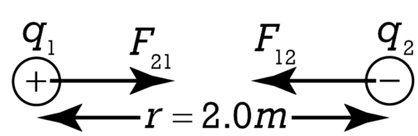
Example #1: Two equal magnitude point charges are located 2.0 meters apart. If the magnitudes of their charges are both 5.0 μC and one is positive and one is negative, what is the electric force acting on each charge caused by the other charge?



Knowns: $q_1 = +5.0\mu C \times \frac{1C}{1\times10^6 \mu C} = 5\times10^{-6}C;\ q_2 = -5.0\mu C = -5\times10^{-6}C;\ r = 2.0m;\ F_e = ?$

$$F_e = \frac{kq_1q_2}{r^2} = \frac{\left(8.99\times10^9\right)\left(5\times10^{-6}\right)\left(-5\times10^{-6}\right)}{2^2} = -0.0561875 \approx -0.056N$$

$$\boxed{F_{\text{each charge}} \approx 0.056N \text{ toward the other charge}}$$



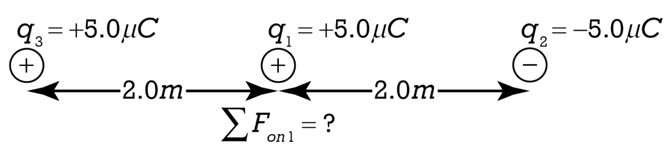
[1] See video "Electric Charge, Law of Charges, and Quantization of Charge" at https://www.flippingphysics.com/charge.html

But what does the negative on $F_e \approx -0.056N$ mean?

I have seen three different versions of Coulomb's Law:

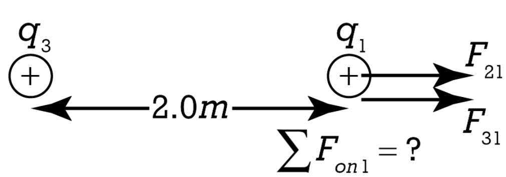
- The one we have been working with: $F_e = \frac{kq_1q_2}{r^2}$
- The magnitude of the electric force: $\left|\vec{F}_e\right| = k\left|\frac{q_1q_2}{r^2}\right|$
  - This ignores direction information, so we are not going to use it.
- And the unit vector version: $\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^{\ 2}}\hat{r}_{12}$
  - $\vec{F}_{12}$ is the electric force by charge 1 on charge 2
  - $\hat{r}_{12}$ is the unit vector directed from charge 1 toward charge 2
  - We have not worked with unit vectors in this algebra based class yet, so we are not going to use this verson of Coulomb's Law.
- Therefore we will use the first one: $F_e = \frac{kq_1q_2}{r^2}$
  - A negative force means an attractive force.
  - A positive force means a repulsive force.
- Note: $F_e = \frac{kq_1q_2}{r^2}$ can be attractive or repulsive however, $F_g = \frac{Gm_1m_2}{r^2}$ is always attractive.

Building on Example #1, Example #2 is .. If we place a third charge, q 3, with the same charge as the positively charged object, q 1, only now 2.0 meters on the opposite side from the other negative charge, q 2, what is the net force acting on q 1, the positive charge in the middle?



Adding the third charge does not affect the electric force of 0.056 newtons which is from q_2 on q_1 and acting to the right. We just need to add the electric force from q_3 on q_1. Because both q_1 and q_3 are positive, force 3-1 will be a positive, repulsive force, on q_1 that would be to the right.

$$F_{31} = \frac{kq_1q_3}{r_{31}^{\ 2}} = \frac{\left(8.99\times10^{9}\right)\left(+5\times10^{-6}\right)\left(+5\times10^{-6}\right)}{2^2} = 0.0561875N \text{ to the right}$$



$$\sum F_{on\,1} = F_{21} + F_{31} = 0.0561875 + 0.0561875 = 0.112375 \approx \boxed{0.11\text{N to the right}}$$

Flipping Physics Lecture Notes:
Charging via Conduction and Induction
https://www.flippingphysics.com/conduction-and-induction.html

An electroscope is an instrument for demonstrating electric charge. This electroscope has a metal ball on top of a vertical metal rod with a hook on the end of it and two thin foils of aluminum hanging from the hook. The metal ball, rod, hook, and two aluminum foils are electrically insulated from the surroundings via the rubber stopper and glass flask.

Charge by Conduction:

- charge the balloon
- bring balloon close to electroscope and foils move apart
- touch balloon to electroscope
- electroscope is charged by conduction
- touch electroscope with hand and foils fall down to original positions

In order to charge the balloon, we rub fur on balloon. This causes electrons to move from the fur to the rubber balloon. The fur now has excess positive charge and the balloon has excess negative charge. This is called charging by friction. Before balloon is close to the electroscope the net charge on electroscope equals zero. The electroscope has an equal number of protons and electrons. The balloon has larger number of electrons then protons.

The negatively charged balloon is brought close to the electroscope, however, the electroscope and the balloon have not touched yet. The aluminum foils are pushed away from one another. The foils must have the same charge. The foils must have an electric force pushing them apart.

We can use the Law of Charges to determine the charge on the foils. We know protons do not easily move because they are in the nucleus of the atoms. That means the electrons are the ones that move. So, the electrons in the electroscope, which are negatively charged, are repelled by the negative charges in the balloon and flow down to the metal foils. That must mean, when the balloon is brought near the electroscope, the metal foils have an excess of electrons, or a net negative charge, and an electric force pushes the metal foils away from one another. That means the top of the electroscope has a net positive charge because there are fewer electrons than protons in the ball at the top of the electroscope. However, realize the electroscope still has a net neutral charge.

When the balloon touches the electroscope, electrons transfer from the balloon to the electroscope. The balloon and electroscope now both have excess electrons, however, total number of excess electrons in the system remains the same. This is called Conservation of Charge. Because the balloon and electroscope have excess electrons, the foils also have excess electrons and repel one another.

Now I touch the electroscope with my finger and the foils fall to their original positions. This is because touching electroscope grounds the electroscope. That means the electroscope is no longer electrically isolated and electrons transfer from electroscope into ground. After grounding, the electroscope has neutral charge. An Ideal Ground is an infinite well of charge carriers. We call it a “ground” because electrical circuits are literally connected to the Earth or the “ground”. And the Earth has, relatively speaking, an infinite number of electrons which we can pull from it, or we can give to it, again relatively speaking, an infinite number of charges. If something goes wrong in a circuit, the “ground” will serve as a way to balance out the charges. In this example, when I touch the electroscope, the electroscope is no longer electrically isolated and the excess electrons on the electroscope flow out of the electroscope into the “ground”, into me, and the electroscope is now electrically neutral, which is why the foils are no longer repelled from one another.

Two items to remember about Charging by Conduction:

1. The two objects have to touch. In this case the two objects are the balloon and the electroscope.
2. The two objects end with the same sign of net charge. In this case they both end with an excess negative charge. (Before the electroscope is grounded and ends with a net neutral charge.)

Charging by Conduction in pictorial form:

And now we switch to Charging by Induction:

- charge the balloon
- bring balloon close to electroscope
- ground the electroscope
- remove the ground
- remove the balloon and electroscope is charged by induction
- ground the electroscope

This time when the electroscope is grounded the negatively charged balloon is held near the electroscope, therefore, electrons in the electroscope flow into the ground. They do that because, according to the Law of Charges, like charges repel one another. So, some of the electrons in the electroscope flow from the electroscope into the ground. The ground essentially provides an escape route for electrons to leave the electroscope. But there are still electrons in the electroscope, just fewer than before, and many are in the metal foils of the electroscope because they are repelled from the electrons in the balloon. That gives the metal foils a net neutral charge and the two foils are not repelled from one another.

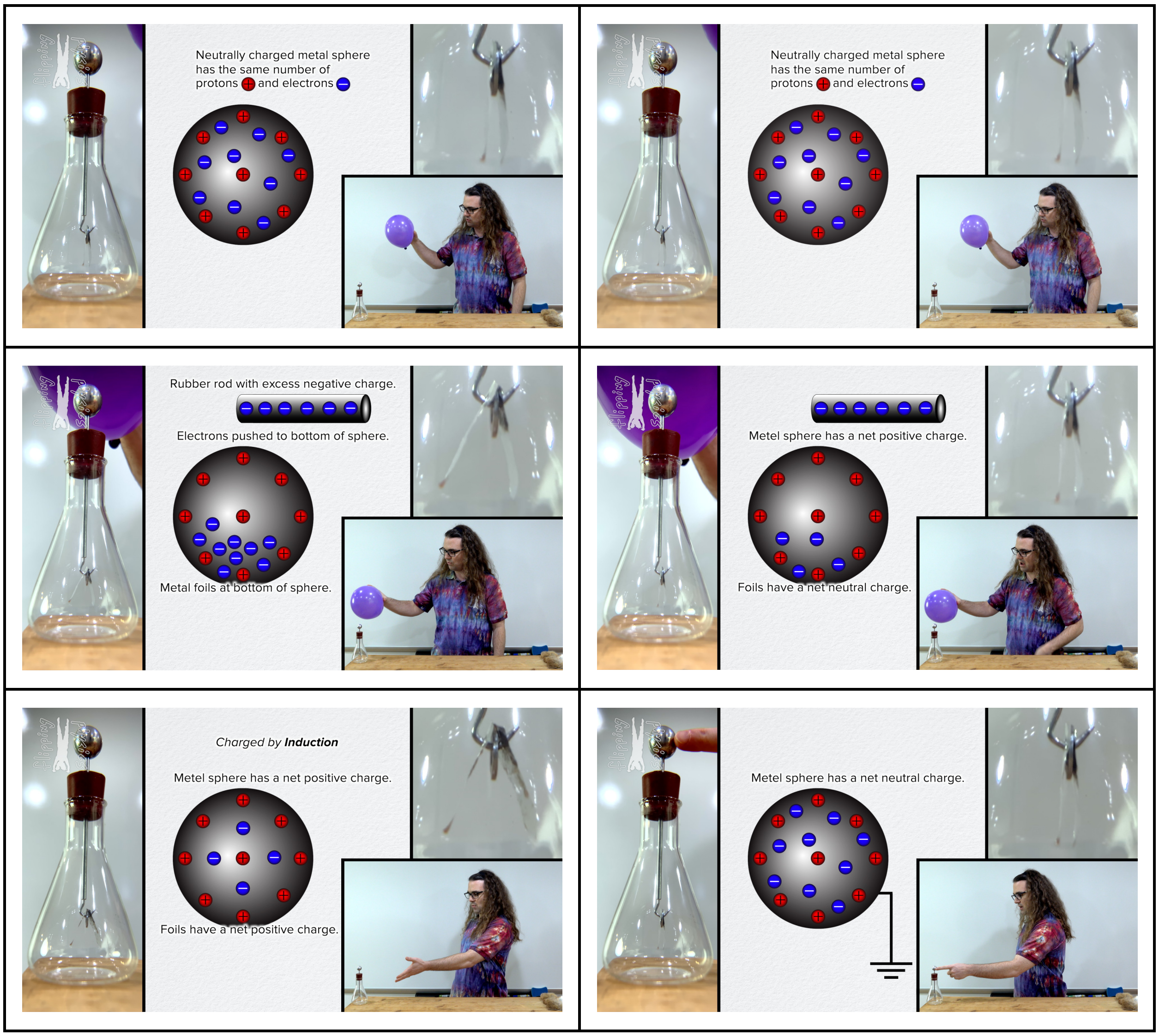
Then when the balloon is removed, the electrons in the electroscope are repelled from one another in the electroscope and get distributed throughout the electroscope. That leaves the electroscope with an excess of protons and a net positive charge. That is why the metal foils are repelled from one another, because the positive charges in the foils repel one another.

And then the electroscope is grounded. In this case, that means electrons are pulled from the ground into the electroscope to balance out the excess protons in the electroscope and leave the electroscope with an equal number of protons and electrons and a net neutral charge.

Two items to remember about Charging by Induction:

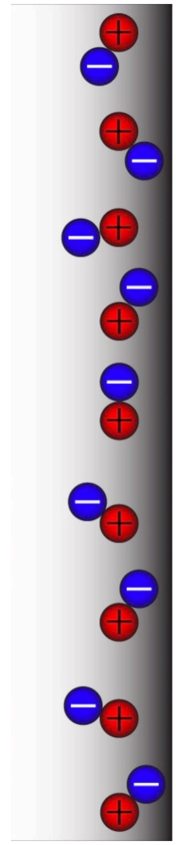
1. The two objects do not have to touch.
2. The two objects end with opposite sign of net charge. In this case the electroscope ends with an excess of positive charge and the balloon ends with an excess negative charge.
   (Before the electroscope is grounded and ends with a net neutral charge.)

Charging by Induction in pictorial form:

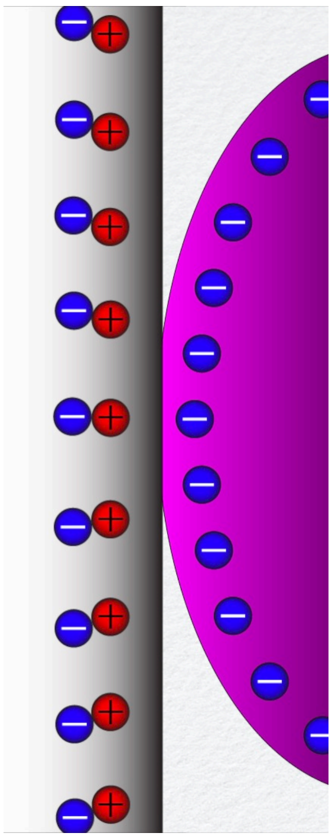


Flipping Physics Lecture Notes:
Polarization of Charge
https://www.flippingphysics.com/polartization-of-charge.html

A charged balloon is attracted to a wall because the molecules in the wall become polarized. Polarization does not mean the wall becomes charged, it simply means the molecules in the wall have aligned themselves such that there will be a net attractive force between the wall and the charged balloon.



To the right is an illustration of electrons and protons randomly oriented in the wall before the charged balloon is brought close to the wall.



When the balloon is brought near the wall (left illustration), the electrons in the wall move away from the electrons in the balloon because, according to the Law of Charges, the electrons in the wall will be repelled from electrons in the balloon because they both have negative charges.

We know like charges repel and unlike charges attract, however, it is important to notice that all the like charges are farther apart than all of the unlike charges. And because the electric force between the charges will be determined by Coulomb's Law: $F_e = \frac{kq_1q_2}{r^2}$, we know the closer the charged objects are to one another, the smaller the "r" value in Coulomb's Law and therefore the larger the electric force.

In other words, because the opposite charges are closer than the like charges, the attractive electric force is larger than the repulsive electric force and the net electric force between the balloon and the wall is an attractive force. This is how a charged object can be attracted to a neutrally charged object.

Two other examples of attracting by polarization are that the charged balloon can pick up little pieces of paper and cause an aluminum can to roll. In both cases, the charged balloon polarizes the other objects, the little pieces of paper and aluminum can, and therefore the objects are attracted to one another.

Please realize the electric forces in the polarization demonstrations are quite small. The masses of the balloon and little pieces of paper are small, so only a small electric force is required to hold them up and it only requires a small force to roll the aluminum can.

The electric force caused by polarization is typically larger for a conductor than an insulator, Because, in insulators, electrons are just pushed to the opposite side of the atom, however, in conductors, the electrons are free to move about more and actually end up farther away. That will produce a larger difference in attractive vs repulsive force and a larger net electric force in a conductor than an insulator.

Flipping Physics Lecture Notes:

Conservation of Charge Example Problems
https://www.flippingphysics.com/conservation-of-charge.html

Conservation of Charge: The total electric charge of an isolated system never changes.

What is an isolated system? We could start with the universe. In other words, the net electric charge of the universe never changes. Add up all the positive charges and subtract all the negative charges and you will always get the same number.

Or we could have a smaller isolated system, like the conductive metal pieces of an electroscope which are electrically isolated from the rest of the universe by the rubber and glass insulators. In other words, the net electric charge of the electroscope will remain constant, as long as it remains isolated.

Example Problem #1: Two charged, conducting objects collide and separate. Before colliding, the charges on the two objects are +3e and -6e. Which of the following are possible values for the final charges on the two objects? Choose all possible answers.

(a) +4e, -7e (b) +2e, -2e (c) -1.5e, -1.5e (d) -3.5e, +2.5e (e) +e, -4e

$q_{1i} = +3e;\ q_{2i} = -6e;\ q_{1f} = ?;\ q_{2f} = ?$ & $q_{total\ i} = q_{1i} + q_{2i} = +3e + (-6e) = -3e = q_{total\ f}$

a) $q_{1f} + q_{2f} = +4e + (-7e) = -3e = q_{total\ f}$ ✓ **it works**

b) $q_{1f} + q_{2f} = +2e + (-2e) = 0 \neq -3e = q_{total\ f}$ **does not work**

c) $q_{1f} + q_{2f} = -1.5e + (-1.5e) = -3e = q_{total\ f}$
But (c) **does not work** because you cannot have half an electron because charge is quantized!!

d) $q_{1f} + q_{2f} = -3.5e + 2.5e = -e = q_{total\ f}$ **does not work**
But (d) also does not work because you cannot have half an electron because charge is quantized!!

e) $q_{1f} + q_{2f} = +e + (-4e) = -3e = q_{total\ f}$ ✓ **it works**

Correct answers are (a) and (e) because those are the only two options which have a total final charge equal to the total initial charge and are integer multiples of the fundamental charge e.

Example Problem #2: Two identical, conducting spheres are held using insulating gloves a distance x apart. Initially the charges on each sphere are +3.0 pC and +6.0 pC. The two spheres are touched together and returned to the same distance x apart. You may assume x is the distance between their centers of charge.

(a) What is the final charge on each sphere?
(b) Is the final electric force between the two spheres increased, decreased, or the same when compared to the initial electric force?

.

$$q_{1i} = +3.0pC;\ q_{2i} = +6.0pC;\ r_i = r_f = x;\ \mathrm{Part(a)}:\ q_{1f} = ?;\ q_{2f} = ?;\ \mathrm{Part(b)}:\ F_{ef}\ ?\ F_{ei}$$

$$q_{total\ f} = q_{total\ i} = q_{1i} + q_{2i} = +3pC + 6pC = +9pC$$

Because the two spheres are identical, after touching, the spheres will have equal charge.

$$q_{1f} = q_{2f} = q_f \Rightarrow q_{total\ f} = q_{1f} + q_{2f} = 2q_f = 9pc \Rightarrow q_f = 4.5pC$$

(a) Both charges end with 4.5 pC of charge.

This is 4.5 picocoulombs of charge or 4.5 x 10^{-9} C which an object is physically able to have. Because then it will have:

$$q_f = n_f e \Rightarrow n_f = \frac{q_f}{e} = \frac{4.5\times10^{-9}C}{1.60\times10^{-19}\frac{C}{\text{charge carrier}}} = 2.8125\times10^{10} \approx 2.8\times10^{10}\text{excess protons}$$

Imagine that. 28 billion more protons than electrons on each sphere. Each sphere will have a heck of a lot more total protons and electrons, however, it has a deficit of 28 billion electrons and therefore has a net charge of 4.5 pC.

And now part (b): The two spheres have like charges, so they are repelled from one another with an electric force with a magnitude of: $F_e = \frac{kq_1q_2}{r^2}$ Therefore …

$$F_{ei} = \frac{kq_{1i}q_{2i}}{r_i^2} = \frac{k\left(3\times10^{-9}\right)\left(6\times10^{-9}\right)}{x^2} = 1.8\times10^{-17}\frac{k}{x^2}$$

$$F_{ef} = \frac{kq_{1f}q_{2f}}{r_f^2} = \frac{k\left(4.5\times10^{-9}\right)\left(4.5\times10^{-9}\right)}{x^2} = 2.025\times10^{-17}\frac{k}{x^2} \approx 2.0\times10^{-17}\frac{k}{x^2}$$

k is the Coulomb Constant and has a constant value: $k = 8.99\times10^{9}\frac{N\bullet m^2}{C^2}$

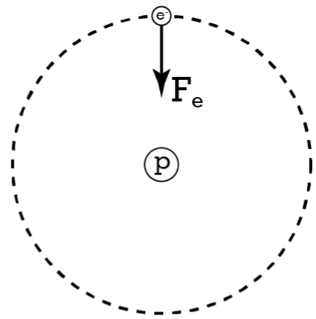
x is also a constant. Therefore: $F_{ef} \approx 2.0\times10^{-17}\frac{k}{x^2} > 1.8\times10^{-17}\frac{k}{x^2} = F_{ei}$

The final electric force is *greater than* the initial electric force.

Flipping Physics Lecture Notes:

Determining the Speed of the Electron in the Bohr Model of the Hydrogen Atom

https://www.flippingphysics.com/electron-speed-bohr.html



Assuming a circular orbit of the electron about the nuclear proton in the Bohr model of the hydrogen atom, determine the speed of the electron. The electron orbits at a radius of 5.29 x 10^{-11} m.

Draw free body diagram of the electron.
Only one force, the electric force, inward toward the proton.

$$r_{orbit} = 5.29\times10^{-11}m;\ v_t = ?\left(\text{magnitude}\right)$$

$$\sum F_{in} = F_e = ma_c \Rightarrow \frac{kq_1q_2}{r^2} = m\left(\frac{v_t^{\,2}}{r}\right) \Rightarrow \frac{kq_1q_2}{r} = mv_t^{\,2} \Rightarrow v_t = \sqrt{\frac{kq_1q_2}{mr}}$$

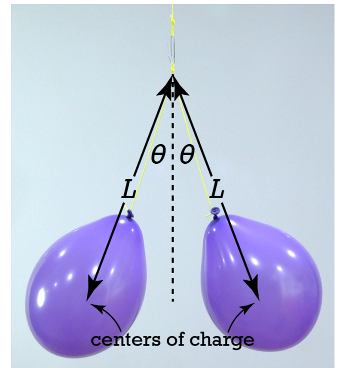
$$\Rightarrow v_t = \sqrt{\frac{\left(8.99\times10^{9}\right)\left(1.60\times10^{-19}\right)^2}{\left(9.11\times10^{-31}\right)\left(5.29\times10^{-11}\right)}} = 2.1853\times10^{6}\frac{m}{s} \approx \boxed{2.19\times10^{6}\frac{m}{s}}$$

$$v_t = 2.1853\times10^{6}\frac{m}{s}\left(\frac{3600sec}{1hour}\right)\left(\frac{1mile}{1609m}\right) \approx 4.89\times10^{6}\frac{mi}{hr}$$

Flipping Physics Lecture Notes:

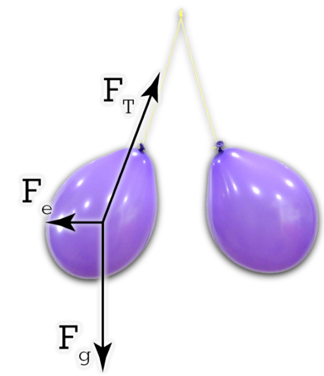
Balloon Excess Charges Experiment

https://www.flippingphysics.com/balloon-charges.html



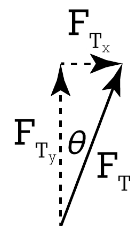
Two 0.0018 kg balloons each have approximately equal magnitude excess charges and hang as shown. If θ = 21° and L = 0.39 m, what is the average number of excess charges on each balloon?

Knowns: $m = 0.0018kg;\ \theta = 21^\circ;\ L = 0.39m;\ q_{avg} = ?$



Draw Free Body Diagram on left balloon.

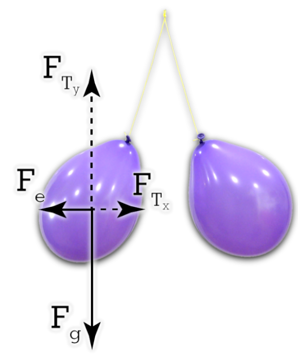
Break Force of Tension into its components.



$$\sin\theta = \frac{O}{H} = \frac{F_{T_x}}{F_T} \Rightarrow F_{T_x} = F_T \sin\theta$$

$$\cos\theta = \frac{A}{H} = \frac{F_{T_y}}{F_T} \Rightarrow F_{T_y} = F_T \cos\theta$$

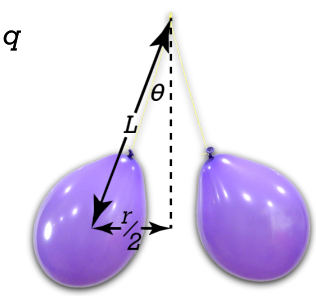
Redraw the Free Body Diagram.



$$\sum F_y = F_{T_y} - F_g = ma_y = m(0) = 0 \Rightarrow F_{T_y} = F_g \Rightarrow F_T \cos\theta = mg \Rightarrow F_T = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_{T_x} - F_e = ma_x = m(0) = 0 \Rightarrow F_{T_x} = F_e \Rightarrow F_T \sin\theta = \frac{kq_1q_2}{r^2} \ \&\ q_1 \approx q_2 \approx q_{avg} \approx q$$

$$\Rightarrow \left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{kqq}{r^2} \Rightarrow mg\tan\theta = \frac{kq^2}{r^2} \Rightarrow q = \sqrt{\frac{mgr^2\tan\theta}{k}} = r\sqrt{\frac{mg\tan\theta}{k}}$$



$$\sin\theta = \frac{O}{H} = \frac{r/2}{L} \Rightarrow r = 2L\sin\theta$$

$$\Rightarrow q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}} = 2(0.39)\sin(21)\sqrt{\frac{(0.0018)(9.81)\tan(21)}{8.99\times10^9}} = 2.42719\times10^{-7} \approx 2\times10^{-7}C$$

$$\Rightarrow q_{avg} \approx 2\times10^{-7}C\left(\frac{1\times10^9 nC}{1C}\right) \approx 200nC$$

Is the force of gravity which exists between the two balloons truly negligible?

$$F_e = \frac{kq_1q_2}{r^2} = \frac{kq^2}{4L^2\sin^2\theta} = \frac{\left(8.99\times10^9\right)\left(2.42719\times10^{-7}\right)^2}{\left(4\right)\left(0.39\right)^2\sin^2\left(21\right)} = 0.00677827 \approx 7\times10^{-3}N$$

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{Gm^2}{4L^2\sin^2\theta} = \frac{\left(6.67\times10^{-11}\right)\left(0.0018\right)^2}{\left(4\right)\left(0.39\right)^2\sin^2\left(21\right)} = 2.76582\times10^{-15} \approx 3\times10^{-15}N$$

$$\frac{F_e}{F_g} = \frac{0.00677827}{2.76582\times10^{-15}} = 2.45073\times10^{12} \Rightarrow F_e \approx 2\times10^{12}F_g$$

Given that the electric force is roughly 2 million million times larger than the force of gravity, I would say it is completely reasonable to assume the force of gravity which exists between the two balloons is negligible.

Now we actually answer the question, "what is the average number of excess charges on each balloon?"

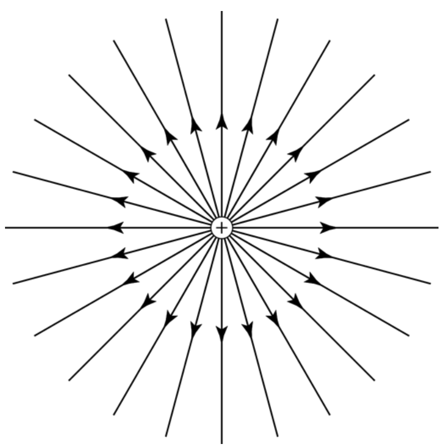
$$q = ne \Rightarrow n = \frac{q}{e} = \frac{2.42719\times10^{-7}C}{1.60\times10^{-19}\frac{C}{\text{charge carrier}}} = 1.51699\times10^{12} \approx \boxed{2\times10^{12}\text{ excess charge carriers}}$$

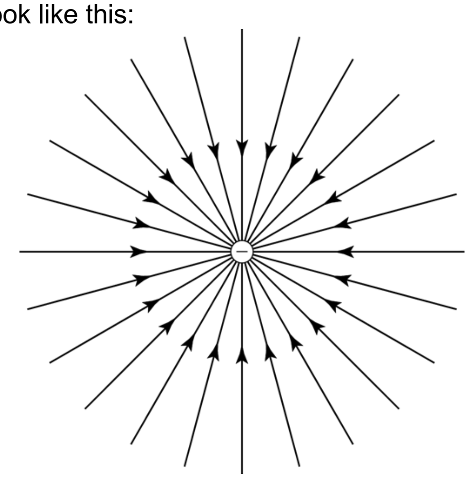
Flipping Physics Lecture Notes:

Electric Fields

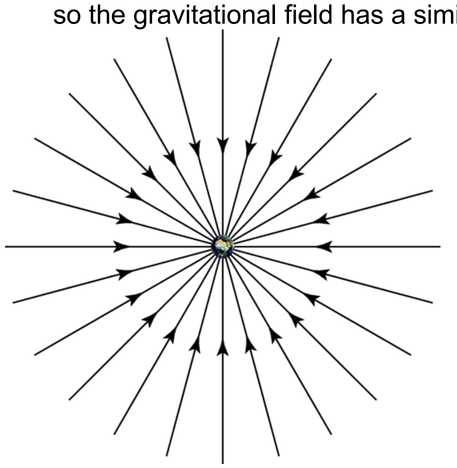
http://www.flippingphysics.com/electric-fields.html

- If we were to place a positive test charge in an electric field, it would experience an electrostatic force. An electric field is the ratio of the electrostatic force the test charge would experience and the charge of the test charge.
    - A positive test charge is a charge which is small enough not to measurably change the electric field it is placed in. Electric field directions are defined according to the direction of the net electrostatic force on a positively charged test charge.
- The equation for an electric field and its units are: $\vec{E} = \frac{\vec{F}_e}{q} \Rightarrow \frac{N}{C}$
- Notice the electric field and the electrostatic force experienced by a positive charge in the electric field will be in the same direction. (Both electric field and electrostatic force are vectors in the equation.)
- That means the electric field which surrounds a point charge is: $\vec{E} = \frac{\vec{F}_e}{q} \Rightarrow E_{\text{point charge}} = \frac{\frac{kqQ}{r^2}}{q} = \frac{kQ}{r^2}$
- And the electric field maps for **isolated** point charges look like this:

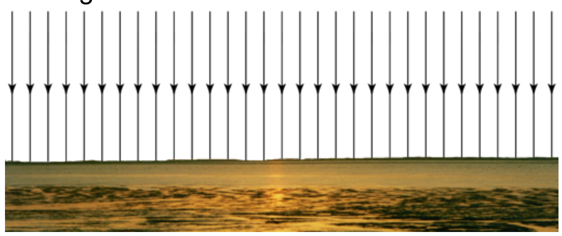


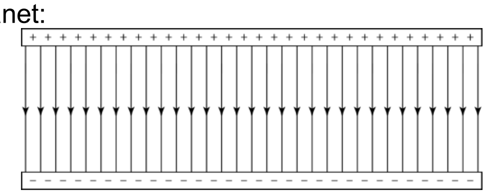


- Notice the similarity to the gravitational field around a planet. The equation has a similar format, so the gravitational field has a similar format: $g = \frac{F_g}{m} \Rightarrow g_{\text{point mass}} = \frac{\frac{GmM}{r^2}}{m} = \frac{GM}{r^2}$

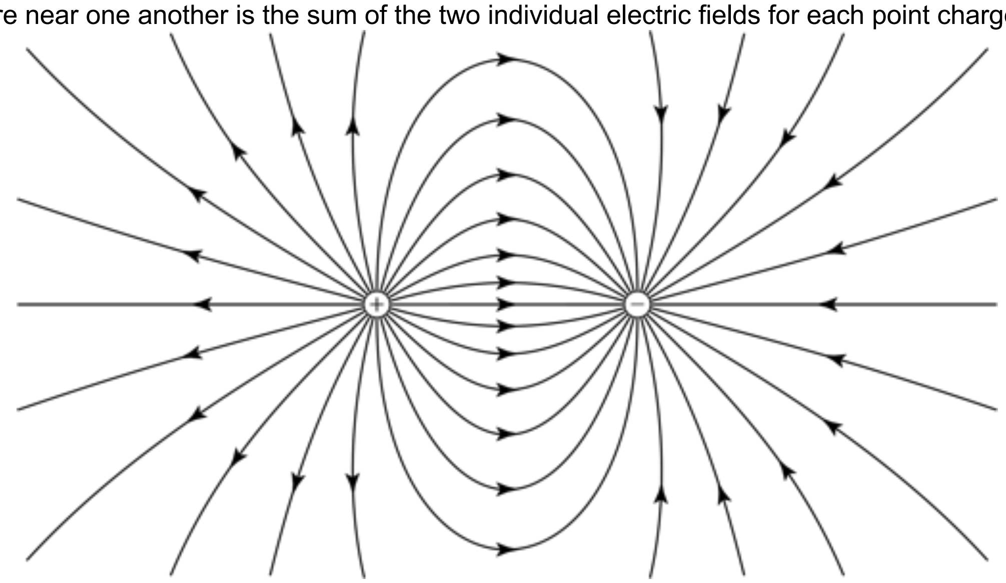


- And the electric field which exists between two large, parallel, oppositely charged plates is similar to the gravitational field close to the surface of a planet:





-
- Remember electric field is a vector which means that the electric field for two point charges which are near one another is the sum of the two individual electric fields for each point charge.



- Electric field maps like the one above are simplified models and vector maps which show the magnitude and direction of the electric field for the entire region.
- Electric Field Lines Basics:
    - In the direction a small, positive, test charge would experience an electrostatic force
    - Electric field lines per unit area is proportional to electric field strength
        - Higher density electric field lines = higher electric field strength
    - Start on a positive charge and end on a negative charge
        - or infinitely far away if more of one charge than another
    - Always start perpendicular to the surface of the charge
    - Electric field lines never cross

Continuous Charge Distribution: A charge that is not a point charge. In other words, a charge which has shape and continuous charge distributed throughout the object.

In order to find the electric field which exists around a continuous charge distribution, we can use Coulomb's Law and the equation definition of an electric field. We consider the charged object to be made up of an infinite number of infinitesimally small point charges dq and add up the infinite number of electric fields via superposition. It's an integral.

$$\vec{F}_e = k\frac{(q_1)(q_2)}{r^2}\hat{r} \;\&\; \vec{E} = \frac{\vec{F}_e}{q} \Rightarrow \vec{E}_{\text{point charge}} = k\frac{\frac{(q)(Q)}{r^2}}{q}\hat{r} = \frac{kQ}{r^2}\hat{r}$$

$$\Rightarrow d\vec{E} = \frac{k(\mathrm{d}q)}{r^2}\hat{r} \Rightarrow \int d\vec{E} = \int \frac{k(\mathrm{d}q)}{r^2}\hat{r} \Rightarrow \vec{E}_{\text{continuous charge distribution}} = k\int \frac{\mathrm{d}q}{r^2}\hat{r}$$

Realize that, for AP Physics C: Electricity and Magnetism, students are only expected to be able to use this equation to determine electric fields around continuous charge distributions with high symmetry. The specific examples students are responsible for are:

- An infinitely long, uniformly charged wire or cylinder at a distance from its central axis
- A thin ring of charge at a location along the axis of the ring
- A semicircular arc or part of a semicircular arc at its center
- A finite wire or line of charges at a distance that is collinear with the line of charge or at a location along its perpendicular bisector.

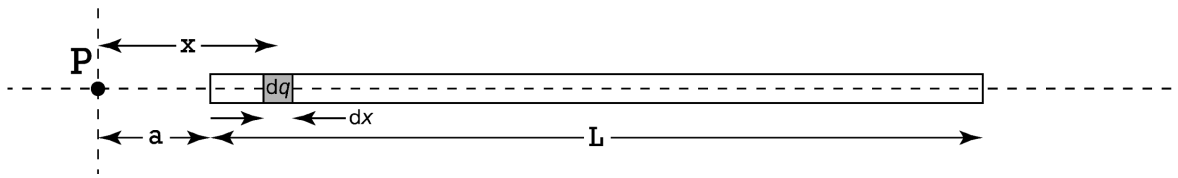
Quick review of charge densities:

linear charge density, $\lambda = \frac{Q}{L}$ in $\frac{C}{m}$ & surface charge density, $\sigma = \frac{Q}{A}$ in $\frac{C}{m^2}$

& volumetric charge density, $\rho = \frac{Q}{V}$ in $\frac{C}{m^3}$

Flipping Physics Lecture Notes:
Charged Rod Electric Field
http://www.flippingphysics.com/rod-electric-field.html

Determine the electric field at point P, which is located a distance “a” to the left of a thin rod with a charge +Q, uniform charge density λ, and length L.



Notice that, if we were to place a positive point charge at point P, it would experience a force to the left from every dq or every infinitesimally small part of the wire. Therefore, we already know the direction of the electric field at point P, it will be to the left or in the negative “i" direction. Now let’s solve for the electric field:

$$\vec{E} = k\int \frac{dq}{r^2}\hat{r} \Rightarrow \vec{E} = k\int \frac{dq}{x^2}\left(-\hat{i}\right) \;\&\; \lambda = \frac{Q}{L} = \frac{dq}{dx} \Rightarrow dq = \lambda dx \;\&\; Q = \lambda L$$

$$\Rightarrow \vec{E} = -k\hat{i}\int \frac{\lambda dx}{x^2} = -k\lambda\hat{i}\int_a^{a+L}\left(\frac{1}{x^2}\right)dx = -k\lambda\hat{i}\int_a^{a+L}\left(x^{-2}\right)dx$$

$$\Rightarrow \vec{E} = -k\lambda\hat{i}\left[\frac{x^{-1}}{-1}\right]_a^{a+L} = k\lambda\hat{i}\left[\frac{1}{x}\right]_a^{a+L} = k\lambda\hat{i}\left[\frac{1}{a+L} - \frac{1}{a}\right] = k\lambda\hat{i}\left[\frac{a-(a+L)}{a(a+L)}\right]$$

$$\Rightarrow \vec{E} = k\lambda\hat{i}\left[\frac{a-a-L}{a(a+L)}\right] = -\frac{k\lambda L}{a(a+L)}\hat{i} \Rightarrow \vec{E} = -\frac{kQ}{a(a+L)}\hat{i}$$

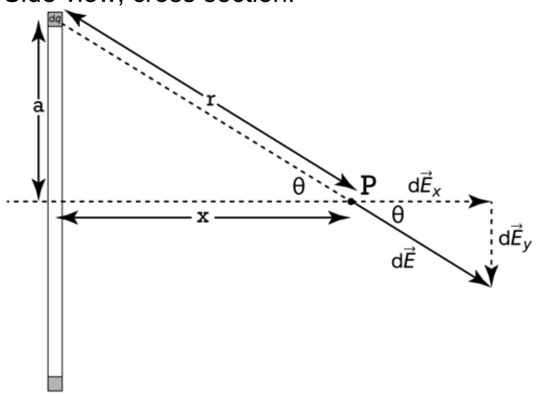
if $a >> L$ then $a + L \approx a$ & $\vec{E} = -\frac{kQ}{a^2}\hat{i}$

In other words, if we get far enough from the charged rod, it acts like a point charge. ☺

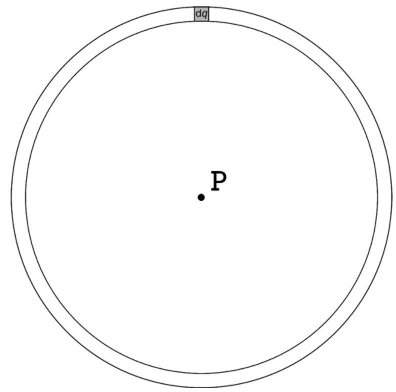
Flipping Physics Lecture Notes:
Thin Ring Electric Field
http://www.flippingphysics.com/thin-ring-electric-field.html

Let's determine the electric field caused by a uniformly charged ring of charge +Q, with radius a, at point P which is located on the axis of the ring a distance x from the center of the ring.

Side view, cross section:



Front view:



Knowns: a, x, Q

$$\vec{E} = k\int \frac{dq}{r^2}\hat{r} \Rightarrow d\vec{E} = k\left(\frac{dq}{r^2}\right)\hat{r} = k\left(\frac{dq}{a^2+x^2}\right)\hat{r}$$

However, all dE's in the vertical plane cancel out because there is an equal but opposite component of dE caused by the dq on the opposite side of the ring. In the figure that is dE_y.

$$d\vec{E}_x = d\vec{E}\cos\theta = k\left(\frac{dq}{a^2+x^2}\right)\hat{\imath}\cos\theta \Rightarrow \vec{E}_P = \int\left(\frac{k}{a^2+x^2}\hat{\imath}\cos\theta\right)dq$$

$$\&\quad \cos\theta = \frac{A}{H} = \frac{x}{r} = \frac{x}{\sqrt{a^2+x^2}} \Rightarrow \vec{E}_P = \int\left[\left(\frac{k}{a^2+x^2}\right)\left(\frac{x}{\sqrt{a^2+x^2}}\right)\hat{\imath}\right]dq$$

Note: $(a^2+x^2)(a^2+x^2)^{\frac{1}{2}} = (a^2+x^2)^{\frac{2}{2}}(a^2+x^2)^{\frac{1}{2}} = (a^2+x^2)^{\frac{3}{2}}$

$$\Rightarrow \vec{E}_P = \int\left(\frac{kx}{(a^2+x^2)^{\frac{3}{2}}}\right)\hat{\imath}\,dq = \left(\frac{kx}{(a^2+x^2)^{\frac{3}{2}}}\right)\hat{\imath}\int dq \Rightarrow \vec{E}_P = \left(\frac{kQx}{(a^2+x^2)^{\frac{3}{2}}}\right)\hat{\imath}$$

Note:

$$\text{if } x >> a \text{ then } a^2+x^2 \approx x^2 \;\&\; \vec{E}_P \approx \left(\frac{kQx}{(x^2)^{\frac{3}{2}}}\right)\hat{\imath} = \left(\frac{kQx}{x^3}\right)\hat{\imath} \qquad \Rightarrow \vec{E}_P \approx \left(\frac{kQ}{x^2}\right)\hat{\imath}$$

That's right, if you get far enough from the uniformly charged ring, it acts like a point particle!
If that sounds familiar, that is because this is true for all continuous charge distributions. If you get far enough away from them, that their own size is small by comparison to the distance, they all have electric fields which are similar to point particles.

Note:

$$\text{if } x << a \text{ then } a^2+x^2 \approx a^2 \;\&\; \vec{E}_P \approx \left(\frac{kQx}{(a^2)^{\frac{3}{2}}}\right)\hat{\imath} \Rightarrow \vec{E}_P \approx \left(\frac{kQx}{a^3}\right)\hat{\imath}$$

And if we use a negative charge, then the force is to the left or towards the center of the ring:

$$\sum F_x = -F_e = ma_x \Rightarrow -qE = -q\left(\frac{kQx}{a^3}\right) = ma_x \Rightarrow a_x = -\left(\frac{kqQ}{ma^3}\right)x$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\left(\frac{kqQ}{ma^3}\right)x \ \ \& \ \ \frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow \omega = \sqrt{\frac{kqQ}{ma^3}} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{ma^3}{kqQ}}$$

$$x(t) = A\cos(\omega t + \phi) \Rightarrow x_{max} = A$$

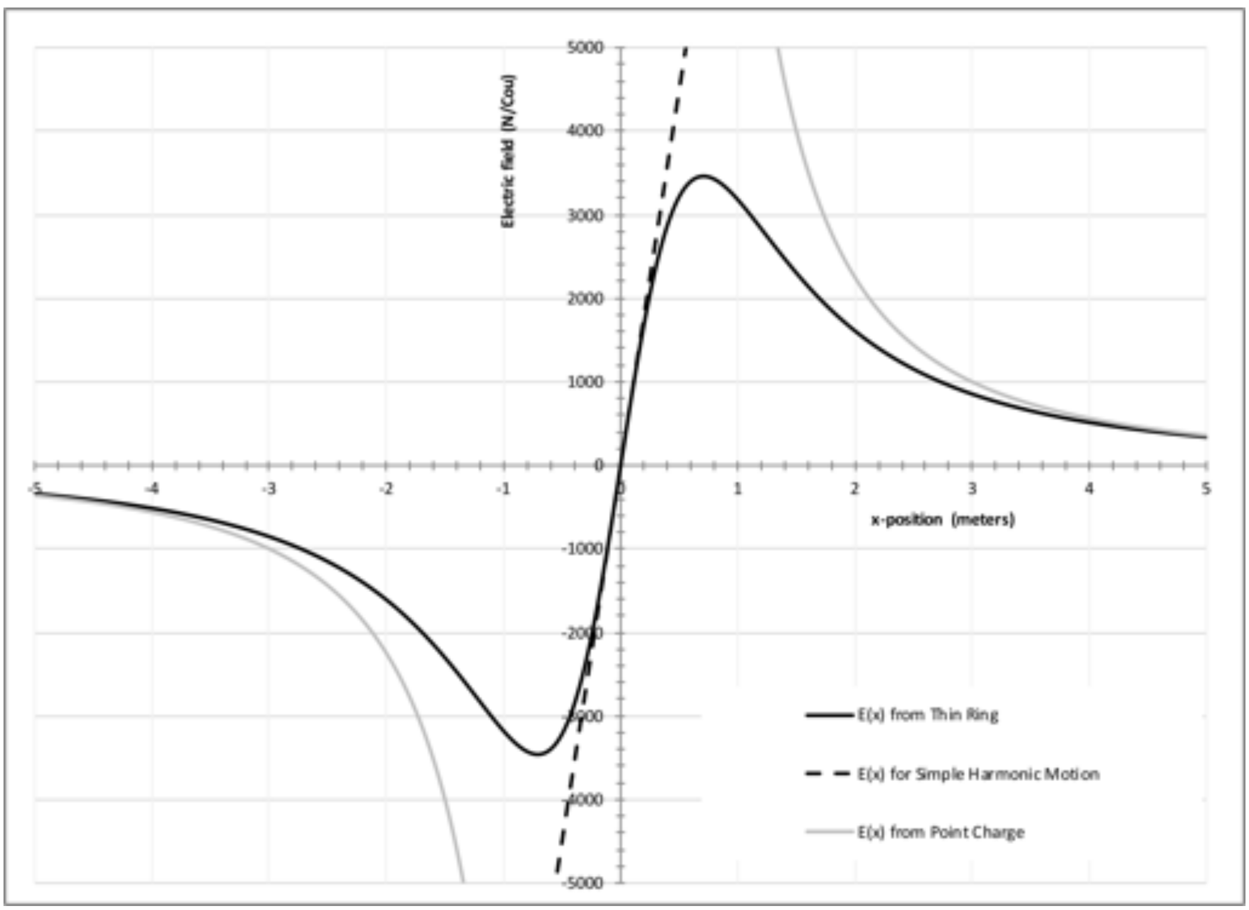
$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi) \Rightarrow v_{max} = A\omega$$

$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi) \Rightarrow a_{max} = A\omega^2$$

That's right, the negative charge will move in simple harmonic motion about the center of the ring.

A bonus graph from Carl Hansen: (Thank you Carl Hansen!)

This following graph shows the equation we derived for the electric field along the axis of a thin ring of a uniform charge distribution, of radius a = 1m, and charge Q = +1μC. The plot shows the linear profile needed for simple harmonic motion close to the origin, and the inverse square law far away. The maximum electric field occurs around x = 0.7m, as a turning point to transition between the two trends.

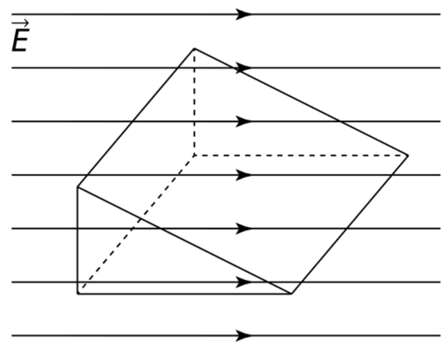


Flipping Physics Lecture Notes:
Electric Flux
http://www.flippingphysics.com/electric-flux.html

Flux is defined as any effect that appears to pass or travel through a surface or substance, however, realize that effect does not need to move. Hence, "appears to".

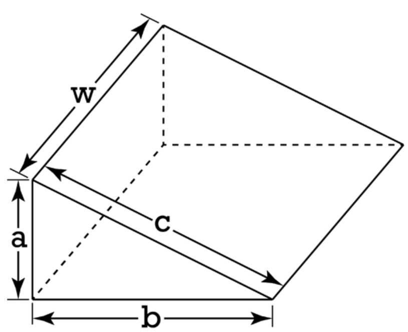
Electric flux is the measure of the amount of electric field which passes through a defined area. The equation for electric flux of a uniform electric field is:

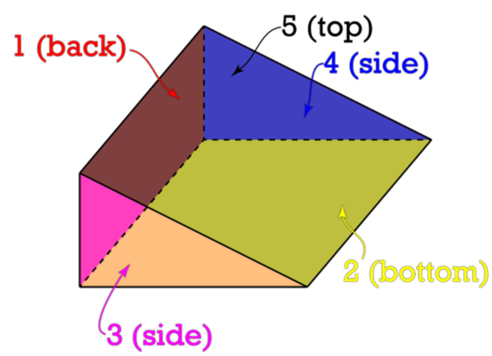
- $\Phi_E = \vec{E} \cdot \vec{A} = EAcos\theta$
- Φ is the uppercase, Greek letter phi
- E is the uniform electric field *(use magnitude)*
- A is the area of the surface through which the uniform electric field is passing *(use magnitude)*
- θ is the angle between the directions of E and A
    - Notice this is the same form as the equation for work. This means you use the magnitudes of E and A, and cosθ determines if the electric flux is positive or negative
    - $W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$
- Electric flux is a scalar
- The units for electric flux are $\frac{N \cdot m^2}{C}$



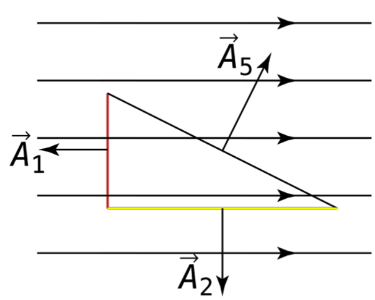
Usually, electric flux is through some sort of closed surface. So, let's do an example and determine the net electric flux of a uniform, horizontal electric field through a right triangular box.

Let's define and label the dimensions and sides of the triangular box as:





And now we can determine the electric flux through each side:



Electric flux for Area 1 (back): θ_1 is 180° because Area 1 is to the left or out of the rectangular box and the electric field is to the right.

$$\Phi_1 = EA_1cos\theta_1 = E(aw)\cos(180°) = -Eaw$$

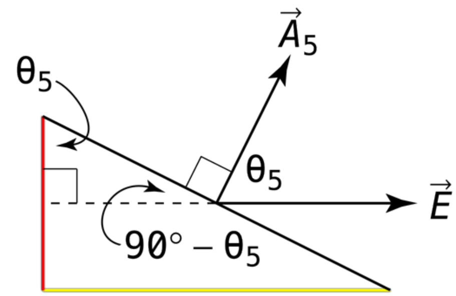
Electric flux for Area 2 (bottom): θ_2 is 90° because Area 2 is down or out of the rectangular box and the electric field is to the right.

$$\Phi_2 = EA_2cos\theta_2 = E(bw)\cos(90°) = 0$$

Electric flux for Areas 3 and 4 (sides): θ_3 and θ_4 are both 90° because Area 3 is out of the page and Area 4 is into the page (and the electric field is to the right).

$$\Phi_3 = EA_3 cos\theta_3 = E\left(\frac{1}{2}ba\right)\cos(90°) = 0 = \Phi_4$$

Electric flux for Area 5 (top): To understand why $\cos\theta_5$ = a/c, we need to draw another diagram.



$$\cos\theta_5 = \frac{A}{H} = \frac{a}{c}$$

$$\Phi_5 = EA_5 cos\theta_5 = E(cw)\left(\frac{a}{c}\right) = Eaw$$

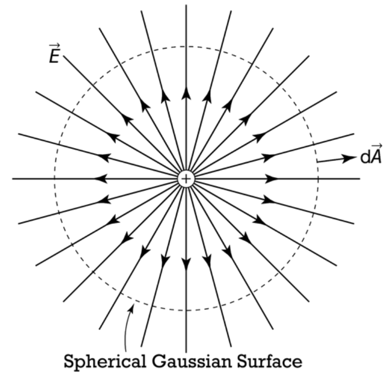
And the total electric flux through the entire triangular box is:

$$\Phi_{total} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 = -Eaw + 0 + 0 + 0 + Eaw = 0$$

Notice that:

- When an electric field is going into a closed surface, the electric flux is negative.
- When an electric field is coming out of a closed surface, the electric flux is positive.

Flipping Physics Lecture Notes:
Gauss's Law - Point Charge Electric Flux
http://www.flippingphysics.com/gauss-law.html



Let’s determine the electric flux passing through a sphere which is concentric to and surrounds a positive point charge.

Notice we cannot use $\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$ because the electric field is not uniform. We need to use the integral equation for electric flux:

$$\Phi_E = \vec{E} \cdot \vec{A} \Rightarrow d\Phi_E = \vec{E} \cdot d\vec{A} \Rightarrow \int d\Phi_E = \int \vec{E} \cdot d\vec{A} \Rightarrow \Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E dA \cos\theta = \int E dA \cos(0) = \int E dA = E \int dA = EA$$

$$\vec{F}_{21} = k\frac{(q_1)(q_2)}{r^2}\hat{r}_{21} \;\&\; \vec{E} = \frac{\vec{F}_e}{q} \Rightarrow E_{+\text{ point charge}} = \frac{\frac{kqQ}{r^2}}{q} = \frac{kQ}{r^2} \;\&\; A_{\text{sphere}} = 4\pi r^2$$

$$\Rightarrow \Phi_E = \left(\frac{kQ}{r^2}\right)\left(4\pi r^2\right) = 4\pi kQ = 4\pi\left(\frac{1}{4\pi\varepsilon_0}\right)Q = \frac{Q}{\varepsilon_0}$$

In other words, the electric flux through a closed Gaussian surface is:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

This is Gauss’ law!

Gauss’ law relates electric flux through a Gaussian surface to the charge enclosed by the Gaussian surface:

- A Gaussian surface is a three-dimensional closed surface
- While the Gaussian surfaces we usually work with are imaginary, the Gaussian surface could actually be a real, physical surface
- Typically, we choose the shapes of our Gaussian surfaces such that the electric field generated by the enclosed charge is either perpendicular or parallel to the various sides of the Gaussian surface. This greatly simplifies the surface integral because all the angles are multiples of 90° and the cosine of those angles have a value of -1, 0, or 1.
- As long as the amount of charge enclosed in a Gaussian surface is constant, the total electric flux through the Gaussian surface does not depend on the size of the Gaussian surface.
- Gauss’ law is the first of Maxwell’s equations which are a collection of equations which fully describe electromagnetism.

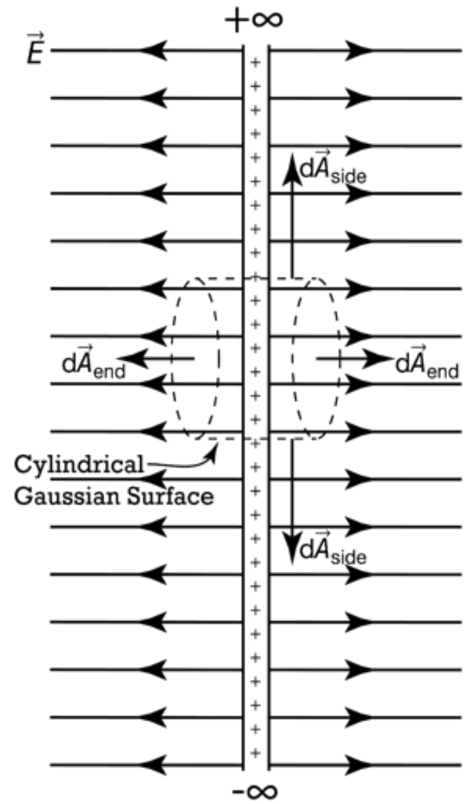
Notice then that, if the net charge inside a closed Gaussian surface is zero, then the net electric flux through the Gaussian surface is zero.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

This is why the net electric flux through the closed rectangular box in the example in our previous lesson was zero.

Flipping Physics Lecture Notes:
Gauss's Law - Charged Plane Electric Field
http://www.flippingphysics.com/gauss-law-plane.html

Determine the electric field which surrounds an infinitely large, thin plane of positive charges with uniform surface charge density, σ:



First off, we know the electric field will be directed normal to and away from the infinite plane of positive charges. This is because the plane is infinitely large; therefore, every component of the electric field, dE, which is parallel to the plane of charges and is caused by infinitesimally small, charged pieces of the plane, dq, will cancel out leaving only electric field components of dE which are perpendicular to the plane and directed away from the plane.

We pick a Gaussian surface such that it is a cylinder with ends parallel to the plane of charges and a side parallel to the electric field and use Gauss' law. The two ends of the Gaussian cylinder are equidistant from the charged plane.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$\Phi_E = \int_{\text{side}} \vec{E} \cdot d\vec{A}_{\text{side}} + \int_{\substack{\text{left}\\\text{end}}} \vec{E} \cdot d\vec{A}_{\text{end}} + \int_{\substack{\text{right}\\\text{end}}} \vec{E} \cdot d\vec{A}_{\text{end}} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = \int_{\text{side}} E dA \cos\theta_{\text{side}} + \int_{\substack{\text{left}\\\text{end}}} E dA \cos\theta_{\text{end}} + \int_{\substack{\text{right}\\\text{end}}} E dA \cos\theta_{\text{end}} = \frac{q_{in}}{\varepsilon_0}$$

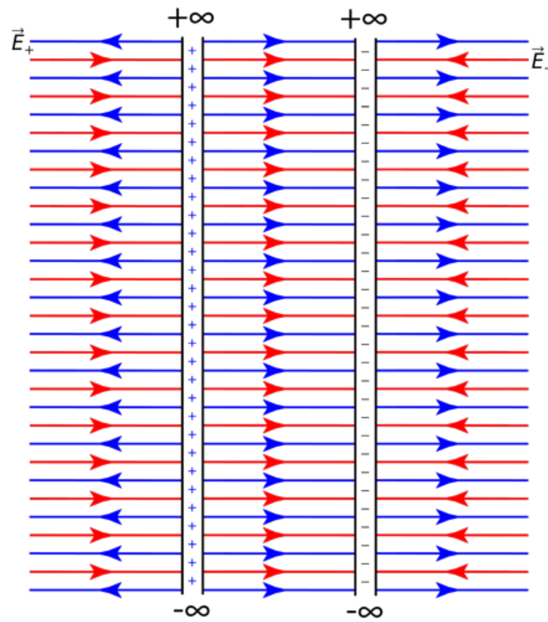
$$\Rightarrow \Phi_E = \int_{\text{side}} E dA \cos(90°) + \int_{\substack{\text{left}\\\text{end}}} E dA \cos(0°) + \int_{\substack{\text{right}\\\text{end}}} E dA \cos(0°) = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = E \int_{\substack{\text{left}\\\text{end}}} dA + E \int_{\substack{\text{right}\\\text{end}}} dA = E(2A_{\text{end}}) = \frac{q_{in}}{\varepsilon_0}$$

$$\& \ \sigma = \frac{Q}{A} = \frac{q_{in}}{A_{\text{end}}} \Rightarrow q_{in} = \sigma A_{\text{end}} \Rightarrow E(2A_{\text{end}}) = \frac{\sigma A_{\text{end}}}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

Notice this electric field is uniform and is independent of the distance from the infinite plane of charges.

And notice what happens if we have two infinite parallel planes of charges, one with positive charge and one with negative charge:



The electric field outside the planes of charges cancels out to give zero electric field outside the planes of charges:

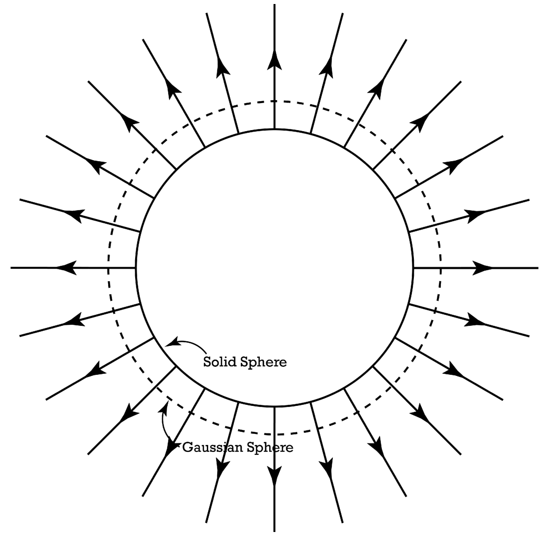
$$E_{\text{outside}} = 0$$

And between the two planes of charges, the electric fields add together:

$$E_{\text{between}} = 2E_{\text{one plate}} = 2\left(\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

And we have begun our journey towards determining the capacitance of a parallel plate capacitor…

Notice that a positively charged particle moving through a constant electric field will experience an electrostatic force in the direction of the electric field. This force will be constant and equal to qE. In other words, the motion of a charged particle through a constant electric field will have similar characteristics to a mass moving through the constant gravitational field near the surface of a planet. This is very similar to projectile motion.



Outside the surface of a uniformly charged sphere, the electric field is the same as if the charged sphere were a point particle.

- Example: Solid, uniformly charged sphere with charge Q and radius, a.
- Create a Gaussian surface which is a concentric sphere with radius r > a.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = \int_{sphere} E\cos\theta\, dA = \int_{sphere} E\cos(0°)\, dA = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E\int_{sphere} dA = EA_{sphere} = E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{r^2} \Rightarrow E = \frac{kQ}{r^2}$$

- This is true of a conductor or an insulator, however, the electric field inside a conductor will be zero, and inside an insulator the electric field depends on the radius and charge distribution, and can be derived in a similar manner.

Flipping Physics Lecture Notes:
Electric Potential
http://www.flippingphysics.com/apcem-electric-potential.html

The electrostatic force is a conservative force, therefore:

$$F_x = -\frac{dU}{dx} \Rightarrow F_e = -\frac{dU_e}{dr} \Rightarrow dU_e = -\vec{F}_e \cdot d\vec{r} \Rightarrow \int dU_e = -\int \vec{F}_e \cdot d\vec{r}$$

$$\Rightarrow \Delta U_e = -\int \vec{F}_e \cdot d\vec{r} \;\;\&\;\; \vec{E} = \frac{\vec{F}_e}{q} \Rightarrow \vec{F}_e = q\vec{E} \Rightarrow \Delta U_e = -\int_A^B q\vec{E} \cdot d\vec{r}$$

$$\Rightarrow \Delta U_e = -q\int_A^B \vec{E} \cdot d\vec{r} = -q\int_A^B E\cos\theta\, dr$$

We have determined the change in electric potential energy experienced by a charged particle which has moved from point A to point B in an electric field. Notice, because the electrostatic force is a conservative force, this change in electric potential energy does not depend on the path taken from point A to point B.

To make a comparison to gravitational potential energy, if we lift an object vertically upward in a constant downward gravitational field, it will experience a positive change in gravitational potential energy.

The same is true for a positively charged object, if we move a positively charged object vertically upward in a constant downward electric field, it will experience a positive change in electric potential energy.

Notice the negative and the dot product in the equation. As we move a charge in a direction opposite the direction of the field, the direction of the displacement of the charge and the direction of the field are opposite to one another, therefore, the angle between those two directions is 180°, the cosine of 180° is negative one, which makes the change in potential energy of a positive charge positive.

$$\Rightarrow \Delta U_e = -q\int_A^B E\cos(180°)\, dr = -q\int_A^B E(-1)\, dr = q\int_A^B E\, dr$$

Next, we need to define **electric potential**. Just like we define the electric field in terms of the force experienced by a small, positive test charge, we define the electric potential in terms of the energy experienced by a small, positive test charge.

- $\vec{E} = \frac{\vec{F}_e}{q} \Rightarrow V = \frac{U_e}{q}$ in volts, $V = \frac{J}{C}$
- The symbol for electric potential is V. Yes, I know. The symbol for electric potential, V, is the same as the symbol for the units for electric potential, volts, V. It's not my fault.
- Electric potential is a **scalar** attribute of a **vector** electric field which does not depend on any electric charges which could be placed in that field.
  - The fact that electric potential is a scalar can be very helpful in this class.
  - This scalar can be either positive or negative for any given location.
  - Just like gravitational potential energy, we need to either assign a location where it equals zero, or follow a convention for assigning the zero potential location.
- Most often we work with **electric potential difference** not just electric potential. Electric potential difference is the difference in the electric potential between two points:[1]

[1] I know. I know. … Duh! … But it had to be said.

$$\Delta V = V_f - V_i = V_B - V_A \ \ \& \ \ V = \frac{U_e}{q}$$

$$\Rightarrow \Delta V = \frac{\Delta U_e}{q} = \frac{-q\int_A^B \vec{E}\cdot d\vec{r}}{q} = -\int_A^B \vec{E}\cdot d\vec{r}$$

- We will often set the initial electric potential, or electric potential at point A, equal to zero.
- Realize we can rearrange every integral to form a derivative:

$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{r} \Rightarrow dV = -\vec{E}\cdot d\vec{r} \Rightarrow E_r = -\frac{dV}{dr}$$

Now that we have volts, the units for electric potential, it is important to realize the units for the electric field can be given in terms of volts as well.

$$\vec{E} = \frac{\vec{F}_e}{q} \text{ in } \frac{N}{C} = \left(\frac{N}{C}\right)\left(\frac{m}{m}\right) = \left(\frac{J}{C}\right)\left(\frac{1}{m}\right) = \frac{V}{m} \Rightarrow \frac{N}{C} = \frac{V}{m}$$

Flipping Physics Lecture Notes:
Work to Move a Charge in an Electric Field in Electron Volts
http://www.flippingphysics.com/work-charge-electron-volt.html

If a charge is moved from point A to point B via an external force, the external force does work on the charge, that changes the electric potential energy of the charge. And, as long as there is no change in the kinetic energy of the charge, that work equals the charge of the charge multiplied by the electric potential difference the charge went through: $W = q\Delta V$

A unit of energy often used for very small amounts of energies, like one would use in atomic and nuclear physics, is the electron volt (eV). An electron volt is defined as the energy a charge-field system gains or losses when a charge of magnitude e (the elementary charge or the magnitude of the charge on an electron or proton) is moved through a potential difference of 1 V:

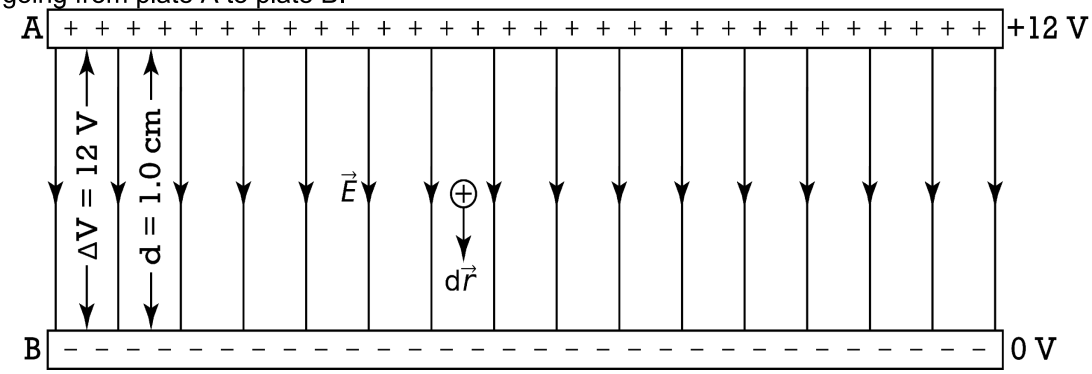
$$W = q\Delta V \Rightarrow W_{\mathrm{eV}} = \left(1.6\times10^{-19}C\right)(1V) = 1.6\times10^{-19}C\cdot V \;\;\&\;\; C\cdot V = C\cdot\frac{J}{C} = J$$

$$\Rightarrow 1eV = 1.6\times10^{-19}J$$

I consider the electron volt to be a misnomer because it sounds like a unit of electric potential (volts), however, it is a unit of energy. It also refers to a positive amount of energy, even though the electron is negative. Be careful of that.

Flipping Physics Lecture Notes:
Parallel Plates Electric Potential Difference using an Integral
http://www.flippingphysics.com/parallel-plates-potential-difference.html

Let's say we have two, large, equal magnitude charged parallel plates, the top plate has a positive charge, and the bottom plate has a negative charge. We have shown the electric field is constant in this case and will be directed downward. Let's say the electric potential difference between the two plates is 12 volts and the distance between the two plates is 1.0 cm. Let's define the top plate as plate A, and the bottom plate as plate B. Let's start by determining the general equation for the electric potential difference when going from plate A to plate B.



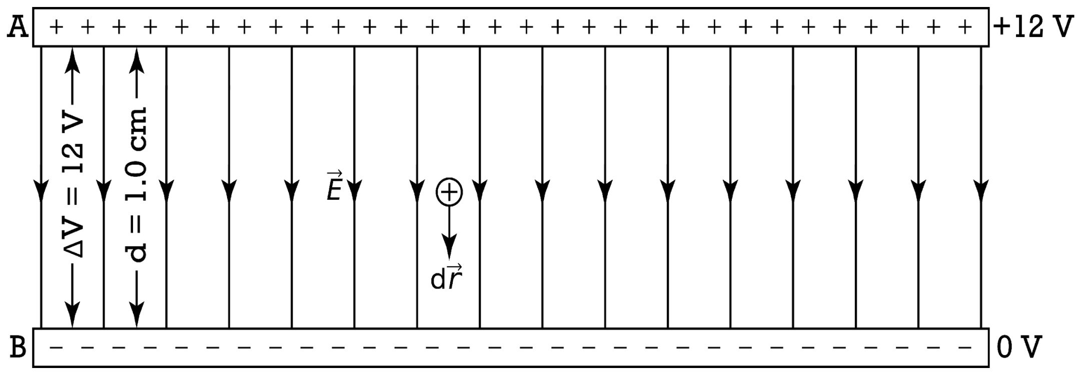
$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{r} = -\int_A^B E\cos\theta\, dr$$

$$\Rightarrow \Delta V = -\int_A^B E\cos(0°)\, dr = -E\int_A^B dr \Rightarrow \Delta V_{\text{constant E}} = -Ed$$

This is a good time to discuss the negative sign in the electric potential equation. In other words, a charge moving in the direction of the electric field will go through a negative potential difference and a charge moving opposite the direction of the electric field will go through a positive electric potential difference.

Flipping Physics Lecture Notes:
Speed of a Proton in a Uniform Electric Field
http://www.flippingphysics.com/proton-speed-electric-field.html

Let's say we have two, large, equal magnitude charged parallel plates, the top plate has a positive charge, and the bottom plate has a negative charge. We have shown the electric field is constant in this case and will be directed downward. Let's say the electric potential difference between the two plates is 12 volts and the distance between the two plates is 1.0 cm. Let's define the top plate as plate A, and the bottom plate as plate B. We have already determined that the electric potential difference between the two plates: $\Delta V_{\text{constant E}} = -Ed$



If we release a proton from the inside surface of plate A, what will the speed of the proton be right before it runs into plate B?

Set initial point at A and final point at B. Do not need a horizontal zero line because gravitational potential energy for subatomic particles is usually negligible and it is in this case. We do not actually know the electric potential energy initial or electric potential energy final; however, we do know the change in the electric potential energy. Also, the charge and mass of a proton are given in the Table of Information provided on the AP Physics C exam.

$$ME_i = ME_f \Rightarrow ME_A = ME_B \Rightarrow U_{elec_A} = KE_B + U_{elec_B}$$

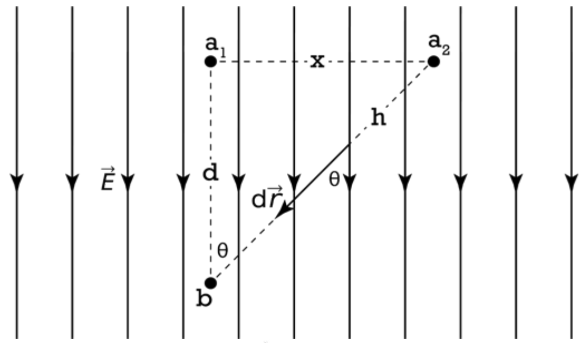
$$\Rightarrow -KE_B = U_{elec_B} - U_{elec_A} = \Delta U_{elec} \Rightarrow -\frac{1}{2}mv_B^2 = q\Delta V$$

$$\& \ \Delta V = \frac{\Delta U_{elec}}{q} \Rightarrow \Delta U_{elec} = q\Delta V \ \& \ \Delta V_{A\to B} = -12V$$

$$\Rightarrow v_B = \sqrt{-\frac{2q\Delta V}{m}} = \sqrt{-\frac{(2)\left(1.6\times10^{-19}\right)(-12)}{1.67\times10^{-27}}} = 47952\frac{m}{s} \approx 48\frac{km}{s}$$

$$v_B = 47952\frac{m}{s}\left(\frac{3600\ s}{1\ \text{hr}}\right)\left(\frac{1\ \text{mi}}{1609\ m}\right) = 107288\frac{\text{mi}}{\text{hr}} \approx 1.1\times10^5\frac{\text{mi}}{\text{hr}}$$

Flipping Physics Lecture Notes:
Equipotential Surfaces
http://www.flippingphysics.com/equipotential.html



Let's look at determining the electric potential difference when moving at an angle relative to a uniform electric field. We already know the electric potential difference when moving from point a_1 to b:

$$\Delta V_{a_1 \to b} = -Ed$$

Let's determine the electric potential difference when moving from point a_2 to b:

$$\Delta V_{a_2 \to b} = -\int_{a_2}^{b} E \cdot dr = -\int_{a_2}^{b} E \cos\theta \, dr = -\int_{a_2}^{b} E\left(\frac{d}{h}\right) dr = -\left(\frac{Ed}{h}\right)\int_{a_2}^{b} dr$$

$$\Rightarrow \Delta V_{a_2 \to b} = -E\left(\frac{d}{h}\right)(h) = -Ed \Rightarrow \Delta V_{a_1 \to b} = \Delta V_{a_2 \to b}$$

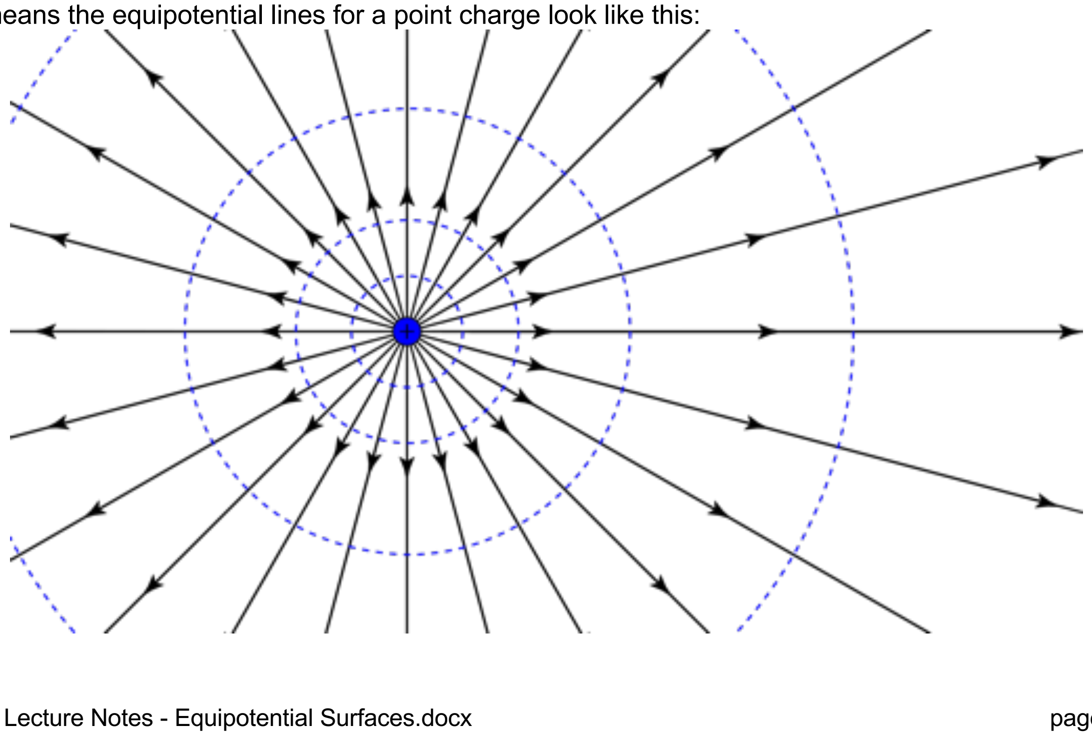
The electric potential difference is the same for both of these because points a_1 and a_2 have the same electric potential.

$$\Delta V_{a_2 \to a_1} = -\int_{a_2}^{a_1} E \cdot dr = -E\int_{a_2}^{a_1} \cos\theta \, dr = -E\int_{a_2}^{b} \cos(90°) \, dr = 0$$

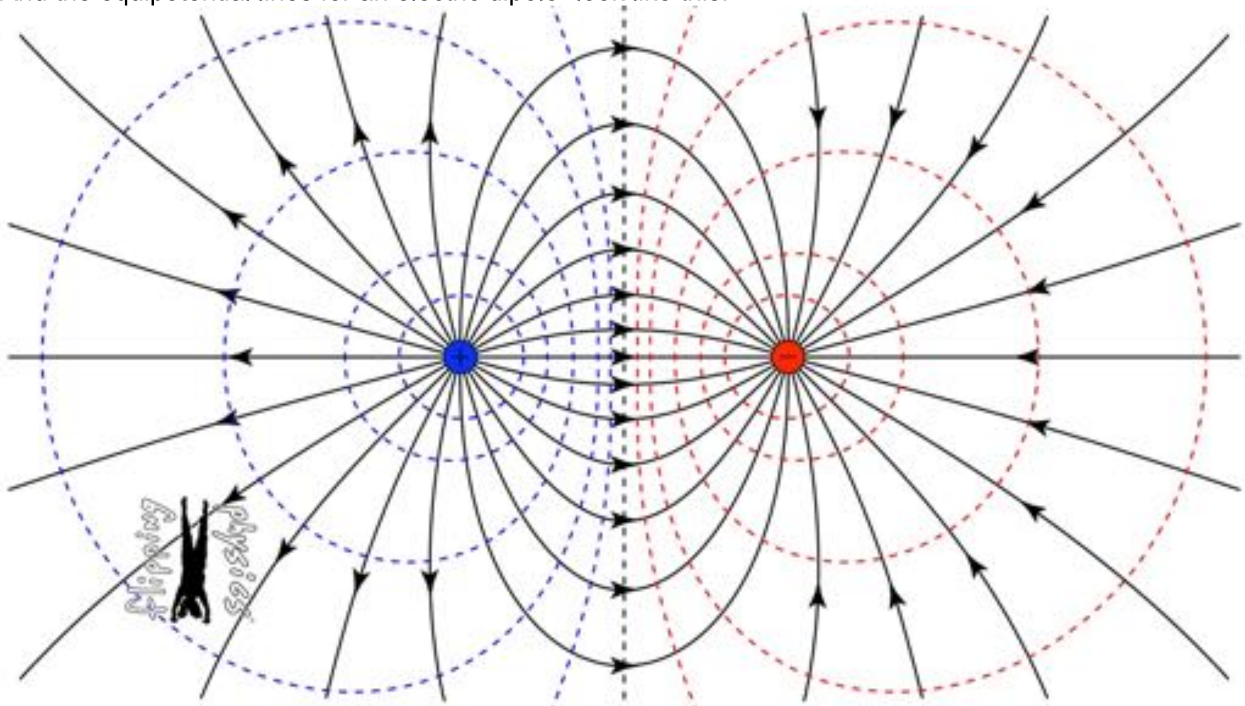
Points a_1 and a_2 are on an equipotential surface. An equipotential surface (or line):

- Has the same electric potential at every point on the surface (or line)
- Is always perpendicular to the electric field
  - Therefore, the electric field has no component along the equipotential line
- Equipotential lines are sometimes called isolines
- And it takes zero work to move a charged object along an equipotential surface
  - $W = q\Delta V \Rightarrow W_{\text{equipotential surface}} = q(0) = 0$
  -

This means the equipotential lines for a point charge look like this:



And the equipotential lines for an electric dipole[1] look like this:



The equation for the electric potential which surrounds and is caused by a point charge is:

$$V_{\text{point charge}} = \frac{kq}{r}$$

This equation assigns our location of zero electric potential to be infinitely far away.

We can use the relationship between electric potential and electric potential energy to determine the electric potential energy which surrounds and is caused by a point charge:

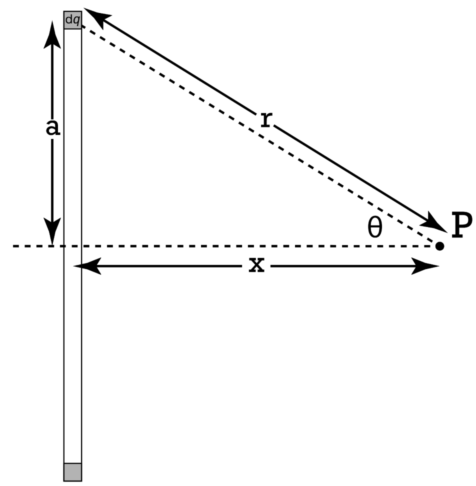
$$V = \frac{U_{\text{elec}}}{q} \Rightarrow U_{\text{elec}} = qV \Rightarrow U_{\text{2 point charges}} = q_1\left(\frac{kq_2}{r}\right)$$

$$\Rightarrow U_{\text{2 point charges}} = \frac{kq_1q_2}{r}$$

[1] This is a simple example of an electric dipole which is a pair of electric charges of equal magnitude, but opposite sign separated by some typically small distance.

Flipping Physics Lecture Notes:
Thin Ring Electric Field using Electric Potential
http://www.flippingphysics.com/thin-ring-electric-potential.html

Because electric potential and electric potential energy are scalar values, determining those values for multiple particles uses superposition. You just add all the values together.



In order to understand how useful it is that electric potential is a scalar and not a vector, let's revisit an example from before.[1] Let's determine the electric potential caused by a uniformly charged, thin ring of charge +Q, with radius a, at point P, which is located on the axis of the ring a distance x from the center of the ring.

Because this is a continuous charge distribution, we need to break the uniformly charged thin ring of charge +Q into an infinite number of infinitesimally small charges, dQ.

$$V_{\text{point charge}} = \frac{kq}{r} \Rightarrow V_{\text{continuous charge distribution}} = \int \left(\frac{k}{r}\right) dq = \frac{k}{r} \int dq = \frac{kQ}{r} \Rightarrow V_P = \frac{kQ}{\sqrt{a^2 + x^2}}$$

And from there we can determine the electric field at point P.

$$E_r = -\frac{dV}{dr} = -\frac{d}{dx}\left(\frac{kQ}{\sqrt{a^2 + x^2}}\right) = -kQ\frac{d}{dx}\left(a^2 + x^2\right)^{-\frac{1}{2}} = -kQ\left(-\frac{1}{2}\right)\left(a^2 + x^2\right)^{-\frac{3}{2}}(2x)$$

$$\Rightarrow E_P = \frac{kQx}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$$

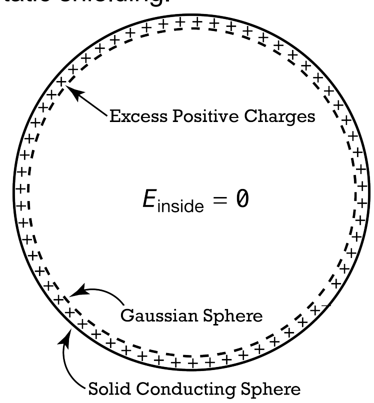
Notice that this derivation of the electric field at point P is much easier than deriving the electric field directly like we did before. Therefore, I would recommend that you remember that, for a continuous charge distribution, you can first determine the electric potential and then the electric field, and that is often easier than solving for the electric field directly.

[1] Thin Ring Electric Field - http://www.flippingphysics.com/thin-ring-electric-field.html

Flipping Physics Lecture Notes:
3 Properties of Conductors in Electrostatic Equilibrium
http://www.flippingphysics.com/electrostatic-equilibrium.html

Conductors are materials where the electrons are free to move rather easily, however, when they are in electrostatic[1] equilibrium, this means the charges are stationary in the object. There are four things you need to remember about conductors in electrostatic equilibrium. These are the first three:

1) The electric field inside a conductor in electrostatic equilibrium equals zero. $E_{\text{inside}} = 0$
   a. If the electric field inside were not equal to zero, charges would have a net electrostatic force acting on them and they would accelerate, therefore the conductor would not be in electrostatic equilibrium.
      i. $E_{\text{inside}} \neq 0 \Rightarrow F_e = qE \neq 0 \Rightarrow$ not in electrostatic equilibrium
   b. Notice that this means that anything inside a conductor in electrostatic equilibrium is shielded from all external electric fields. This is called electrostatic shielding.
2) All excess charges are located on the surface (or surfaces) of the conductor.
   a. Solid conducting sphere example:
      i. Draw a Gaussian surface as a concentric sphere with a radius slightly smaller than the radius of the sphere.
      ii. Using Gauss' law, because there is no electric field inside the conductor in electrostatic equilibrium, we know the left-hand side of the equation equals zero.
      iii. Therefore, there must be zero net charge inside the Gaussian sphere and all the excess charges must be outside the Gaussian sphere.
      iv. Therefore, all the excess charges are on the surface of the conductor.



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \ \& \ E_{\text{inside}} = 0 \Rightarrow 0 = \frac{q_{\text{enclosed}}}{\varepsilon_0} \Rightarrow q_{\text{enclosed}} = 0$$

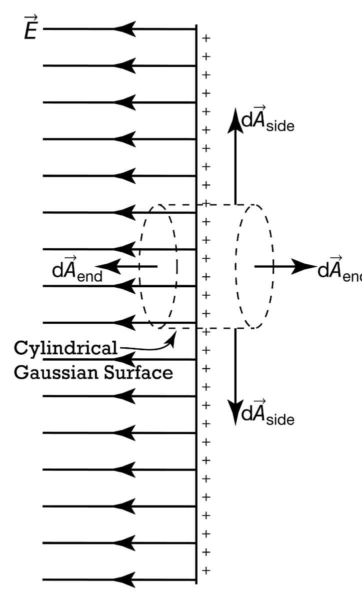
3) The electric field just outside the surface of a conductor in electrostatic equilibrium is:

$$E_{\substack{\text{just}\\\text{outside}}} = \frac{\sigma_{\text{local}}}{\varepsilon_0} \ \& \perp \text{ to surface}$$

   a. If the electric field had a component parallel to the surface of the conductor, the charges would move, and the conductor would no longer be in electrostatic equilibrium. Therefore, the electric field at the surface of a conductor in electrostatic equilibrium must be perpendicular to the surface.
      i. Because equipotential surfaces are always perpendicular to the electric field, the surface of a conductor in electrostatic equilibrium must be an equipotential surface.

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r} = -\int_A^B E\cos\theta \, dr = -\int_A^B E\cos(90°)\, dr = 0$$

   b. If we zoom way in on the surface of the conductor in electrostatic equilibrium, we can draw a Gaussian cylinder with its cylindrical axis normal to the surface of the conductor.



[1] Electrostatics is the study of electromagnetic phenomena that occur when there are no moving charges.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$\Rightarrow \Phi_E = \int_{\text{side}} E dA \cos\theta_{\text{side}} + \int_{\substack{\text{left}\\\text{end}}} E dA \cos\theta_{\text{end}} + \int_{\substack{\text{right}\\\text{end}}} E dA \cos\theta_{\text{end}} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$\Rightarrow \int_{\text{side}} E dA \cos(90°) + \int_{\substack{\text{left}\\\text{end}}} E dA \cos(0°) + \int_{\substack{\text{right}\\\text{end}}} (0)\, dA \cos\theta_{\text{end}} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

$$\& \ \sigma = \frac{Q}{A} \Rightarrow \sigma_{\text{local}} = \frac{q_{\text{in}}}{A_{\text{end}}} \Rightarrow q_{\text{in}} = \sigma_{\text{local}} A_{\text{end}}$$

$$\Rightarrow E \int_{\substack{\text{left}\\\text{end}}} dA = E A_{\text{end}} = \frac{\sigma_{\text{local}} A_{\text{end}}}{\varepsilon_0} \Rightarrow E = \frac{\sigma_{\text{local}}}{\varepsilon_0}$$

The fourth thing you need to remember about conductors in electrostatic equilibrium is in my video:

- "Irregularly Shaped Conductors in Electrostatic Equilibrium"
- http://www.flippingphysics.com/electrostatic-equilibrium-irregular-shape.html

Flipping Physics Lecture Notes:
Irregularly Shaped Conductors in Electrostatic Equilibrium
http://www.flippingphysics.com/electrostatic-equilibrium-irregular-shape.html

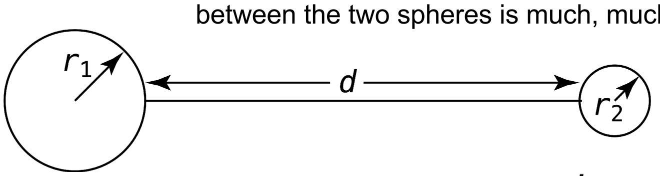
Conductors are materials where the electrons are free to move rather easily, however, when they are in electrostatic[1] equilibrium, this means the charges are stationary in the object. There are four things you need to remember about conductors in electrostatic equilibrium. The first three are described in my video:

- "3 Properties of Conductors in Electrostatic Equilibrium"
- http://www.flippingphysics.com/electrostatic-equilibrium.html

The fourth is:

For an irregular shape, the local surface charge density is at its maximum where the radius of curvature is at its minimum. In other words, the largest number of excess charges per area will be where the radius of curvature is the smallest. $\sigma_{local} = \text{maximum} \ @ \ r_{curvature} = \text{minimum}$

a. To prove this, we have two conducting spheres connected by a long conducting wire with the whole system in electrostatic equilibrium.
    i. This system is a conductor in electrostatic equilibrium. In other words, when two conductors are brought into contact with one another, the charges redistribute such that both conductors are at the same electric potential. Please realize this happens so quickly that the time for this to occur is considered to be negligible.
b. The radius of sphere 2 is smaller than the radius of sphere 1, and the distance, d, between the two spheres is much, much larger than either radius.



$$r_2 < r_1 \ \& \ d >> r_1 \ \& \ V_1 = V_2 \Rightarrow \frac{kq_1}{r_1} = \frac{kq_2}{r_2} \Rightarrow \frac{q_1}{r_1} = \frac{q_2}{r_2}$$

$$\Rightarrow q_1 = \left(\frac{r_1}{r_2}\right) q_2 \ \& \ \frac{r_1}{r_2} > 1 \Rightarrow q_1 > q_2$$

$$E_1 = \frac{kq_1}{(r_1)^2} \ \& \ E_2 = \frac{kq_2}{(r_2)^2} \Rightarrow \frac{E_1}{E_2} = \frac{\frac{kq_1}{(r_1)^2}}{\frac{kq_2}{(r_2)^2}} = \left(\frac{kq_1}{(r_1)^2}\right)\left(\frac{(r_2)^2}{kq_2}\right) = \frac{q_1 (r_2)^2}{q_2 (r_1)^2}$$

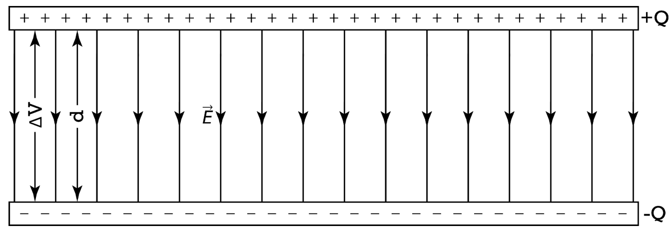
$$\Rightarrow \frac{E_1}{E_2} = \frac{\left(\left(\frac{r_1}{r_2}\right) q_2\right)(r_2)^2}{q_2 (r_1)^2} = \frac{r_2}{r_1} \Rightarrow E_2 = \left(\frac{r_1}{r_2}\right) E_1 \ \& \ \frac{r_1}{r_2} > 1$$

$$\Rightarrow E_2 > E_1 \ \& \ E = \frac{\sigma_{local}}{\varepsilon_0} \Rightarrow \sigma_2 > \sigma_1 \Rightarrow \text{if } r_2 < r_1 \text{ then } \sigma_2 > \sigma_1$$

[1] Electrostatics is the study of electromagnetic phenomena that occur when there are no moving charges.

Flipping Physics Lecture Notes:
Capacitance
http://www.flippingphysics.com/capacitance.html

A capacitor is a way to store electric potential energy in an electric field. The simplest form of a capacitor is a parallel plate capacitor.

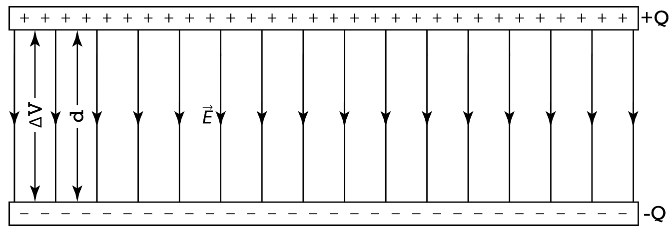


Capacitance, C, is defined as the magnitude of the charge stored on one plate divided by the electric potential difference between the two plates: $C \equiv \frac{Q}{\Delta V}$

- Capacitance is always positive.
    - Q, is the charge on the positive plate.
    - ΔV is the positive electric potential difference between the two plates.
- The net charge on a capacitor is zero.
    - $Q_{\text{total}} = +Q + (-Q) = 0$
- $C \equiv \frac{Q}{\Delta V} \Rightarrow \text{Capacitance in } \frac{\text{coulombs}}{\text{volts}} = F\text{, farads}$
    - $\text{charge, } Q \Rightarrow \text{coulombs, } C \;\;\&\;\; \text{capacitance, } C \Rightarrow \text{farads, } F$
    - It is not my fault the symbol for capacitance is C and capacitance is charge per electric potential difference and the units for charge are coulombs for which the symbol is C.
- The three-line equal sign, $\equiv$, means "is defined as". This is not a derivation. We made it up. We have simply decided to define the charge on a capacitor divided by the electric potential difference of the capacitor as "capacitance".
- Energy is stored in the electric field of the capacitor.
- The capacitance of a capacitor depends only on the capacitor's physical characteristics. For example, the capacitor's shape and material used to separate the plates of the capacitor.

The basic idea is:
- Start with an uncharged capacitor.
    - No charge on either plate.
    - No electric field between the plates.
- Attach the terminals of a battery to the two plates of the capacitor.
- Charges flow from one plate to the other plate of the capacitor.
- We now have a charged capacitor.
    - Both plates have equal magnitude charge.
    - There is an electric field and an electric potential difference between the plates.
    - Energy is stored in the electric field of the capacitor.

A capacitor is a way to store electric potential energy in an electric field. The simplest form of a capacitor is a parallel plate capacitor.

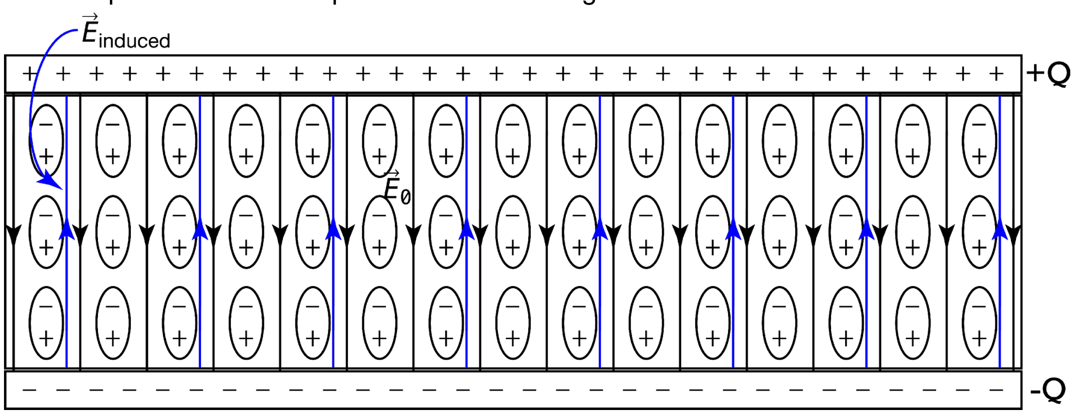


Let's derive the equation for the capacitance of a parallel plate capacitor. We have already derived two equations for two parallel, infinitely large, charged plates with equal magnitude, but opposite sign.

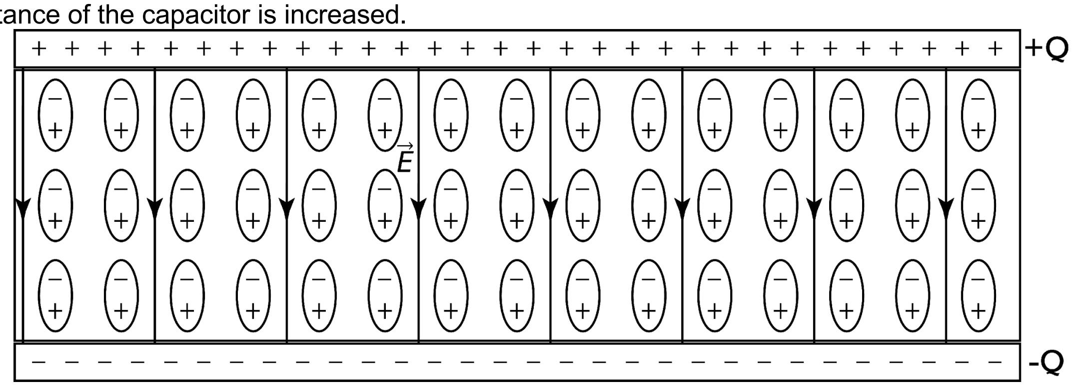
$$E_{\parallel \text{ plates}} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;\&\; \Delta V_{\text{constant E}} = -Ed \Rightarrow \|\Delta V\| = Ed = \left(\frac{Q}{A\varepsilon_0}\right)d$$

$$\&\; C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} \Rightarrow C_{\parallel \text{ plate}} = \frac{\varepsilon_0 A}{d}$$

This assumes there is a vacuum between the two plates. Usually, we place an insulating material between the plates of a capacitor. This is both to help physically separate the two plates and because it increases the capacitance of the capacitor. This insulating material is called a dielectric.



The charged particles in the dielectric are polarized and induce their own electric field (above in blue) which is opposite the direction of the original electric field of the capacitor E_0 (above in black). The net electric field (below in black) is decreased. Because the electric field is decreased, the electric potential difference across the capacitor is decreased, the charge of the capacitor remains the same, and the capacitance of the capacitor is increased.



$$E = E_0 - E_{\text{induced}} \Rightarrow E \downarrow \;\&\; \|\Delta V\| = Ed$$
$$\Rightarrow \Delta V \downarrow \;\&\; Q \text{ is constant } \;\&\; C = \frac{Q}{\Delta V} \Rightarrow C \uparrow$$

The way we define the effect of a dielectric is with the dielectric constant. The symbol for the dielectric constant is the lowercase Greek letter kappa, κ. It looks basically like a lowercase k. The dielectric constant equals the ratio of the electric permittivity of the dielectric to the electric permittivity of free space.

- $$\kappa = \frac{\varepsilon}{\varepsilon_0}$$
- The dielectric constant has no units.
- Electric permittivity is the measurement of how much a material is polarized when it is placed in an electric field.
    - The easier it is for electrons to change configurations in a material, the larger the dielectric constant of that material.
- The dielectric constant is also sometimes called *relative permittivity*.

We can also determine the relationship between the electric field between the parallel plates of the capacitor with a vacuum and with a dielectric.

$$E_{\text{vacuum}} = \frac{\sigma}{\varepsilon_0} \;\&\; E_{\text{dielectric}} = \frac{\sigma}{\varepsilon} \Rightarrow \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} = \frac{\frac{\sigma}{\varepsilon_0}}{\frac{\sigma}{\varepsilon}} = \frac{\varepsilon}{\varepsilon_0} = \kappa \Rightarrow \kappa = \frac{E_{\text{vacuum}}}{E_{\text{dielectric}}} \Rightarrow \kappa = \frac{E_0}{E}$$

And then use that to determine the relationship between the capacitance of the capacitor with a vacuum and the capacitance of the capacitor with a dielectric.

$$C_{\parallel \text{ plate}} = C_0 = \frac{\varepsilon_0 A}{d} \;\&\; \kappa = \frac{\varepsilon}{\varepsilon_0} \Rightarrow \varepsilon = \kappa\varepsilon_0$$
$$\&\; C_{\text{dielectric}} = C = \frac{\varepsilon A}{d} = \frac{\kappa\varepsilon_0 A}{d} \Rightarrow C_{\text{dielectric}} = \frac{\kappa\varepsilon_0 A}{d} \;\&\; C = \kappa C_0$$

According to the College Board, students are responsible for determining the capacitance only of the following shapes: parallel-plate capacitors, spherical capacitors, and cylindrical capacitors.

Flipping Physics Lecture Notes:
Energy Stored in a Capacitor
http://www.flippingphysics.com/capacitor-energy.html

To derive the equation for the energy stored in a capacitor, start with an uncharged capacitor, move one, infinitesimally small charge from one plate to the other plate. Because the electric potential difference between the plates is zero, moving this first charge takes no work. However, moving the next charge does take work because there is now an electric potential difference between the two plates. The work it takes to move a charge equals the change in electric potential energy of the capacitor and it equals the magnitude of the charge which is moved times the electric potential difference the charge is moved through which is the electric potential difference across the capacitor which now has an infinitesimally small electric potential difference across it. We need to identify the infinitesimally small charge as dq and the amount of work it takes to move that charge dW. And take the integral of both sides.

$$Q_i = 0 \;\;\&\;\; W = \Delta U_{\text{elec}} = q\Delta V \Rightarrow dW = \Delta V dq$$

$$\& \;\; C = \frac{Q}{\Delta V} \Rightarrow \Delta V = \frac{Q}{C} \Rightarrow Q = C\Delta V$$

$$W = \int_0^Q \Delta V\, dq = \int_0^Q \frac{q}{C} dq = \left[\frac{q^2}{2C}\right]_0^Q = \frac{Q^2}{2C} - \frac{0^2}{2C} \Rightarrow U_C = \frac{Q^2}{2C}$$

$$\Rightarrow U_C = \frac{(C\Delta V)^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(\frac{Q}{\Delta V}\right)\Delta V^2 = \frac{1}{2}Q\Delta V$$

$$\Rightarrow U_C = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}Q\Delta V$$

Note: The energy stored in the capacitor is stored in the electric field of the capacitor and is equal to the amount of work needed to move the charges from one plate to the other.

The capacitor in the disposable camera:

$$C = 120\mu F;\; \Delta V = 330V$$

$$\Rightarrow U_C = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\left(120\times10^{-6}\right)(330)^2 = 6.532 \approx 6.5J$$

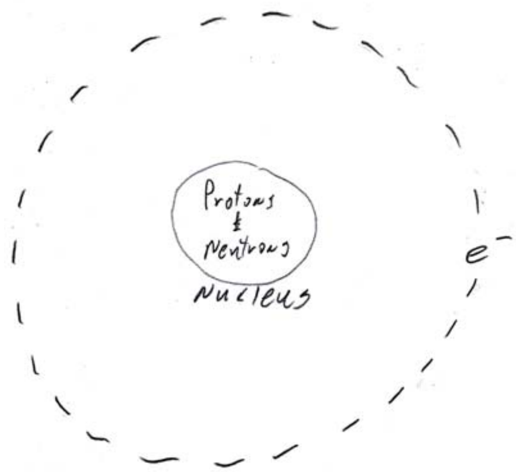
$$\Rightarrow U_C = 6.532J \times \left(\frac{1\,eV}{1.6x10^{-19}J}\right) = 4.0825\times10^{19} \approx 41\times10^{18}eV$$

$$\Rightarrow U_C \approx 41\times10^9\times10^9 eV \Rightarrow U_C \approx 41 \text{ billion billion } eV$$

College Prep Physics II – Video Lecture Notes – Chapter 19
Video Lecture #1
Introduction to Conventional Current and Direct Current & Example Problem

Current, I: The movement of charges. The rate at which charges pass by a point in a wire.

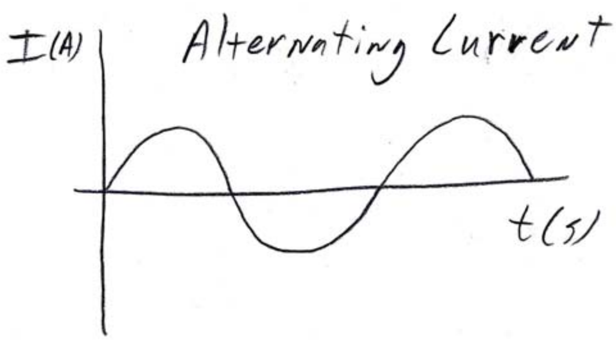
Bohr Model of the Atom: Protons and Neutrons in the nucleus with electrons in orbital shells. Electrons are much easier to remove from the atom; therefore it is generally electrons that flow in wires.

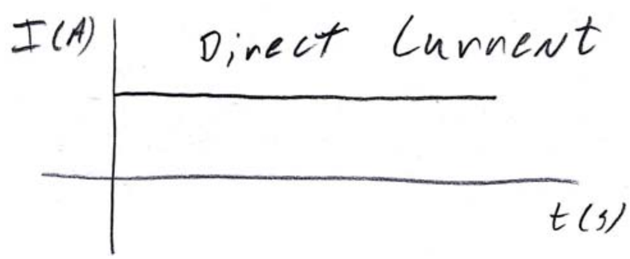


$$I = \frac{\Delta Q}{\Delta t} \Rightarrow \frac{Coulombs}{second} = \frac{C}{s} \Rightarrow Amperes, Amps, A$$

(Base SI Dimension)

Conventional Current: The direction that positive charges would flow. The reality is that negative charges flow in a negative direction.





Alternating Current, AC: Direction and magnitude of the current changes. Has a frequency like a sine or cosine wave. Less power loss over distance.

Direct Current, DC: Direction and magnitude of the current is constant. Large power loss over distance.

Many electronic devices have an AC/DC power converter to convert the alternating current that comes to your house to direct current. That is what the "brick" attached to your electronic devices is for.

Example Problem: A charge of 13.0 mC passes through a cross-section of wire in 4.5 seconds. (a) What is the current on the wire? (b) How many electrons pass through the wire in this time?

$\Delta Q = 13.0mC \times \frac{1C}{1000mC} = 0.013C$; $\Delta t = 4.5s$; a) I = ? b) # of electrons = ?

a) $I = \frac{\Delta Q}{\Delta t} = \frac{0.013}{4.5} = 0.002\overline{8}\,A \approx \boxed{0.0029A = 2.9mA}$

b) $Q = ne \Rightarrow n = \frac{Q}{e} = \frac{0.013}{1.6\times10^{-19}} = 8.125\times10^{16}e^- \approx \boxed{8.1\times10^{16}e^-}$

$n \approx 81\times10^{15}e^- = 81Pe^- = 81{,}000{,}000{,}000{,}000{,}000\,e^-$ (That is a lot of electrons, eh?)

Current density, J, is current per unit area:

- $J = \frac{I}{A} = \frac{nAv_d q}{A} \Rightarrow J = nv_d q$ & $J = \sigma E$
  - Materials which have this property are considered to be ohmic and follow Ohm's Law.
  - σ is the conductivity of the material.
    - Conductivity is a measure of how little a material opposes the movement of electric charges.
    - Conductivity is a fundamental property of a material.
- $\|\Delta V\| = Ed \Rightarrow \|\Delta V\| = EL$
  - An electric potential difference across a wire is what causes current in the wire and we are assuming the electric field created in the wire is uniform. Rather than using d for the distance in the electric field, we use L for the length of the wire.
- $\Rightarrow E = \frac{\Delta V}{L} \Rightarrow J = \sigma\left(\frac{\Delta V}{L}\right) \Rightarrow \Delta V = \frac{JL}{\sigma} = \frac{IL}{A\sigma} \Rightarrow \Delta V = \left(\frac{L}{\sigma A}\right)I$

The *resistance* of a wire, R, is defined as $R = \frac{L}{\sigma A}$

- However, usually resistance is defined in terms of *resistivity*, ρ.
  - Resistivity is a measure of how strongly a material opposes the movement of electric charges.
  - Resistivity is a fundamental property of a material.
  - $\rho = \frac{1}{\sigma} \Rightarrow R = \frac{\rho L}{A}$ & $E = \rho J$
  - This equation requires the resistor to have uniform geometry.
- Which brings us to the more common version of Ohm's law:
  - $\Delta V = \left(\frac{L}{\sigma A}\right)I = I\left(\frac{\rho L}{A}\right) \Rightarrow \Delta V = IR$
  - Again, not all materials are ohmic and follow Ohm's law.
- $\Rightarrow R = \frac{\Delta V}{I} \Rightarrow \text{ohms}, \Omega = \frac{\text{volts}, V}{\text{amperes}, A}$
- $R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L} \Rightarrow \frac{\Omega \cdot m^2}{m} = \Omega \cdot m$

Resistance and *resistivity* are two terms which students often mix up:

- Resistance has units of ohms, Ω, and is a property of an object.
- Resistivity has units of Ω·m and is property of a material.
- Two objects can have the same resistivity but different resistances if they are made of the same material; however, they have different lengths or cross-sectional areas.

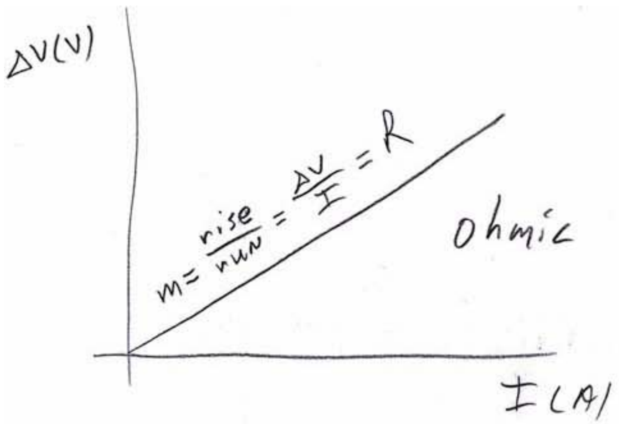
The resistivity of a conducting material typically decreases with decreasing temperature. Think of superconductors. Superconducting materials have zero resistivity, and require very, very low temperatures.

- In this class, unless otherwise stated, the resistivity of conducting materials is considered to be constant regardless of temperature.
- Resistors usually convert electric potential energy to thermal energy which can increase the temperature of the resistor and can increase the temperature of the resistor's environment.

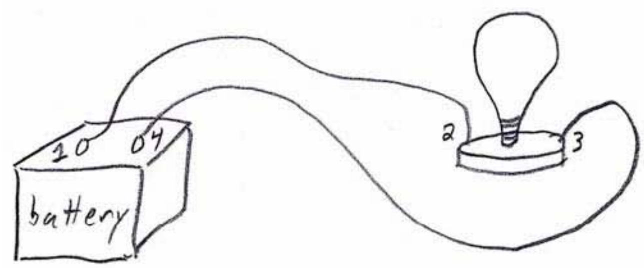
College Prep Physics II – Video Lecture Notes – Chapter 19
Video Lecture #2
Defining Resistance, Ohmic vs. Non-Ohmic, Electrical Power

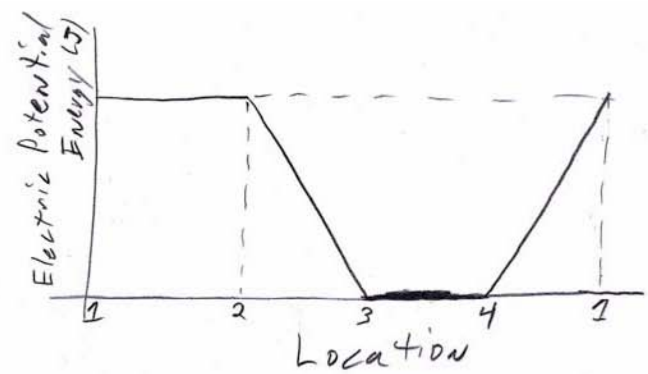
Resistance, R, is the resistance to current flow. $R = \frac{\Delta V}{I} \Rightarrow \boxed{\Delta V = IR}$ (Ohm's Law)

$R = \frac{\Delta V}{I} \Rightarrow \frac{Volts}{Amps} = \Omega$ or Ohms (Capital Omega, an upside down horse shoe, it's unlucky.)



Materials that follow Ohm's Law are called Ohmic. If they don't they are Non-Ohmic.
We will consider all resistors to be Ohmic, unless otherwise stated.





Electric Power: The rate at which electrical potential energy is being converted to heat, light and sound.

From 1-2 and 3-4 the charges are moving along the wire and we consider wires to have zero resistance unless otherwise stated.
From 2-3 the electric potential energy of the electrons is converted to heat, light and sound.
From 4-1 the electrons are being given electric potential energy by the battery.

Derivation of Electric Power Equation:

$$P = \frac{W}{t} = \frac{\Delta PE_{electric}}{t} \Rightarrow \frac{J}{s} = Watts \text{ \& } \Delta V = \frac{\Delta PE_{ele}}{q} \Rightarrow \Delta PE_{ele} = q\Delta V$$

Therefore: $P = \frac{\Delta PE_{electric}}{t} = \frac{q\Delta V}{t} = \left(\frac{q}{t}\right)\Delta V = I\Delta V$ & $\Delta V = IR$

Gives: $P = I\Delta V = I(IR) = I^2 R$ & $\Delta V = IR \Rightarrow I = \frac{\Delta V}{R}$

Gives: $P = I^2 R = \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2 R}{R^2} = \frac{\Delta V^2}{R}$ Therefore: $\boxed{P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}}$

College Prep Physics II – Video Lecture Notes – Chapter 19
Video Lecture #3
Finding the cost to power light bulbs and Defining Kilowatt Hour

$\$0.12$

Example Problem: 3 Light Bulbs; P_f = 15 watts; P_i = 60 watts; ΔV = 120 V; $\frac{\$0.12}{kW \cdot hr}$

$\frac{3hr}{day}$ (Bulbs are powered for this much time)

(yes, there are 24 hours in a day, however, the light bulbs are not on 24 hours each day.)

(A standard household circuit in the United States has a potential difference of 120 V.)

$$\Delta P = P_f - P_i = 15 - 65 = -50\,watts \Rightarrow \|\Delta P\| = 50\,watts$$

$$\Delta V = IR \Rightarrow R = \frac{\Delta V}{I} = \frac{120}{I} = ?????$$ (we don't know the current.)

$$P = \frac{\Delta V^2}{R} \Rightarrow R = \frac{\Delta V^2}{P} = \frac{120^2}{15} = \boxed{960\Omega}$$

Power saved by three bulbs: $P_{saved} = 3 \times 50 = 150\,watts \times \frac{1kW}{1000\,watts} = 0.15kW$

$$0.15kW \times \frac{3hr}{day} = \frac{0.45kW \cdot hr}{day} \Rightarrow \left(\frac{0.45kW \cdot hr}{day}\right)\left(\frac{\$0.12}{kW \cdot hr}\right)\left(\frac{365.242days}{1year}\right) = \frac{\$19.723}{year}$$

$$\$108 \times \frac{1year}{\$19.723} = 5.4758 \approx \boxed{5.5\,years}$$

What is a KiloWatt Hour?

$$(kW \cdot hr)\left(\frac{1000W}{1kW}\right)\left(\frac{3600s}{1hr}\right) = 3{,}600{,}000W \cdot s = 3{,}600{,}000\frac{J}{s} \cdot s = 3{,}600{,}000s = 3.6MJ$$

$1kW \cdot hr = 3.6MJ$ (you will not be given this as a conversion, you must derive it, every time.)

Flipping Physics Lecture Notes:

Resistivity

https://www.flippingphysics.com/resistivity.html

An open circuit does not contain a closed loop for current to flow and therefore current does not flow. A closed circuit does contain a closed loop for current to flow and therefore current does flow.

We tested various materials and discovered conductors such as aluminum, stainless-steel, and gold do allow current to flow. However, insulators such as plastic rubber and glass, do not allow current to flow. This is because conductors have electrons which are loosely bound to their atoms which allows current to flow. Whereas insulators have electrons which are tightly bound to their atoms which does not allow current to flow.

We have already learned about resistance, R, which is how an object limits current flow. Resistance is a physical property of an object.

$$\Delta V = IR \Rightarrow R = \frac{\Delta V}{I}$$

Today we learn about resistivity, ρ, which is a material property. Resistivity is a fundamental property of the material to limit electric current flow.

Because the difference between Resistance and Resistivity can be difficult for students to remember, I will repeat myself:

- Resistance is the property of an object.
- Resistivity is the property of a material.

Resistance and Resistivity are related by the following equation:

$$R = \frac{\rho L}{A}$$

- R = Resistance
- ρ = Resistivity. (I know. I am sorry. It's not density.)
- L = Length
- A = cross sectional area

The units for resistivity are: $\Omega \bullet m$

$$R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L} = \frac{\Omega \bullet m^2}{m} = \Omega \bullet m$$

Resistivities for some common materials at 20°C are:

Material	***ρ* @ 20°C (Ω·m)**		**Type**
Silver	1.6×10^{-8}	0.000000016	Conductor
Copper	1.7×10^{-8}	0.000000017	Conductor
Gold	2.4×10^{-8}	0.000000024	Conductor
Aluminum	2.8×10^{-8}	0.000000028	Conductor
Stainless Steel	6.9×10^{-7}	0.00000069	Conductor
Germanium	4.6×10^{-1}	0.46	Semiconductor
Silicon	6.4×10^{2}	620	Semiconductor
Glass	$10\times10^{10} – 10\times10^{14}$	100,000,000,000 – 1,000,000,000,000,000	Insulator
Hard Rubber	10×10^{13}	100,000,000,000,000	Insulator
Air	$1.3\times10^{16} – 3.3\times10^{16}$	13,000,000,000,000,000 – 33,000,000,000,000,000	Insulator

Resistivities compiled from electronics-notes.com[1] and sciencenotes.org[2].

Resistivity is temperature dependent.

- Conductors: As temperature increases, resistivity increases.
- Semiconductors: As temperature increases, resistivity decreases.

[1] https://www.electronics-notes.com/articles/basic_concepts/resistance/electrical-resistivity-table-materials.php
[2] https://sciencenotes.org/table-of-electrical-resistivity-and-conductivity/

Flipping Physics Lecture Notes:

Graphing Resistivity

https://www.flippingphysics.com/graphing-resistivity.html

Given a length of nichrome wire and a variable power supply which displays both current and electric potential difference, what data would you need to collect and what would need to go on the axes of a graph such that the resistivity of nichrome would be the slope of the best-fit line of the data?

Let's start with Ohm's Law and solve for resistance and we also have an equation for resistance in terms of resistivity which we can set equal to one another.

$$\Delta V = IR \Rightarrow R = \frac{\Delta V}{I} = \frac{\rho L}{A}$$

Then we remove all variables from the denominators. We know have:

$$\Delta V A = \rho\left(IL\right)$$

Comparing that to the slope intercept form equation for a line:

$$y = mx + b$$

And you can see what variables go on the y and x axes:

$$\Rightarrow y = \Delta V A;\ m = slope = \rho;\ x = IL;\ b = 0$$

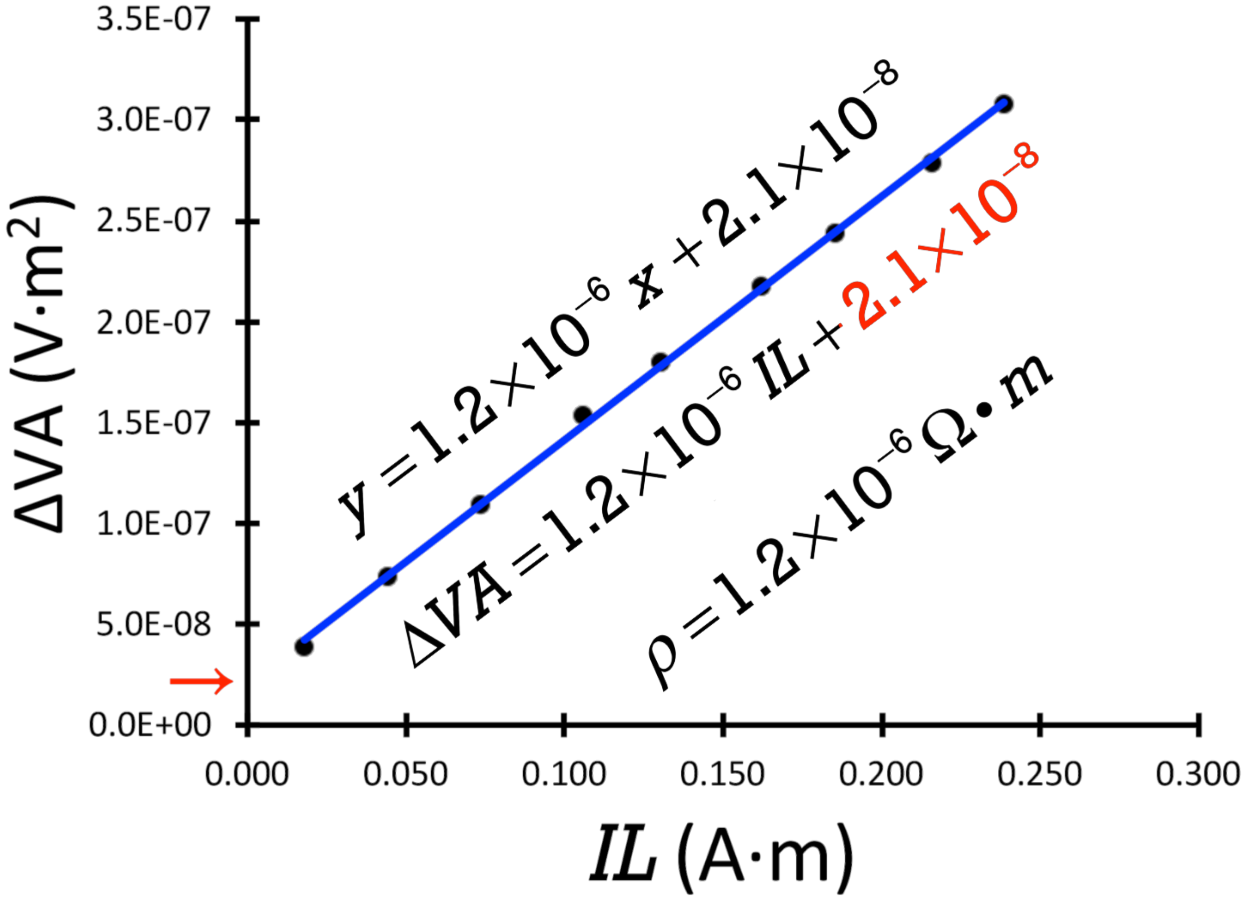
We will need the cross-sectional area of the wire. We know the wire is a 32-gauge wire. Which, according to American Wire Gauge standards, has a diameter of 0.202 mm, and a radius which is half that or 0.101 mm or 0.000101 m. Therefore, the cross-sectional area is the area of a circle or πr^2.

$$Diameter = 0.202mm \Rightarrow r = \frac{Dia}{2} = \frac{0.202mm}{2} = 0.101mm = 0.000101m$$

$$Area = \pi r^2 = \pi\left(0.000101\right)^2 = 3.20474\times10^{-8}\,m^2$$

And, if we adjust the length of the wire, then we can measure the electric potential difference across the wire and current through the wire.

ΔV (V)	Current (A)	Length (m)	ΔVA ($V\cdot m^2$)	IL (A·m)
1.2	0.36	0.050	3.8E-08	0.018
2.3	0.44	0.100	7.4E-08	0.044
3.4	0.49	0.150	1.1E-07	0.074
4.8	0.53	0.200	1.5E-07	0.11
5.6	0.52	0.250	1.8E-07	0.13
6.8	0.54	0.300	2.2E-07	0.16
7.6	0.53	0.350	2.4E-07	0.19
8.7	0.54	0.400	2.8E-07	0.22
9.6	0.53	0.450	3.1E-07	0.24



The resistivity we get from our experiment is roughly 1.2 x 10^{-6} Ω·m, which is right in the range we expect because the published value for the resistivity for nichrome at 20°C is in the range 1.0 – 1.5 x 10^{-6} Ω·m.[1]

Notice that b, the y-intercept, does not actually work out to be zero. It ends up being a small, positive number. That is because our solution assumes the wires we use in the experiment have zero resistance. The wires do have a small amount of resistance, which causes the y-intercept to have a small, positive number.

Another item to note is that I purposefully reduced the electric potential difference as the length of the wire was reduced. This is because resistivity of conductors increases with temperature. I did not want the nichrome wire to heat up too much during the experiment.

[1] https://hypertextbook.com/facts/2007/HarveyKwan.shtml

Flipping Physics Lecture Notes:

Electric Potential Difference and Circuit Basics
https://www.flippingphysics.com/electric-potential-difference.html

A mass, m, in a gravitational field can have gravitational potential energy, U_g.
Similarly, a charge, q, in an electric field can have electric potential energy, U_e.
We can determine the electric potential energy per unit charge, V, it is called electric potential:

$$V = \frac{U_e}{q}$$

However, typically we are interested in the *change* in electric potential energy per unit charge, which is called the electric potential difference, ΔV:

$$\Delta V = \frac{\Delta U_e}{q}$$

In other words, between any two points in an electric field there can exist an electric potential difference which represents the difference between the electric potential energies of those two locations per unit charge. This means you do not need a charge for that electric potential difference to be there. That electric potential difference is always there, essentially waiting for a charge to then provide that charge with a change in electric potential energy. You can determine the change in electric potential energy on a charge by multiplying the charge by the electric potential difference.

$$\Delta V = \frac{\Delta U_e}{q} \Rightarrow \Delta U_e = q\Delta V$$

It is not unusual for people to drop the "electric" from electric potential difference and just call it potential difference. I will do my best not to do that and always clearly identify ΔV as *electric* potential difference.

The units for electric potential difference are joules per coulomb:

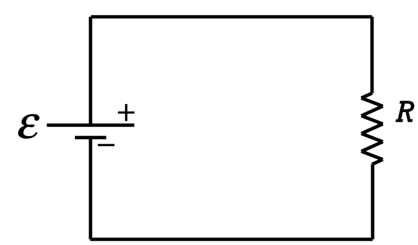
$$\Delta V = \frac{\Delta U_e}{q} \Rightarrow \frac{J}{C} = Volts, V$$

- Joules per coulomb are called Volts. The symbol for Volts is V.
- Electric potential difference is also commonly called the "voltage".
- Electric Potential Energy is a scalar; therefore, Electric Potential Difference is also a scalar.
- Please be careful to distinguish electric potential difference, ΔV, from the units for electric potential difference, V. I know they use the same symbol, which is irksome.
- Volts are named after the Italian physicist Alessandro Volta (1745–1827). He was a pioneer of electricity and power, and is credited with the invention of the electric battery.

Speaking of an electric battery, the maximum possible voltage a battery can provide between its terminals is called the electromotive force or emf. In a non-ideal battery, the emf differs from the battery's terminal voltage, ΔV_t, because the terminal voltage (the electric potential difference measured between the terminals of the battery) will be less than the emf of the battery because the internal resistance of the battery decreases the terminal voltage of the battery. Until further notice, however, all batteries are "ideal" and therefore, emf and ΔV_t are identical. In summary:

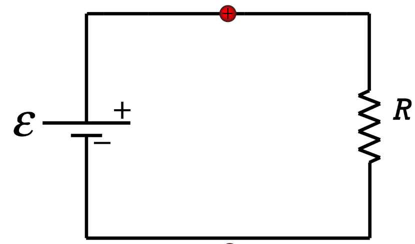
- emf (electromotive force) is the ideal, maximum voltage across a battery.
- ΔV_t (terminal voltage) is the voltage measured at the terminals of the battery.
- In a real battery, ΔV_t < emf due to internal resistance in the battery.
- However, for now, we assume all batteries are ideal and we are assuming ΔV_t = emf. ☺
- Oh, and emf has its own symbol, Ɛ.
- Also, I will point out that electromotive force is an "historical term" which, unfortunately, we still use even though it's a misnomer because it is *not* … a … force.

Let's take a look at a basic circuit with a battery, 2 connecting wires, and a resistor. The circuit diagram looks like this:

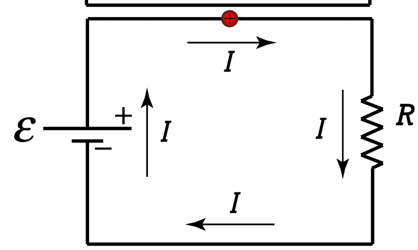


- The battery, which is indicated with the emf symbol, Ɛ, has a positive terminal which is indicated with a long line and a + and a negative terminal which is indicated with short line and a -.
- The resistor is labelled R for resistor.
- Please remember that, unless otherwise stated, all wires are considered to have zero resistance.

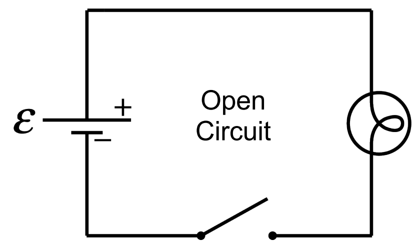
We need to determine which direction current will flow in the circuit. To do that we place a small, positive test charge at a location in the circuit (as shown in the diagram) and discuss, using the Law of Charges, which direction that small, positive test charge will experience an electric force.



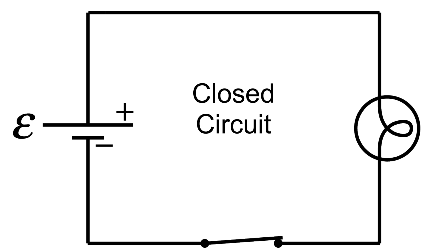
- We are using a positive test charge because conventional current is defined by the direction positive charges would flow. We do this even though we know negative charges (electrons) actually flow opposite the direction of conventional current. Yea!
- According to the Law of Charges, the positive charge is repelled away from the positive terminal of the battery (like charges repel) and the positive charge is attracted to the negative terminal of the battery (unlike charges attract). In other words…
- Current flows in a clockwise direction in this circuit.



Now let's add a switch to the circuit and replace the resistor with a light bulb so we can see evidence of current flow. As you can see in the video, with the switch open, there is no closed loop for the current to flow through, so current does not flow, the light bulb does not glow, and this is called an open circuit.

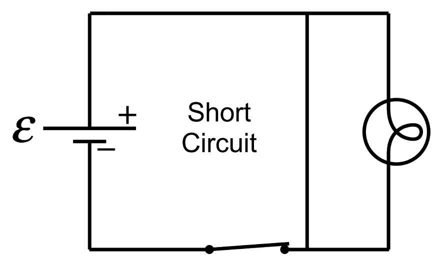


When we close the switch, there is now a closed loop for the current to flow through, current flows, the light bulb glows and this is called a closed circuit. The light bulb (or the resistor in the previous circuit) is called the "electrical load" of the circuit. The "electrical load" is the part of the circuit which is converting eclectic potential energy to heat, sound, and (in the case of the lightbulb) light. Because the switch and all the wires are "ideal" and considered to have zero resistance, those items are not a part of the electrical load. The battery is not a part of the electrical load because the battery is an electrical power source; the battery is a source of electric potential energy.



If we were to add a wire to this circuit which bypasses the load, this would be a short circuit. A short circuit is a circuit which has a very small resistance and therefore a very large current. Short circuits are usually the result of some sort of accident and should be avoided because, with a very small resistance and a constant electric potential difference, the electric power, or the rate at which electric potential energy is converted to heat, light, and sound, can be very large and dangerous.

$$P = \frac{\Delta V^2}{R}$$



Flipping Physics Lecture Notes:
Resistance and Ohm's Law
http://www.flippingphysics.com/resistance-ohms-law.html

Current density, J, is current per unit area:

- $J = \frac{I}{A} = \frac{nAv_d q}{A} \Rightarrow J = nv_d q$ & $J = \sigma E$
  - Materials which have this property are considered to be ohmic and follow Ohm's Law.
  - σ is the conductivity of the material.
    - Conductivity is a measure of how little a material opposes the movement of electric charges.
    - Conductivity is a fundamental property of a material.
- $\|\Delta V\| = Ed \Rightarrow \|\Delta V\| = EL$
  - An electric potential difference across a wire is what causes current in the wire and we are assuming the electric field created in the wire is uniform. Rather than using d for the distance in the electric field, we use L for the length of the wire.
- $\Rightarrow E = \frac{\Delta V}{L} \Rightarrow J = \sigma\left(\frac{\Delta V}{L}\right) \Rightarrow \Delta V = \frac{JL}{\sigma} = \frac{IL}{A\sigma} \Rightarrow \Delta V = \left(\frac{L}{\sigma A}\right)I$

The *resistance* of a wire, R, is defined as $R = \frac{L}{\sigma A}$

- However, usually resistance is defined in terms of *resistivity*, ρ.
  - Resistivity is a measure of how strongly a material opposes the movement of electric charges.
  - Resistivity is a fundamental property of a material.
  - $\rho = \frac{1}{\sigma} \Rightarrow R = \frac{\rho L}{A}$ & $E = \rho J$
  - This equation requires the resistor to have uniform geometry.
- Which brings us to the more common version of Ohm's law:
  - $\Delta V = \left(\frac{L}{\sigma A}\right)I = I\left(\frac{\rho L}{A}\right) \Rightarrow \Delta V = IR$
  - Again, not all materials are ohmic and follow Ohm's law.
- $\Rightarrow R = \frac{\Delta V}{I} \Rightarrow \text{ohms}, \Omega = \frac{\text{volts}, V}{\text{amperes}, A}$
- $R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L} \Rightarrow \frac{\Omega \cdot m^2}{m} = \Omega \cdot m$

Resistance and *resistivity* are two terms which students often mix up:

- Resistance has units of ohms, Ω, and is a property of an object.
- Resistivity has units of Ω·m and is property of a material.
- Two objects can have the same resistivity but different resistances if they are made of the same material; however, they have different lengths or cross-sectional areas.

The resistivity of a conducting material typically decreases with decreasing temperature. Think of superconductors. Superconducting materials have zero resistivity, and require very, very low temperatures.

- In this class, unless otherwise stated, the resistivity of conducting materials is considered to be constant regardless of temperature.
- Resistors usually convert electric potential energy to thermal energy which can increase the temperature of the resistor and can increase the temperature of the resistor's environment.

Flipping Physics Lecture Notes:
Electric Power
http://www.flippingphysics.com/electric-power.html

Now we get to discuss *electric power*, which is the rate at which electric potential energy is converted to other types of energy such as heat, light, and sound.

$$P = \frac{dU}{dt} \Rightarrow P_{elec} = \frac{dU_{elec}}{dt} = \frac{d(q\Delta V)}{dt} = \frac{dq}{dt}\Delta V \Rightarrow P = I\Delta V$$

$$\& \ \Delta V = IR \Rightarrow P = I(IR) = I^2R$$

$$\& \ I = \frac{\Delta V}{R} \Rightarrow P = \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2}{R}$$

$$\Rightarrow P = I\Delta V = I^2R = \frac{\Delta V^2}{R}$$

A unit which is often used when it comes to electricity is the kilowatt-hour:

$$1kW \cdot hr\left(\frac{1W}{1000kW}\right) = 1000W \cdot hr = 1000\left(\frac{J}{s}\right)hr\left(\frac{3600s}{1hr}\right) = 3.6\times10^6 J$$

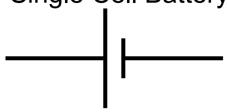
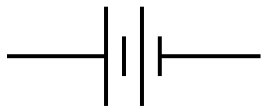
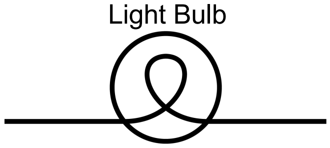
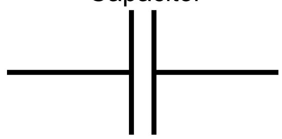
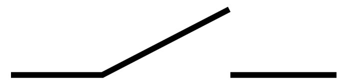
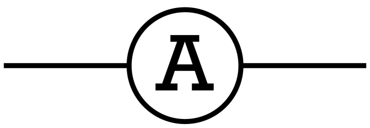
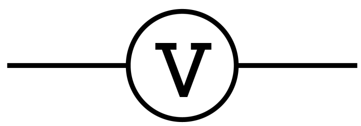
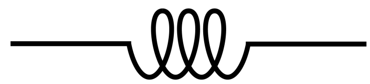
In other words, the kilowatt-hour is a misnomer (or maybe just misleading). It sounds like a unit of power; however, it is a unit of energy. And we know: $1kW \cdot hr = 3.6MJ$

A light bulb is a common item used in physics. It is a resistor which converts electric potential energy to light, heat, and sound energy. The brightness of a light bulb increases with increasing power; therefore, the brightness of a light bulb is often used to demonstrate the power in an electric circuit. Speaking of electric circuits…

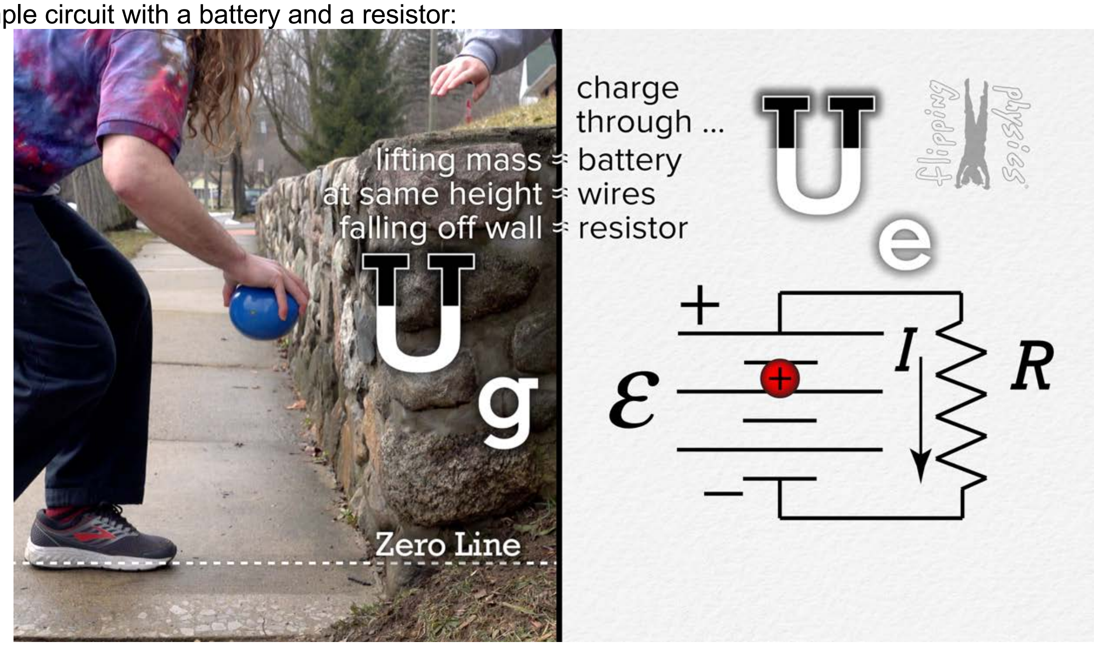
Flipping Physics Lecture Notes:
Electric Circuit Basics
http://www.flippingphysics.com/electric-circuit-basics.html

The Basics of Electric Circuits:

- An electric circuit is typically composed of electrical loops which can include wires, batteries, resistors, light bulbs, capacitors, switches, ammeters, voltmeters, and inductors.
- Typical symbols for elements in electric circuits are:

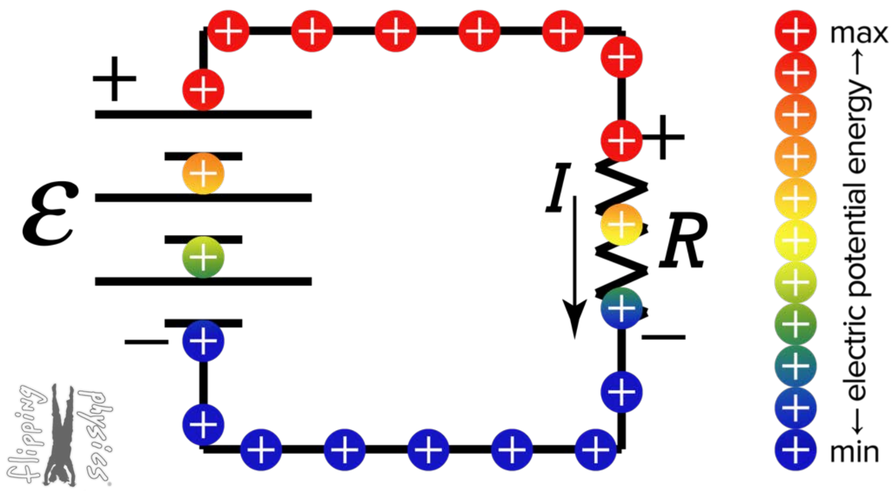
Single Cell Battery	Double Cell Battery	Resistor
		
Light Bulb	Capacitor	Switch
		
Ammeter	Voltmeter	Inductor
		

A simple circuit with a battery and a resistor:



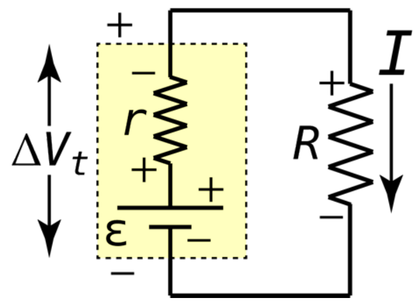
- The long line of the battery is the positive terminal, and the short line is the negative terminal.
- *Electromotive force*, emf, ε, is the ideal electric potential difference, or voltage, across the terminals of the battery.
    - Yes, the symbol, lowercase Greek letter epsilon, is the same as electric permittivity. ☹
    - Yes, electromotive force is not a force. The term is another misnomer. ☹

- According to the law of charges, positive charges are repelled from the positive terminal and attracted to the negative terminal; therefore, current is clockwise in this circuit.
- A battery adds electric potential energy to electric charges.
  - Like lifting a mass adds gravitational potential energy to masses.
  - A battery is essentially an electric potential energy pump.
- A resistor converts electric potential energy to heat energy. (And maybe light sound energy)
  - Like a mass falling off a wall converts gravitational potential energy to kinetic energy.
- Unless otherwise stated, wires are considered to be ideal and have zero resistance; therefore, there is no change in electric potential energy of charges as they move along a wire.
  - Like a mass at rest maintaining a constant height and therefore a constant gravitational potential energy at either the top or bottom of the wall.



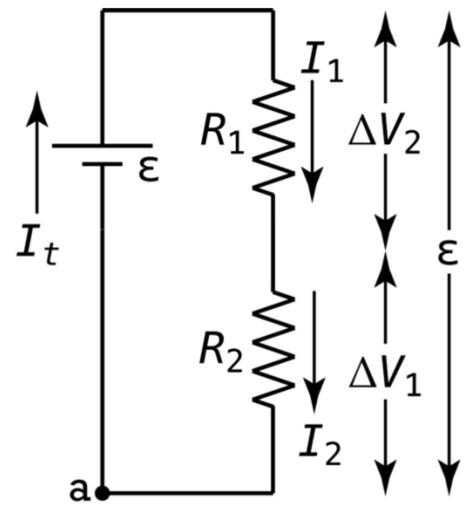
Flipping Physics Lecture Notes:
Terminal Voltage
http://www.flippingphysics.com/terminal-voltage.html

- *Terminal Voltage*, ΔV_t, is the measured voltage across the terminals of the battery.
    - Because all real batteries have some internal resistance, when a battery is supplying current to a circuit, the terminal voltage of a real battery is less than the emf.
        - The symbol for the internal resistance of a real battery is typically, r.
    - One way to illustrate a real battery in an electric circuit is shown in yellow.



    - $\Delta V_t = \varepsilon - \Delta V_r \Rightarrow \Delta V_t = \varepsilon - Ir$
    - As current increases, the terminal voltage decreases.
    - The only way to get the terminal voltage to be equal to the emf is to have no current flowing through the battery.

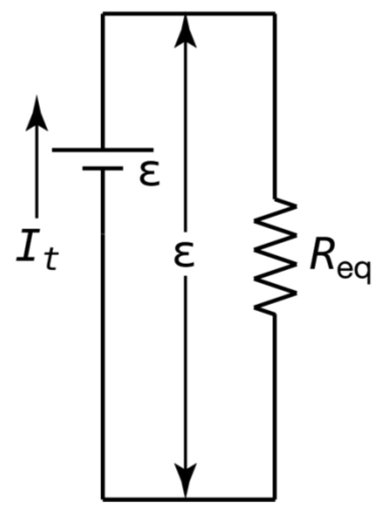
Flipping Physics Lecture Notes:
Resistor Series and Parallel Circuits
http://www.flippingphysics.com/resistor-series-parallel.html



When an anthropomorphic[1] charge has no choice but to go through two circuit elements, those two circuit elements are in *series*. For example, a charge which goes through resistor 1 has no choice but to also go through resistor 2. There is no other path for the anthropomorphic charge to choose.

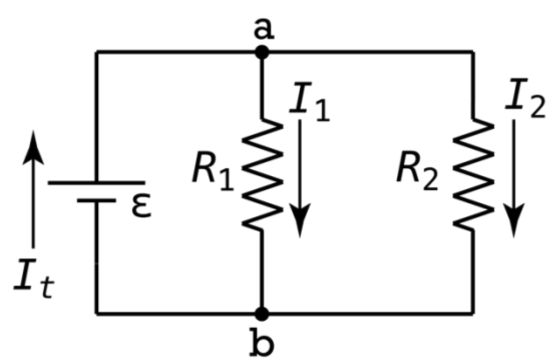
The currents through the three circuit elements must all be equal:
$$I_t = I_1 = I_2$$
The "t" in the subscript refers to the current at the terminals of the battery which is the current delivered by the battery to the circuit.

The electric potential difference across the battery equals the summation of the electric potential difference across the two resistors:
$$\Delta V_{\text{bottom wire}\rightarrow\text{top wire}} = \varepsilon = \Delta V_1 + \Delta V_2$$
(If you'd prefer to look at this in terms of the electric potential difference around the loop in the circuit:)
$$\Delta V_{\text{loop}} = V_f - V_i = V_a - V_a = 0 = \varepsilon - \Delta V_1 - \Delta V_2 \Rightarrow \varepsilon = \Delta V_1 + \Delta V_2$$



We know Ohm's law: $\Delta V = IR$; therefore, …
$$\Rightarrow \varepsilon = I_t R_{eq} = I_1 R_1 + I_2 R_2$$
$$\Rightarrow R_{eq} = R_1 + R_2$$
The "eq" in the subscript means equivalent. In other words, R_{eq} is one resistor with the equivalent resistance of the two resistors.

Therefore, the equation for the equivalent resistance of n resistors in series is:
$$R_{\text{eq series}} = \sum_n R_n = R_1 + R_2 + \ldots$$



When an anthropomorphic charge has the choice between two circuit elements and then the paths through those two circuit elements reconverge without going through another circuit element, the two circuit elements are in *parallel*.

When circuit elements are in parallel, their electric potential differences are equal:
$$\varepsilon = \Delta V_1 = \Delta V_2$$

Note the junctions at points a and b. Due to conservation of charge, the net current going into a junction equals the net current coming out of a junction. For junction a:

[1] *Anthropomorphism:* Giving human characteristics or behaviors to non-human objects.

$$I_{\text{in}} = I_{\text{out}} \Rightarrow I_t = I_1 + I_2$$

We can then use Ohm's law:

$$\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow \frac{\varepsilon}{R_{\text{eq}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

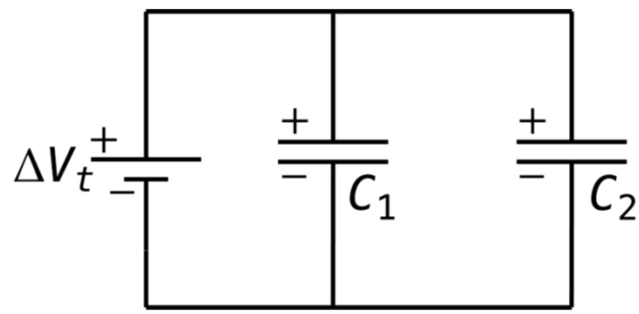
And we get the equivalent resistance for the two resistors in parallel:

$$\Rightarrow R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

And the equivalent resistance for n resistors in parallel:

$$\Rightarrow R_{\text{eq parallel}} = \left(\sum_n \frac{1}{R_n}\right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)^{-1}$$

When we add a resistor in series, the equivalent resistance increases.
When we add a resistor in parallel, the equivalent resistance decreases.



Let's look at two capacitors in parallel:
We know the electric potential differences are all equal.

$$\Delta V_t = \Delta V_1 = \Delta V_2$$

Because the charges moved to the top plates of the capacitors need to go to either capacitor 1 or capacitor 2, the charge moved by the battery to the plates of the capacitors equals the sum of the charges on the capacitors:

$$Q_t = Q_1 + Q_2$$

We can then use the definition of capacitance:

$$C = \frac{Q}{\Delta V} \Rightarrow Q = C\Delta V$$

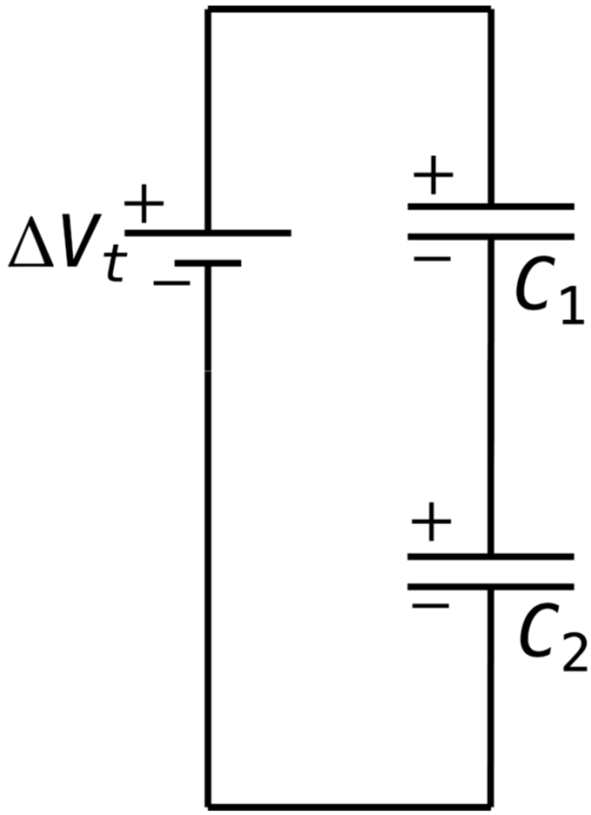
To derive the equivalent capacitance of two capacitors in parallel:

$$\Rightarrow C_{\text{eq}}\Delta V_t = C_1\Delta V_1 + C_2\Delta V_2 \Rightarrow C_{\text{eq}} = C_1 + C_2$$

And the equivalent capacitance of n capacitors in parallel:

$$\Rightarrow C_{\text{eq parallel}} = \sum_n C_n = C_1 + C_2 + \dots$$

And we can now look at two capacitors in series:



The electric potential is the same as resistors in series:

$$\Delta V_t = \Delta V_1 + \Delta V_2$$

And the charges on each capacitor are equal:

$$Q_t = Q_1 = Q_2$$

This is because the magnitude of the charge moved by the battery to the top plate of capacitor 1 and the bottom plate of capacitor 2 are equal in magnitude. And those plates polarize the charges on the wire between the two capacitors and the bottom of capacitor 1 and the top of capacitor 2. This causes all four plates of the two capacitors to have equal magnitude charges. This is an illustration of conservation of charge.

And we can solve for electric potential difference in terms of capacitance and charge:

$$Q = C\Delta V \Rightarrow \Delta V = \frac{Q}{C}$$

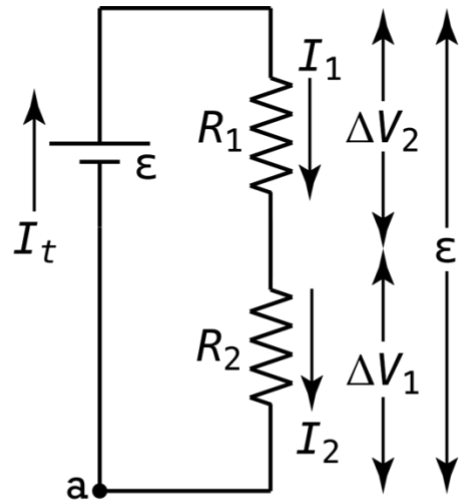
And use that to solve for the equivalent capacitance of two capacitors:

$$\Rightarrow \frac{Q_t}{C_{\text{eq}}} = \frac{Q_1}{C_2} + \frac{Q_2}{C_2} \Rightarrow \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

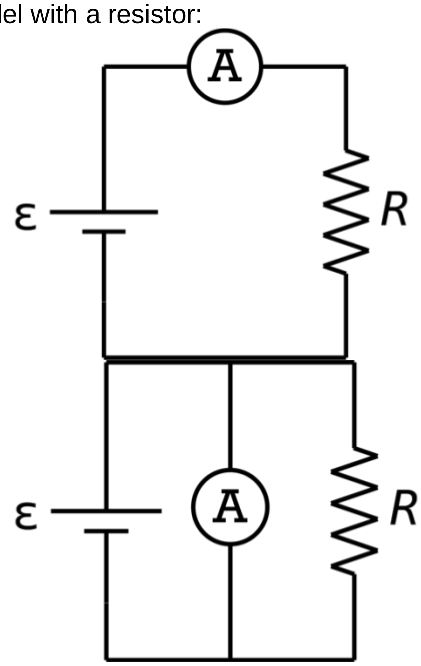
And the equivalent capacitance of n capacitors:

$$\Rightarrow C_{\text{eq series}} = \left(\sum_n \frac{1}{C_n}\right)^{-1} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)^{-1}$$

Notice the equations for resistors and capacitors are reversed. That means that:
When we add a capacitor in parallel, the equivalent capacitance increases.
When we add a capacitor in series, the equivalent capacitance decreases.

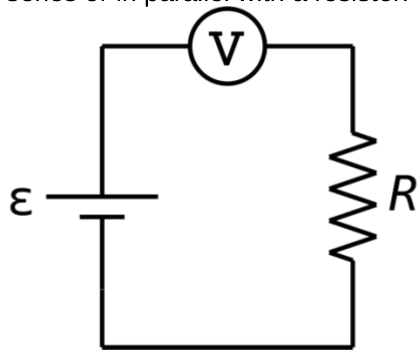


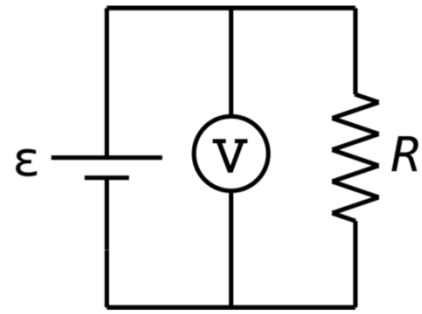
Let's discuss how to use the tools which measure current and electric potential difference. Starting with the ammeter which measures current or amperes. We need to decide if an ammeter needs to be put in series or parallel with the circuit element it is meant to measure the current through. So, let's look at what happens when we attempt to measure the current through a resistor using an ammeter in series and in parallel with a resistor:



Hopefully you recognize that placing an ammeter in parallel with a resistor will not measure the current through the resistor because the current through the ammeter and the resistor are not the same. Therefore, an ammeter needs to be placed in series with a circuit element to measure the current through that circuit element. Also, the resistance of an ammeter needs to be *very* small. In the above example, if the resistance of the ammeter is not *very* small, it will increase the equivalent resistance of the circuit and decrease the current through the resistor you are trying to measure the current through. Unless otherwise indicated, ammeters in this class are considered to have zero resistance.

And now let's attempt to measure the electric potential difference across a resistor using a voltmeter either in series or in parallel with a resistor:





Hopefully you recognize that placing a voltmeter in series with a resistor will not measure the electric potential difference across the resistor because the voltage across the voltmeter and the resistor are not the same. Therefore, a voltmeter needs to be placed in parallel with a circuit element to measure the voltage across that circuit element. Also, the resistance of a voltmeter needs to be *very* large. In the above example, if the resistance of the voltmeter is not *very* large, it will decrease the equivalent resistance of the circuit, increase the current delivered by the battery, and change the overall properties of the circuit. Unless otherwise indicated, voltmeters in this class are considered to have infinite resistance.

To review:

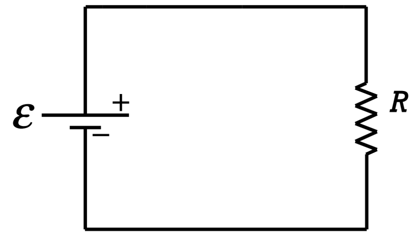
• Ammeters:	• Voltmeters:
○ Measure current	○ Measure electric potential difference
○ Placed in *series* with the circuit element	○ Placed in *parallel* with circuit element
○ Have nearly *zero* resistance*	○ Have nearly *infinite* resistance*

* You may see this called impedance in product literature for Voltmeters and Ammeters, due to the fact that there is more to the behavior of these devices than just resistance. For the purpose of this class and the AP Physics C Electricity and Magnetism exam, it will be called resistance unless otherwise noted.

Flipping Physics Lecture Notes:

Kirchhoff's Rules of Electrical Circuits
https://www.flippingphysics.com/kirchhoff.html

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0$$



Kirchhoff's Two Rules for circuits are very basic rules which are used to understand circuits. Let's start with Kirchhoff's Loop Rule which states that the net electric potential difference around a closed loop equals zero.

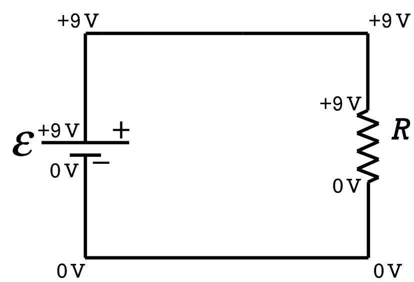
The Loop Rule is essentially conservation of electric potential energy in a circuit. Because electric potential difference equals change in electric potential energy per unit charge, the net change in electric potential energy in a closed loop then equals zero.

$$\sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = 0 \; \& \; \Delta V = \frac{\Delta U_e}{q} \Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \frac{\Delta U_e}{q} = 0 \Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \Delta U_e = 0$$

Using a gravitational potential energy analogy here, this is like saying, if you drop a mass off a wall, then pick up the mass and return it to its original location, the change in gravitational potential energy of that mass equals zero. We know this to be true because the mass returns back to the same height as where it started, so the mass will have the same gravitational potential energy at the end as it did at the beginning, no matter where we place the horizontal zero line.

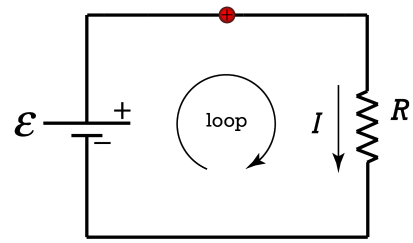
Going back to electric potential energy, this means, after a charge goes through one full, closed loop around a circuit, the electric potential energy of the charge will return back to its original value. But because we are using electric potential, we are really talking about the electric potential energy per unit charge at each location.

Let's say we have a 9-volt battery. That means we know the electric potential difference across the battery equals 9 volts. As we go from the negative to the positive terminals of the battery, the electric potential will go up. Technically we do not know the electric potential at any point, only the *difference* in the electric potential, however, it is customary to assume the minimum electric potential is zero. That means we are assuming the negative terminal of the battery is at zero volts and the positive terminal of the battery is at positive 9 volts.



Because ideal wires have zero resistance, that means the electric potential in the upper left corner must also be 9 volts, the electric potential in the upper right corner equals 9 volts, and the electric potential at the top of the resistor is 9 volts. Also, the electric potential in the lower left corner must be the same as the negative terminal of the battery, so electric potential in the lower left corner equals 0 volts. Therefore, electric potential in the lower right corner is 0 volts, and the electric potential at the bottom of the resistor equals 0 volts. This means the electric potential difference across the resistor also has a magnitude of 9 volts. In other words, in this circuit with two circuit elements, the two elements, the battery and the resistor, both have the same magnitude electric potential difference.

In a previous lesson we determined that a positive charge in the circuit would be repelled from the positive terminal of the battery and attracted to the negative terminal of the battery, therefore the current in this circuit is clockwise. This means the current is down through the resistor.

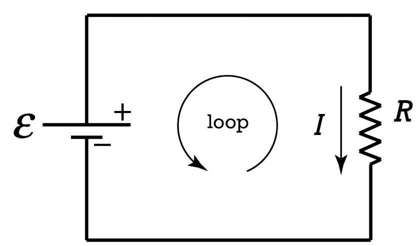


There is only one closed loop in our present circuit, so it might not seem obvious that we need to do this, however, we need to define a loop direction. Often the loop direction is the same as the direction which goes from the negative terminal to the positive terminal of the battery and through the battery, therefore, our loop direction for this circuit is clockwise.

This means as we go in the direction of the loop across the battery, the electric potential goes up because we go from the negative to the positive terminal of the battery. Therefore, when we sum the electric potential differences in our Kirchhoff's loop equation, the electric potential difference across the battery is positive. When we go in the direction of the loop across the resistor, as we illustrated before, the electric potential goes down. Therefore, in our loop equation, the electric potential difference across the resistor is negative. We know the electric potential difference across the battery equals the electromotive force or the emf of the battery. And the electric potential difference across the resistor equals current times resistance. Therefore, we can determine the current in the circuit in terms of the emf of the battery and the resistance of the resistor.

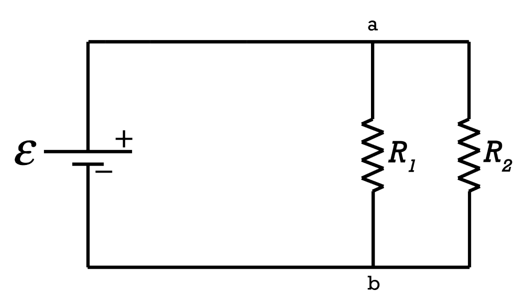
$$\Delta V_{Battery} = \varepsilon \ \&\ \Delta V_{Resistor} = IR$$

$$\Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = \Delta V_{battery} - \Delta V_{Resistor} = 0 = \varepsilon - IR \Rightarrow \varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R}$$



If we had chosen counterclockwise as the loop direction, all of our electric potential differences in Kirchhoff's Loop Rule would have been reversed. Because the loop direction goes from the positive to the negative terminals of the battery, the electric potential difference across the battery is negative, because the electric potential is going down. Because the loop direction through the resistor is opposite the direction of the current direction we defined through the resistor, the electric potential goes up through the resistor and the electric potential difference the across resistor is positive.

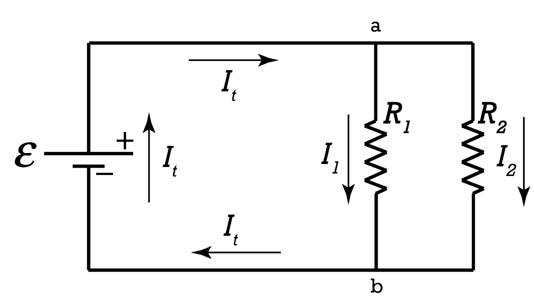
$$\Rightarrow \sum_{\substack{\text{closed}\\\text{loop}}} \Delta V = -\Delta V_{battery} + \Delta V_{Resistor} = 0 = -\varepsilon + IR \Rightarrow \varepsilon = IR \Rightarrow I = \frac{\varepsilon}{R}$$



Realize, we get the same result for the current in the circuit regardless of which loop direction we choose. If we had chosen an incorrect direction for current, the current ends up being negative, which tells you that you chose the incorrect current direction.

Now let's add a resistor to the circuit and talk about Kirchhoff's Junction Rule which is the result of conservation of charge in the circuit. The rule is sum of the currents entering a junction must equal the sum of the currents leaving a junction, which is conservation of charge:

$$\sum I_{in} = \sum I_{out}$$



Junctions are locations in circuits where at least three circuit paths meet. That means in our circuit we have two junctions which are labelled *a* and *b*. Therefore, the current going into both of those junctions equals the current coming out of those junctions. This means we need to define current directions. We do this the same way we did before, we place a positive test charge in the circuit and see which direction the Law of Charges defines electric force direction on the charge. This means current will go to the right through the top wire, to the left through the bottom wire, and down through both resistors. Let's label those currents as current 1 through resistor 1, current 2 through resistor 2, and current t through the battery because it is the current through the terminals of the battery.

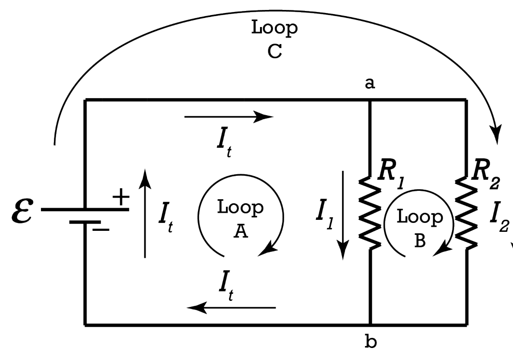
Kirchhoff's Junction Rule equations for this circuit are for:

junction a: $I_t = I_1 + I_2$

junction b: $I_t = I_1 + I_2$

Yes, these two equations are actually the same.

But how do we know *a* and *b* are junctions and the four exterior "corners" of the circuit are not junctions? I know it may seem obvious because there are not at least three circuit paths at any of those locations, however, this is a simple circuit. Again, we return back to placing a positive test charge on the wire. Notice that a charge which approaches point a could go in the wire leading to resistor 1 or in the wire leading to resistor 2. Because junctions are defined as having three circuit paths, any time a charge comes to a fork in the wire, the charge could go down either wire, that makes it a junction. When a charge enters a corner, there is no other choice but to continue along the same wire, therefore none of the corners are junctions.



Let's identify the loops in this second circuit and determine their Kirchhoff's Loop Rule equations. We can define the first loop as the same as the previous circuit, but let's call it loop A with a clockwise direction. There is another loop that contains resistor 1 and resistor 2. Let's call that loop B and also have that be clockwise. Lastly there is a loop all the way around the outside; It includes the battery and resistor 2. Let's call that loop C and have it also be clockwise.

Kirchhoff's Loop Rule equations look like this:

$$\sum_{Loop\ A} \Delta V = \Delta V_t - \Delta V_{R_1} = \varepsilon - I_1 R_1 = 0 \Rightarrow \varepsilon = I_1 R_1 \Rightarrow I_1 = \frac{\varepsilon}{R_1}$$

$$\sum_{Loop\ C} \Delta V = \Delta V_t - \Delta V_{R_2} = \varepsilon - I_2 R_2 = 0 \Rightarrow \varepsilon = I_2 R_2 \Rightarrow I_2 = \frac{\varepsilon}{R_2}$$

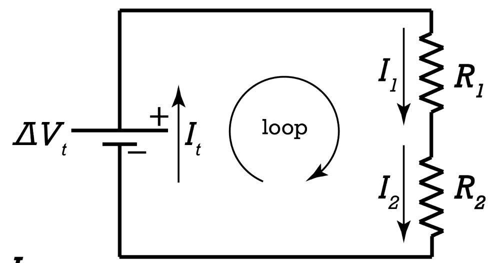
$$\sum_{Loop\ B} \Delta V = \Delta V_{R_1} - \Delta V_{R_2} = I_1 R_1 - I_2 R_2 = 0 \Rightarrow I_1 R_1 = I_2 R_2$$

But notice the third equation is actually just a combination of the previous two: $\varepsilon = I_1 R = I_2 R_2$

Flipping Physics Lecture Notes:

Resistors in Series and Parallel Equation Derivations

https://www.flippingphysics.com/series-parallel-resistors.html



We start with a circuit with a battery and two resisters in series. Because a positive charge would be repelled from the positive terminal of the battery and attracted to the negative terminal of the battery, the current in this circuit is clockwise or up through the battery and down through each resistor. Let's label those currents as the terminal current through the battery and current 1 and current 2 through their respective resistors. Hopefully you recognize that each charge on the wire has to go through all three of these circuit elements, therefore all of these currents are equal: $I_t = I_1 = I_2$

According to Kirchhoff's Loop Rule, a charge moving all the way around a loop in a circuit must end with the same electric potential energy it started with, therefore, the electric potential difference all the way around a loop is equal to zero. If we define the loop in a clockwise direction in our circuit, Kirchhoff's Loop Rule looks like this:

$$\Delta V_{loop} = 0 = \Delta V_t - \Delta V_1 + \Delta V_2 \Rightarrow \Delta V_t = \Delta V_1 + \Delta V$$

Because electric potential difference equals current times resistance, we can substitute current times resistance for each of the electric potential differences. For the battery, the terminal voltage equals the current at the terminals of the battery times the equivalent resistance of the electrical load. The electrical load in this circuit is the two resistors in series.

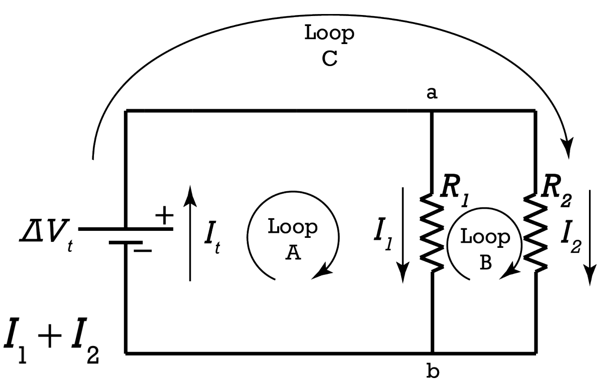
$$\Delta V = IR$$

$$\Rightarrow I_t R_{eq} = I_1 R_1 + I_2 R_2$$

All the currents are the same, so they cancel out and the equivalent resistance for two resistors in series equals the sum of those two resistors. We could perform this experiment with as many resistors in series as we wanted to, and the equivalent resistance would always be the sum of the resistances.

$$\Rightarrow R_{eq} = R_1 + R_2 \Rightarrow \boxed{R_{series} = R_1 + R_2 + R_3 + \ldots}$$

Note: In series circuit elements currents are the same and electric potential differences add.



Now let's do a circuit with a battery and two resistors in parallel. Again, the current directions are up through the battery and down through each of the resistors. There are two junctions in the circuit; junction a and junction b. Using Kirchhoff's Junction Rule, which is a result of conservation of charge, the fact that every charge that goes into the junction must come out of the junction, for junction a we get:

$$\sum I_{in} = \sum I_{out} \Rightarrow I_t = I_1 + I_2$$

We can define three loops for Kirchhoff's Loop Rule as shown in the figure. Remembering that electric potential goes up as you go from the negative to the positive terminals of the battery and, as you go in the direction of current across a resistor, the electric potential goes down; these are the equations for loop A and loop C:

$$\Delta V_{Loop\ A} = 0 = \Delta V_t - \Delta V_1 \Rightarrow \Delta V_t = \Delta V_1$$

$$\Delta V_{Loop\ C} = 0 = \Delta V_t - \Delta V_2 \Rightarrow \Delta V_t = \Delta V_2$$

Notice then that all of the electric potential differences in this circuit are the same. And because electric potential difference equals current times resistance, current equals electric potential difference divided by

$$\Rightarrow \Delta V_t = \Delta V_1 = \Delta V_2 \ \& \ \Delta V = IR \Rightarrow I = \frac{\Delta V}{R}$$

resistance. Therefore, we can combine these equations to solve for the equivalent resistance of the two resistors in parallel:

$$I_t = I_1 + I_2 \Rightarrow \frac{\Delta V_t}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

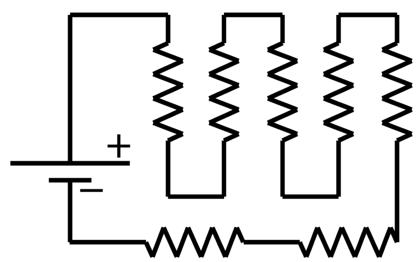
We could perform this experiment with as many resistors in parallel as we want, and the equivalent resistance will always be equal to the inverse of the sum of the inverses of all the resistors in parallel.

$$R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\right)^{-1}$$

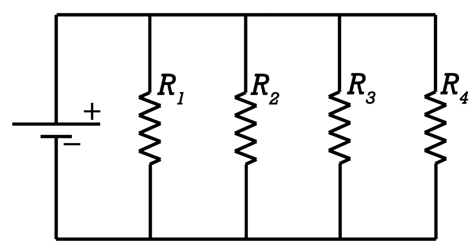
Note: In parallel circuit elements currents add and electric potential differences are the same.

So, notice that adding resistors in series increases the net resistance of the resistors and adding resistors in parallel decreases the net resistance of the resistors. Think of it this way, by adding a resistor in series, you are adding resistance to the path the charges to go through which, no matter how small that resistance is, still increases the resistance. When you add a resistor in parallel, you are adding an additional path for the charges have to go through and therefore, no matter how large the resistor is which you are adding in parallel, the addition of another pathway for the charges to travel decreases the overall resistance.

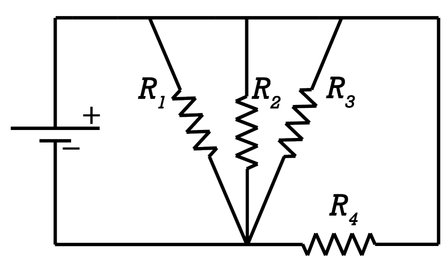
It may seem pretty obvious in simple circuits like the ones we just went through; however, it is important to identify when circuit elements are in series, parallel, or neither. Let's start with series. If every charge that goes through one element also has to go through the other element, those two circuit elements are in series. For example, all of the resistors in the following circuit are in series. This is because every charge in the circuit has to pass through every one of the resistors in the circuit.



Circuit elements which are in parallel all have the same electric potential difference. For example, all the resistors in the following circuit have the same potential at the top and bottom of the resistor, so their electric potential differences are the same. Another way to look at this is that if the charges are split between resistors and then all the charges come back together again, the resistors are in parallel.

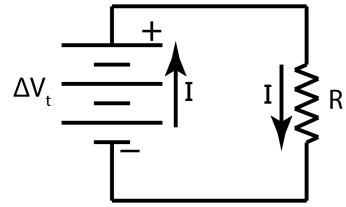


If you look at the next circuit, it appears to be different, however, the top of each resistor (or right side in the case of resistor 4) are all at the same electric potential and the bottom of each resistor (or left side in the case of resistor 4) are at the same electric potential, therefore, all four of these resistors are still in parallel. In fact, this is the same circuit as before, it has simply been drawn slightly differently.

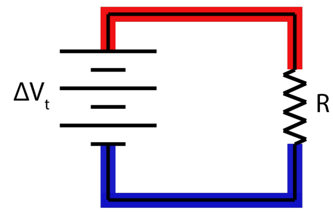


Flipping Physics Lecture Notes:

Basic Series and Parallel Resistor Circuit Demos and Animations
https://www.flippingphysics.com/series-parallel-resistors-basic.html



Before we analyze resistors in series and in parallel, let's get our bearings using a circuit with a battery and one resistor. First off, realize the current will go through the battery from the negative to the positive terminals of the battery. The current will therefore be up through the battery and down through the resistor. Because there is only one loop in the circuit, there is only one current, which we will simply label I.



Because there are only two elements in the circuit, both elements have the same magnitude electric potential difference equal to the terminal voltage, ΔV_t. You can see this by using the electric potential color-coding technique. Starting at the positive terminal of the power supply we draw red on the wire until we come to another circuit element. Everything in red is at the same electric potential. Starting at the negative terminal of the power supply we draw blue on the wire until we come to another circuit element. Everything in blue is at the same electric potential. Both the power supply and the resistor have an electric potential difference which is between red and blue, so both the power supply and the resistor have the same magnitude electric potential difference.

Now we can solve for the current in the circuit: $\Delta V = IR \Rightarrow I = \frac{\Delta V_t}{R} = \frac{5.0V}{5.0\Omega} = 1.0A = 1.0\frac{C}{s}$

With a real example of a 5.0 volt power supply and a 5.0 Ω resistor, we should expect to observe 1.0 amps of current through the circuit or 6.2 million million million electrons every second.

$$q = ne \Rightarrow n = \frac{q}{e} = \frac{1C}{1.6\times10^{-19}\frac{C}{electron}} = 6.25\times10^{18} \approx 6.2\times10^{18} electrons$$

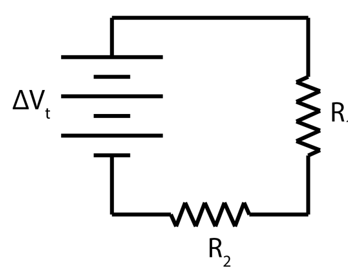
Unfortunately, we only get 0.97 Amps through the power supply. Measuring the electric potential difference across the resistor shows that the resistor actually has 4.9 volts across it. Likely this means two things. One is that we have real wires which have resistance instead of ideal wires which do not have resistance. And the power supply is actually delivering slightly less than what it is displaying.

We can also determine the rate at which the resistor is converting electric potential energy to heat:

$$P = I\Delta V = (1)(5) = 5.0 watts$$

This means our resistor is converting 5.0 joules of energy to heat every second, which is why the resistor is getting HOT!

Please watch the video to see the animations relating charged particle location and electric potential energy. I could try to describe it here, but it's an animation. You should watch it instead.

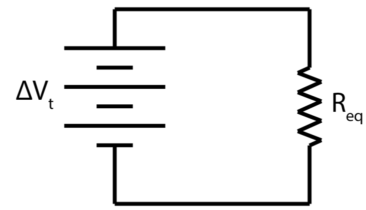


Next, let's analyze two resistors in series with a 5.0 V power supply. Both resistors have a resistance of 5.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.

First, we know these are two resistors in series because there are no other paths for charges to follow. Every charge which goes through resistor one must eventually go through resistor two. Because the two resistors are in series, their resistances add to get their equivalent resistance:

In Series: $R_{eq} = R_1 + R_2 = R + R = 2R = (2)(5) = 10\Omega$



Now we can replace the two resistors in our circuit with one equivalent resistor, R_{eq}. Notice how this is the same setup as our original circuit, one resistor and one power supply, therefore:

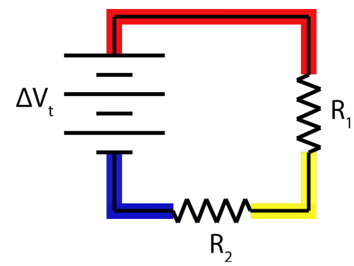
$$\Delta V_t = IR_{eq} \Rightarrow I = \frac{\Delta V_t}{R_{eq}} = \frac{5}{10} = 0.50A$$

The observed value for the current in the circuit is 0.49 A, which is to be expected based on our observed value for the first circuit. We have answered part (a), the accepted current through all circuit elements equals 0.50 A.

Because we know the current through and resistance of each resistor, we can now determine (b), the electric potential difference across each resistor, 2.5 V.

$$\Delta V_1 = I_1 R_1 = (0.5)(5) = 2.5V \text{ \& } \Delta V_2 = I_2 R_2 = (0.5)(5) = 2.5V$$

Because both resistors have the same resistance and current through them, the electric potential differences across each resistor are equal.



Also, I want to use the electric potential color-coding technique to show that the electric potential differences across the two resistors add up to the terminal voltage. The highest electric potential wire is in red, the lowest electric potential wire is in blue, and the wire with a middle value electric potential is in yellow. Therefore, the terminal voltage is from blue to red, and across the resistors the electric potential goes from red to yellow plus yellow to blue. This means that:

$$\Delta V_t = \Delta V_1 + \Delta V_2 \Rightarrow 2.5 + 2.5 = 5.0V$$

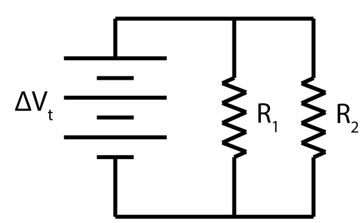
Again, we showed this because 2.5 volts plus 2.5 volts equals 5 volts.

For part (c), the power dissipated by each resistor is:

$$P_1 = P_2 = \frac{\Delta V_1^2}{R_1} = \frac{\Delta V_2^2}{R_2} = \frac{(2.5)^2}{5} = 1.25 \approx 1.2 watts$$

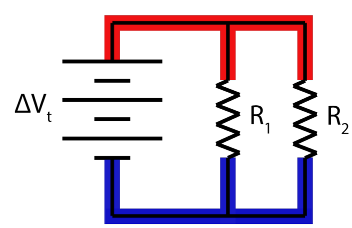
Therefore, the total power dissipated by both resistors is 2.5 watts; which is the same as the power delivered by the power supply:

$$P_{both\ resistors} = P_1 + P_2 = 1.25 + 1.25 = 2.5 watts \text{ \& } P_{power\ source} = I\Delta V_t = (0.5)(5) = 2.5 watts$$



Next, let's analyze two resistors in parallel with a 5.0 V power supply. Both resistors have a resistance of 10.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.



Let's start by color-coding the circuit to show that the two resistors are in fact in parallel. You can see all electric potential differences are the same, therefore the resistors are in parallel.

Part (b) $\Delta V_t = \Delta V_1 = \Delta V_2 = 5.0V$

We can determine the equivalent resistance of the two resistors by using the equation for resistors in parallel:

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = \left(\frac{2}{10}\right)^{-1} = \left(\frac{1}{5}\right)^{-1} = 5.0\Omega$$

The equivalent resistance for two 10.0 Ω resistors in parallel is 5.0 Ω. In other words, we can replace the two 10.0 Ω resistors with one 5.0 Ω resistor and the current through the circuit should remain unchanged.

But notice, this is exactly the same as the circuit we started with. Therefore, the current delivered by the power supply is 1.0 A and the power delivered by the power supply is 5.0 watts.

We know the electric potential across each circuit element is the same, so we can determine the current through each resistor:

Part (a) $$\Delta V = IR \Rightarrow I_1 = \frac{\Delta V_1}{R_1} = I_2 = \frac{\Delta V_2}{R_2} = \frac{5}{10} = 0.50A$$

And we know the currents in parallel add: $I_t = I_1 + I_2 = 0.5 + 0.5 = 1.0A$
Which confirms our previous calculation for the current thorugh the battery.
And now we can calculate the power dissipated by each resistor:

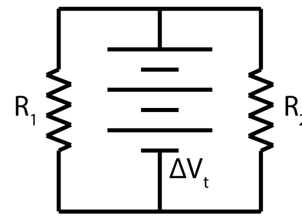
Part (c) $$P = I^2R \Rightarrow P_1 = P_2 = I_1^{\,2}R_1 = I_2^{\,2}R_2 = (0.5)^2(10) = 2.5watts$$

This makes sense because the total power delivered by the power supply should equal the total power dissipated by the resistors:

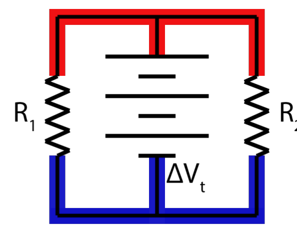
$$P_{both\ resistors} = P_1 + P_2 = 2.5 + 2.5 = 5.0watts\ \&\ P_{power\ source} = I\Delta V_t = (1)(5) = 5.0watts$$

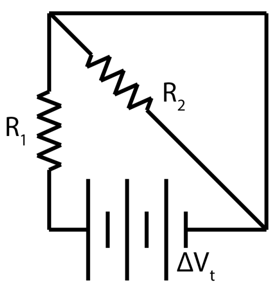
Next, let's analyze two resistors in this circuit with a 5.0 V power supply.
Both resistors have a resistance of 10.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.



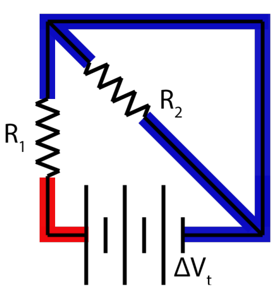
Let's start by color-coding the electric potential of the wires to determine if the resistors are in series or parallel. Hopefully you now recognize, because the electric potential color-coding is exactly the same, that this is the same circuit we just did, only drawn slightly differently. All the answers are the same.





Next, analyze two resistors in this circuit with a 5.0 V power supply. Both resistors have a resistance of 5.0 Ω. Let's determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.

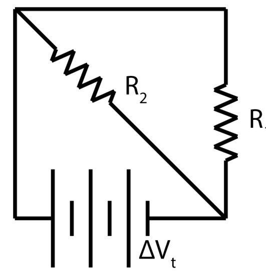


Let's start by color-coding the electric potential of the wires to determine if the resistors are in series or parallel. Notice the electric potential difference across resistor 2 is zero, this means no current will flow across resistor 2. This is because the wire in the upper right corner completely shorts resistor 2 out of the circuit. Because that wire has zero resistance, all charges will flow along that wire and none will flow through resistor 2. In other words, this circuit behaves the same as our very first circuit, other than resistor 2 having no current, no electric potential difference, and therefore no power dissipated. Therefore, our answers are:

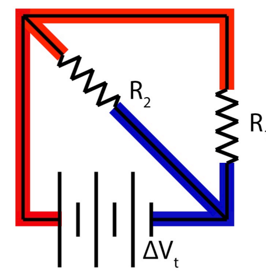
a) I_1 = 1.0 A and I_2 = 0
b) ΔV_1 = 5.0 V and ΔV_2 = 0
c) P_1 = 5.0 Watts and P_2 = 0

Okay, one last example. Analyze two resistors in this circuit with a 5.0 V power supply. Both resistors have a resistance of 10.0 Ω. Let’s determine:

a) Current through each circuit element.
b) Electric potential difference across each resistor.
c) Power dissipated by each resistor.



Again, we start by color-coding the electrical potential differences of the circuit.



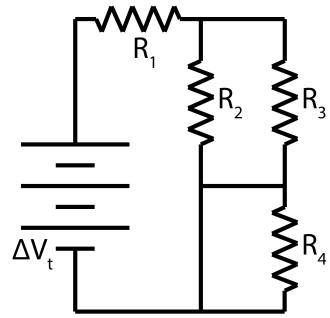
Both resistors have the same electric potential difference. This is two resistors in parallel with a power supply. This is actually the same as two circuits we already analyzed.

Please, be careful to look at circuits carefully before throwing equations at them. You need to first determine which circuit elements are in series and parallel, then you can start using equations.

Flipping Physics Lecture Notes:

Resistor Circuit Example
http://www.flippingphysics.com/resistor-circuit-example.html

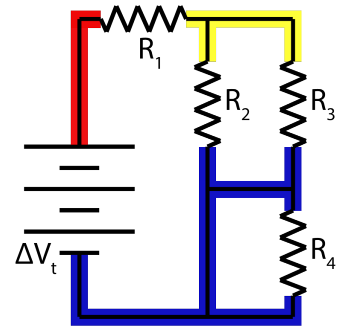
The circuit shown has four identical 5.0 Ω resistors and a 5.0 V battery. What is the current through, electric potential difference across, and power dissipated by resistor 4?



We know $\Delta V_4 = I_4 R_4$. Because we know the resistance of resistor 4, if we know either the current through or electric potential difference across resistor 4, we can determine the other unknown.

We also know $P_4 = I_4 \Delta V_4 = I_4^2 R_4 = \frac{\Delta V_4^2}{R_4}$. So again, if we know either current through or electric potential diference across resistor 4, then we can determine the power dissipated by resistor 4.

Let's do the electric potential difference color coding technique to help us determine the electric potential difference across resistor 4.



From this we can see that the electric potential difference across resistor 4 is zero, therefore the current through resistor 4 is also zero, and the power dissipated by resistor 4 is zero.

$$\Delta V_4 = 0$$

$$\Delta V_4 = I_4 R_4 \Rightarrow I_4 = \frac{\Delta V_4}{R_4} = \frac{0}{R_4} = 0$$

$$P_4 = I_4^2 R_4 = (0)^2 (0) = 0$$

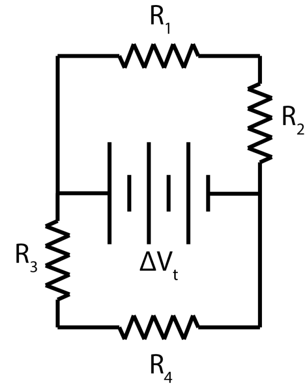
Resistor 4 is short circuited in this circuit.

Also, please enjoy the animation of the charges moving through the circuit in the video. It really helps with understanding how the charges are moving in the circuit.

Flipping Physics Lecture Notes:

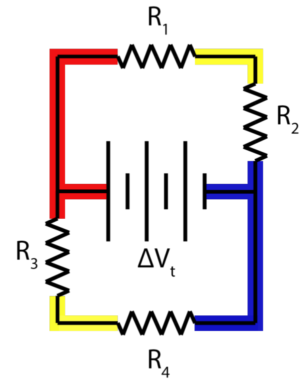
Intermediate Series and Parallel Resistor Circuit

https://www.flippingphysics.com/series-parallel-resistors-intermediate.html



The circuit shown has four 5.0 Ω resistors and a 5.0 V power source. Determine …

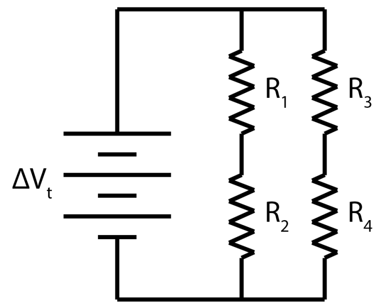
a) The equivalent resistance of all four resistors.
b) Current through each circuit element.
c) Electric potential difference across each resistor.
d) Power dissipated by each resistor.



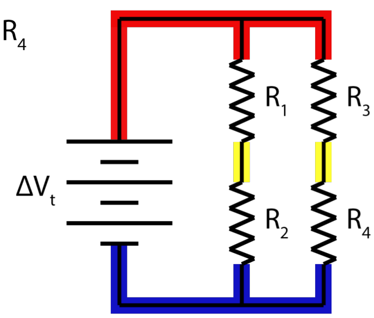
Start by color-coding the electric potential along all the wires of the circuit. From the color-coded circuit diagram, you can now see several things:

1) R_1 and R_2 are in series.
2) R_3 and R_4 are in series.
3) Those two sets of series resistors are in parallel.

In other words, we can redraw the circuit diagram like this:



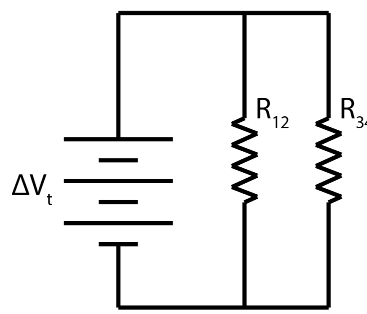
In fact, it is even easier to see they are the same circuit when we color-code the electric potential of the circuit:



I do want to point out that the only reason we know the wires between R_1 and R_2, and R_3 and R_4 are all at the same electric potential, and therefore the same color yellow, is because all four resistors have equal resistance. If they did not have the same number of ohms, then the electric potential difference across each resistor would be different and the electric potential between the resistors would not be the same.

Now we can begin solving the problem. Let's start by determining the equivalent resistance of the resistor pairs.

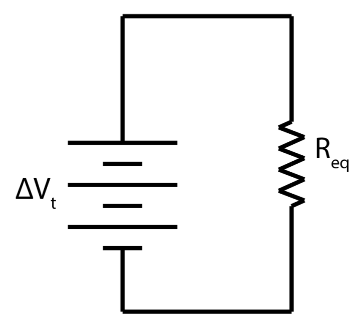
$$R_{12} = R_1 + R_2 = 5 + 5 = 10\Omega \quad \& \quad R_{34} = R_3 + R_4 = 5 + 5 = 10\Omega$$



We can replace R_1 and R_2 with equivalent resistor R_{12}.
We can replace R_3 and R_4 with equivalent resistor R_{34}.

And now we have two resistors in parallel and can deterimen the equivalent resistance of those two resistors:

$$R_{eq} = \left(\frac{1}{R_{12}} + \frac{1}{R_{34}}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = \left(\frac{2}{10}\right)^{-1} = \left(\frac{1}{5}\right)^{-1} = 5.0\Omega$$



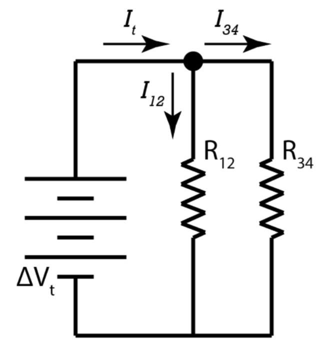
Part (a): We can replace all four resistors with one equivalent resistance of, R, or **5.0 Ω.**

Now we can determine the current delivered by the power source to the equivalent resistor:

$$\Delta V = IR \Rightarrow I_t = \frac{\Delta V_t}{R_{eq}} = \frac{5}{5} = \boxed{1.0A}$$

So, the power source is delivering 1 coulomb of charge every second to the circuit. And we know the electric potential difference across the power source is the same magnitude as the electric potential difference across the two resistors R_{12} and R_{34}.

$$\Delta V_t = \Delta V_{12} = \Delta V_{34} = 5.0V$$



Therefore, we can determine the current through R_{12} and R_{34}.

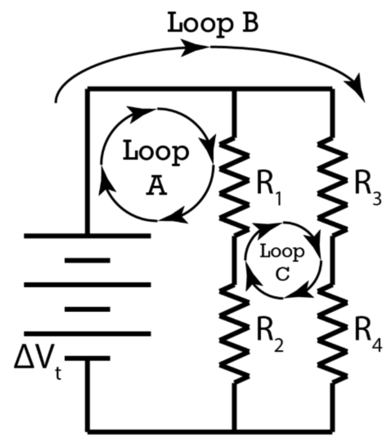
(b) $$I_{12} = \frac{\Delta V_{12}}{R_{12}} = \frac{5}{10} = \boxed{0.50A = I_1 = I_2} \ \& \ I_{34} = \frac{\Delta V_{34}}{R_{34}} = \frac{5}{10} = \boxed{0.50A = I_3 = I_4}$$

These currents make sense because we know Kirchhoff's Junction Rule states that the current going into a junction equals the current going out of a junction:

$$I_t = I_{12} + I_{34} = 0.5A + 0.5A = 1A$$

Now, we can determine the electric potential difference across each resistor:

(c) $$\boxed{\Delta V_1 = I_1 R_1 = (0.5)(5) = 2.5V = \Delta V_2 = \Delta V_3 = \Delta V_4}$$



This makes sense because we know Kirchhoff's Loop Rule states that the electric potential difference around any loop equals zero:

$$\Delta V_{loop\ A} = 0 = \Delta V_t - \Delta V_1 - \Delta V_2 = 5 - 2.5 - 2.5 = 0$$

$$\Delta V_{loop\ B} = 0 = \Delta V_t - \Delta V_3 - \Delta V_4 = 5 - 2.5 - 2.5 = 0$$

$$\Delta V_{loop\ c} = 0 = \Delta V_1 + \Delta V_2 - \Delta V_3 - \Delta V_4 = 2.5 + 2.5 - 2.5 - 2.5 = 0$$

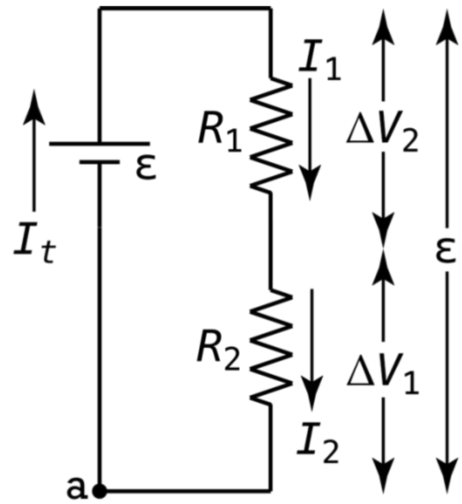
And lastly, we can determine the rate at which energy is dissipated in each circuit element:

$$\boxed{\begin{aligned} P_t &= I_t \Delta V_t = (1)(5) = 5.0 watts \\ P_1 &= I_1^2 R_1 = (0.5)^2 (5) = 1.25 watts \approx 1.2 watts = P_2 = P_3 = P_4 \end{aligned}}$$

Which makes sense because the power added to the circuit from the power source needs to equal the power dissipated by the circuit:

$$P_t = P_1 + P_2 + P_3 + P_4 = 1.25 + 1.25 + 1.25 + 1.25 = 5.0 watts$$

Flipping Physics Lecture Notes:
Resistor Series and Parallel Circuits
http://www.flippingphysics.com/resistor-series-parallel.html



When an anthropomorphic[1] charge has no choice but to go through two circuit elements, those two circuit elements are in *series*. For example, a charge which goes through resistor 1 has no choice but to also go through resistor 2. There is no other path for the anthropomorphic charge to choose.

The currents through the three circuit elements must all be equal:

$$I_t = I_1 = I_2$$

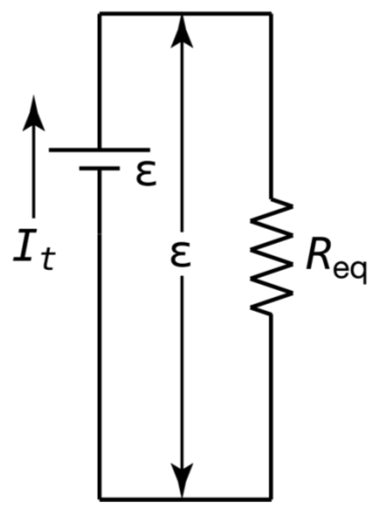
The "t" in the subscript refers to the current at the terminals of the battery which is the current delivered by the battery to the circuit.

The electric potential difference across the battery equals the summation of the electric potential difference across the two resistors:

$$\Delta V_{\text{bottom wire}\rightarrow\text{top wire}} = \varepsilon = \Delta V_1 + \Delta V_2$$

(If you'd prefer to look at this in terms of the electric potential difference around the loop in the circuit:)

$$\Delta V_{\text{loop}} = V_f - V_i = V_a - V_a = 0 = \varepsilon - \Delta V_1 - \Delta V_2 \Rightarrow \varepsilon = \Delta V_1 + \Delta V_2$$



We know Ohm's law: $\Delta V = IR$; therefore, …

$$\Rightarrow \varepsilon = I_t R_{\text{eq}} = I_1 R_1 + I_2 R_2$$

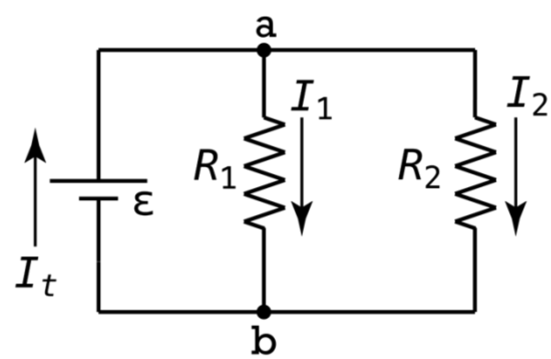
$$\Rightarrow R_{\text{eq}} = R_1 + R_2$$

The "eq" in the subscript means equivalent. In other words, R_{eq} is one resistor with the equivalent resistance of the two resistors.

Therefore, the equation for the equivalent resistance of n resistors in series is:

$$R_{\text{eq series}} = \sum_n R_n = R_1 + R_2 + \ldots$$

When an anthropomorphic charge has the choice between two circuit elements and then the paths through those two circuit elements reconverge without going through another circuit element, the two circuit elements are in *parallel*.



When circuit elements are in parallel, their electric potential differences are equal:

$$\varepsilon = \Delta V_1 = \Delta V_2$$

Note the junctions at points a and b. Due to conservation of charge, the net current going into a junction equals the net current coming out of a junction. For junction a:

[1] *Anthropomorphism:* Giving human characteristics or behaviors to non-human objects.

$$I_{\text{in}} = I_{\text{out}} \Rightarrow I_t = I_1 + I_2$$

We can then use Ohm's law:

$$\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} \Rightarrow \frac{\varepsilon}{R_{\text{eq}}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

And we get the equivalent resistance for the two resistors in parallel:

$$\Rightarrow R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

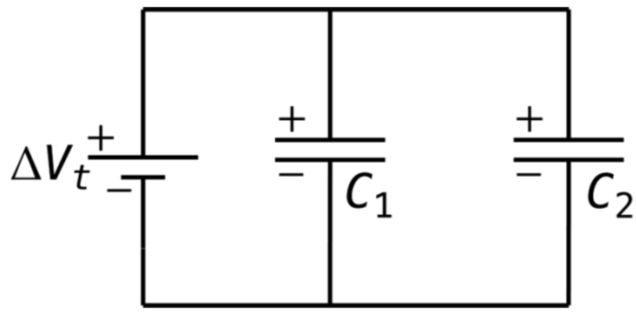
And the equivalent resistance for n resistors in parallel:

$$\Rightarrow R_{\text{eq parallel}} = \left(\sum_n \frac{1}{R_n}\right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)^{-1}$$

When we add a resistor in series, the equivalent resistance increases.
When we add a resistor in parallel, the equivalent resistance decreases.

Let's look at two capacitors in parallel:
We know the electric potential differences are all equal.

$$\Delta V_t = \Delta V_1 = \Delta V_2$$



Because the charges moved to the top plates of the capacitors need to go to either capacitor 1 or capacitor 2, the charge moved by the battery to the plates of the capacitors equals the sum of the charges on the capacitors:

$$Q_t = Q_1 + Q_2$$

We can then use the definition of capacitance:

$$C = \frac{Q}{\Delta V} \Rightarrow Q = C\Delta V$$

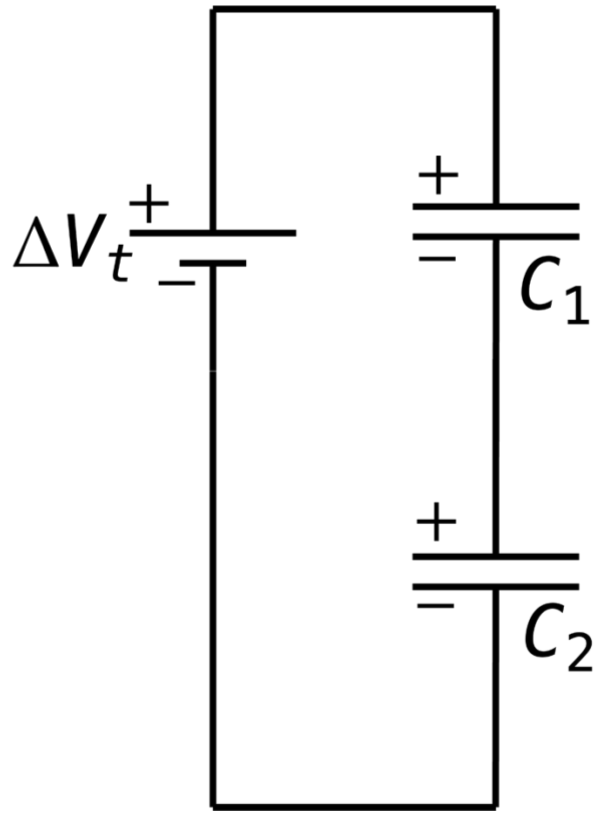
To derive the equivalent capacitance of two capacitors in parallel:

$$\Rightarrow C_{\text{eq}}\Delta V_t = C_1\Delta V_1 + C_2\Delta V_2 \Rightarrow C_{\text{eq}} = C_1 + C_2$$

And the equivalent capacitance of n capacitors in parallel:

$$\Rightarrow C_{\text{eq parallel}} = \sum_n C_n = C_1 + C_2 + \dots$$

And we can now look at two capacitors in series:



The electric potential is the same as resistors in series:

$$\Delta V_t = \Delta V_1 + \Delta V_2$$

And the charges on each capacitor are equal:

$$Q_t = Q_1 = Q_2$$

This is because the magnitude of the charge moved by the battery to the top plate of capacitor 1 and the bottom plate of capacitor 2 are equal in magnitude. And those plates polarize the charges on the wire between the two capacitors and the bottom of capacitor 1 and the top of capacitor 2. This causes all four plates of the two capacitors to have equal magnitude charges. This is an illustration of conservation of charge.

And we can solve for electric potential difference in terms of capacitance and charge:

$$Q = C\Delta V \Rightarrow \Delta V = \frac{Q}{C}$$

And use that to solve for the equivalent capacitance of two capacitors:

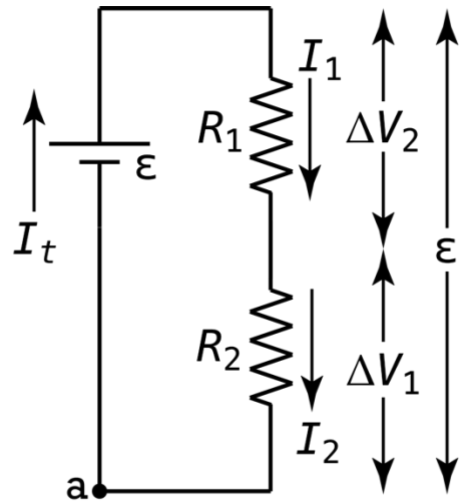
$$\Rightarrow \frac{Q_t}{C_{\text{eq}}} = \frac{Q_1}{C_2} + \frac{Q_2}{C_2} \Rightarrow \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

And the equivalent capacitance of n capacitors:

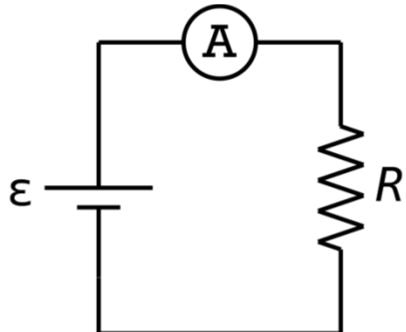
$$\Rightarrow C_{\text{eq series}} = \left(\sum_n \frac{1}{C_n}\right)^{-1} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)^{-1}$$

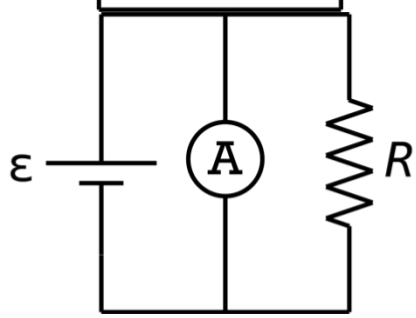
Notice the equations for resistors and capacitors are reversed. That means that:
When we add a capacitor in parallel, the equivalent capacitance increases.
When we add a capacitor in series, the equivalent capacitance decreases.

Flipping Physics Lecture Notes:
Ammeter and Voltmeter - Where Do They Go?
http://www.flippingphysics.com/ammeter-voltmeter.html



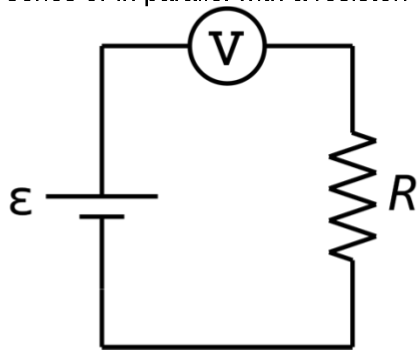
Let's discuss how to use the tools which measure current and electric potential difference. Starting with the ammeter which measures current or amperes. We need to decide if an ammeter needs to be put in series or parallel with the circuit element it is meant to measure the current through. So, let's look at what happens when we attempt to measure the current through a resistor using an ammeter in series and in parallel with a resistor:

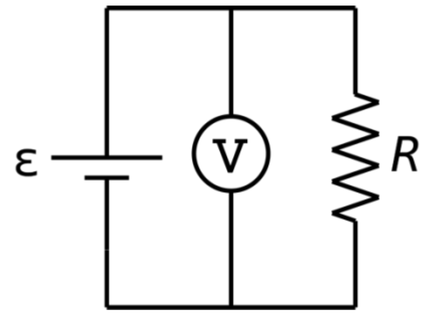




Hopefully you recognize that placing an ammeter in parallel with a resistor will not measure the current through the resistor because the current through the ammeter and the resistor are not the same. Therefore, an ammeter needs to be placed in series with a circuit element to measure the current through that circuit element. Also, the resistance of an ammeter needs to be *very* small. In the above example, if the resistance of the ammeter is not *very* small, it will increase the equivalent resistance of the circuit and decrease the current through the resistor you are trying to measure the current through. Unless otherwise indicated, ammeters in this class are considered to have zero resistance.

And now let's attempt to measure the electric potential difference across a resistor using a voltmeter either in series or in parallel with a resistor:





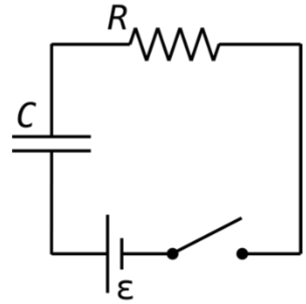
Hopefully you recognize that placing a voltmeter in series with a resistor will not measure the electric potential difference across the resistor because the voltage across the voltmeter and the resistor are not the same. Therefore, a voltmeter needs to be placed in parallel with a circuit element to measure the voltage across that circuit element. Also, the resistance of a voltmeter needs to be *very* large. In the above example, if the resistance of the voltmeter is not *very* large, it will decrease the equivalent resistance of the circuit, increase the current delivered by the battery, and change the overall properties of the circuit. Unless otherwise indicated, voltmeters in this class are considered to have infinite resistance.

To review:

• Ammeters:	• Voltmeters:
○ Measure current	○ Measure electric potential difference
○ Placed in *series* with the circuit element	○ Placed in *parallel* with circuit element
○ Have nearly *zero* resistance*	○ Have nearly *infinite* resistance*

* You may see this called impedance in product literature for Voltmeters and Ammeters, due to the fact that there is more to the behavior of these devices than just resistance. For the purpose of this class and the AP Physics C Electricity and Magnetism exam, it will be called resistance unless otherwise noted.

Flipping Physics Lecture Notes:
RC Circuit Basics
http://www.flippingphysics.com/rc-circuit.html

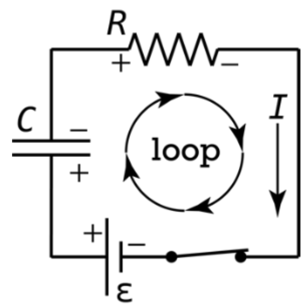


Up until this point we have assumed all changes in electric current, electric potential difference, and charge on capacitor plates were instantaneous. Today, we put a resistor and a capacitor together and learn how those variables change as a function of time. This is called an RC circuit. We start with a circuit composed of an uncharged capacitor, a resistor, a battery, and an open switch, all connected in series.

At time initial, $t_i = 0$, we close the switch.
We are *charging a capacitor through a resistor.*

Let's start by adding a loop in the direction of current flow in the circuit. Then use Kirchhoff's Loop Rule starting in the lower right-hand corner of the circuit:

$$\Delta V_{loop} = 0 = +\varepsilon - \Delta V_C - \Delta V_R$$



We can use the definition of capacitance to solve for the electric potential difference across the capacitor:

$$C = \frac{Q}{\Delta V} \Rightarrow \Delta V_C = \frac{Q}{C}$$

And we know Ohm's law: $\Delta V_R = IR$

$$\Rightarrow \Delta V_{loop} = 0 = \varepsilon - \frac{q}{C} - iR$$

Notice we are using lowercase "q" for charge because the charge is changing as a function of time. I wish we had a similar notation for current, I, however, if we used lowercase "i", I am sure it would be more confusing. So, please realize charge, q, and current, I, are both changing as a function of time in the above equation.

Now let's look at limits, starting with $t_i = 0$:

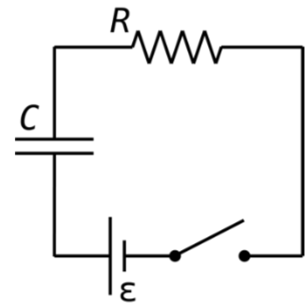
- The initial charge on the capacitor is zero: $q_i = 0$
- This means the initial electric potential difference across the capacitor is also zero:
$$\Rightarrow \Delta V_{C_i} = \frac{q}{C} = \frac{0}{C} = 0$$
- We can now use the loop equation to solve for the initial current through the circuit.
$$\Rightarrow 0 = \varepsilon - \frac{0}{C} - i_{initial}R \Rightarrow i_{initial}R = \varepsilon \Rightarrow i_{initial} = \frac{\varepsilon}{R} = i_{max}$$
- Because the charge on the capacitor will increase as a function of time, electric potential difference across the capacitor will also increase. This means the current in the circuit will decrease. In other words, the initial current in the circuit is also the maximum current.

And now the limit of "after a long time" or the $t_f \approx \infty$.

- The final current in the circuit is zero: $i_{final} \approx 0$

- This means the final electric potential difference across the resistor is also zero:
  $$\Rightarrow \Delta V_{R_f} = i_{\text{final}} R = (0) R = 0$$
- And we can use the loop equation to solve for the final charge on the capacitor:
  $$\Rightarrow 0 = \varepsilon - \frac{q_f}{C} - (0) R \Rightarrow q_f = \varepsilon C = q_{\max}$$
- Because we know the charge has been increasing this whole time, we know this is the maximum charge on the capacitor.

We already determine the limits of charge and current when charging a capacitor in an RC circuit.



Now, let's figure out what happens between $t_i = 0$ and $t_f \approx \infty$, however, before we do, I want to point out that AP Physics C: Electricity and Magnetism students are responsible for knowing how to derive these equations. So, yes, you do need to understand these derivations and be able to do them on your own. And here we go … starting with our Kirchhoff's Loop Rule equation:

$$0 = \varepsilon - \frac{q}{C} - iR \Rightarrow iR = \varepsilon - \frac{q}{C} \Rightarrow i = \frac{\varepsilon}{R} - \frac{q}{RC} \Rightarrow \frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC}$$

$$\Rightarrow \frac{dq}{dt} = \frac{1}{RC}(\varepsilon C - q) \Rightarrow \frac{dq}{dt} = -\frac{1}{RC}(q - \varepsilon C)$$

(The above step is the one I find students forget most often. Yes, factor out a negative one on the right-hand side of the equation. Write it down. Remember it. No, it is not an obvious step you need to take.)

$$\Rightarrow \left(\frac{1}{q - \varepsilon C}\right) dq = -\frac{1}{RC} dt \Rightarrow \int_0^q \left(\frac{1}{q - \varepsilon C}\right) dq = -\int_0^t \left(\frac{1}{RC}\right) dt = -\frac{1}{RC}\int_0^t dt$$

$$\Rightarrow [\ln(q - \varepsilon C)]_0^q = [\ln(q - \varepsilon C)] - [\ln(0 - \varepsilon C)] = \ln\left(\frac{q - \varepsilon C}{-\varepsilon C}\right) = -\frac{t}{RC}$$

$$\Rightarrow e^{\left[\ln\left(\frac{q - \varepsilon C}{-\varepsilon C}\right)\right]} = e^{-\frac{t}{RC}} \Rightarrow \frac{q - \varepsilon C}{-\varepsilon C} = e^{-\frac{t}{RC}} \Rightarrow q - \varepsilon C = (-\varepsilon C)\, e^{-\frac{t}{RC}}$$

$$\Rightarrow q = \varepsilon C - \varepsilon C e^{-\frac{t}{RC}} \Rightarrow q(t) = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow q(t) = q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

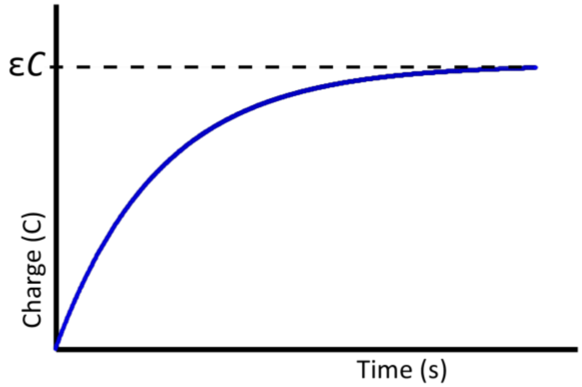
Applicable known equations:

$$\ln x - \ln y = \ln\left(\frac{x}{y}\right) \ \ \& \ \ e^{\ln x} = x$$

Notice this equation fits our limits for charge:

$$q(0) = \varepsilon C\left(1 - e^{-\frac{0}{RC}}\right) = \varepsilon C\left(1 - e^{0}\right) = \varepsilon C(1 - 1) = 0$$

$$q(\infty) = \varepsilon C\left(1 - e^{-\frac{\infty}{RC}}\right) = \varepsilon C\left(1 - e^{-\infty}\right) = \varepsilon C(1 - 0) = \varepsilon C = q_{max}$$



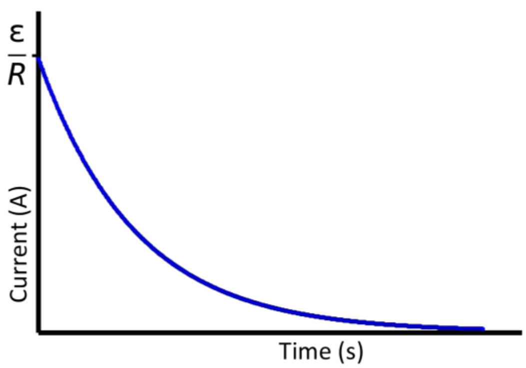
And we can derive the current through the circuit as a function of time:

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(\varepsilon C - \varepsilon C e^{-\frac{t}{RC}}\right) = \frac{d}{dt}\left(-\varepsilon C e^{-\frac{t}{RC}}\right) = -\varepsilon C \frac{d}{dt}\left(e^{-\frac{t}{RC}}\right) = -\varepsilon C\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}$$

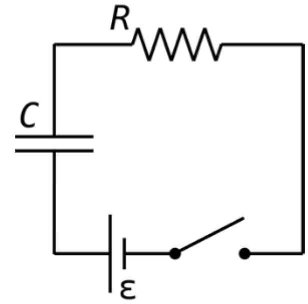
$$\Rightarrow i(t) = \left(\frac{\varepsilon}{R}\right)e^{-\frac{t}{RC}} \Rightarrow i(t) = i_{\max} e^{-\frac{t}{RC}}$$

Again, this fits our limits for current:

$$\Rightarrow i(0) = \left(\frac{\varepsilon}{R}\right)e^{-\frac{0}{RC}} = \frac{\varepsilon}{R} = i_{\max} \quad \& \quad \Rightarrow i(\infty) = \left(\frac{\varepsilon}{R}\right)e^{-\frac{\infty}{RC}} = 0$$



Flipping Physics Lecture Notes:
RC Circuit Time Constant
http://www.flippingphysics.com/rc-circuit-time-constat.html



We have already determined the equations for charge and current as functions of time while charging a capacitor in an RC circuit.

Now we get to talk about the time constant!
In the equations for charge and current as functions of time, there appears this expression: $e^{-\frac{t}{RC}}$

The time constant equals whatever appears in the denominator of that fraction. In other words, for an RC circuit, the time constant equals resistance times capacitance. The symbol for the time constant is the lowercase Greek letter tau, τ: $\tau = RC$

Before we discuss further what the times constant is, let's determine its units:

$$\tau = RC \Rightarrow \Omega F = \left(\frac{V}{A}\right)\left(\frac{C}{V}\right) = \frac{C}{A} = \frac{C}{\frac{C}{s}} = \frac{1}{\frac{1}{s}} = s$$

$$R = \frac{\Delta V}{I} \Rightarrow \Omega = \frac{V}{A} \;\&\; C = \frac{Q}{\Delta V} \Rightarrow F = \frac{C}{V} \;\&\; I = \frac{dq}{dt} \Rightarrow A = \frac{C}{s}$$

The units for the time constant are seconds; it is the *time* constant.

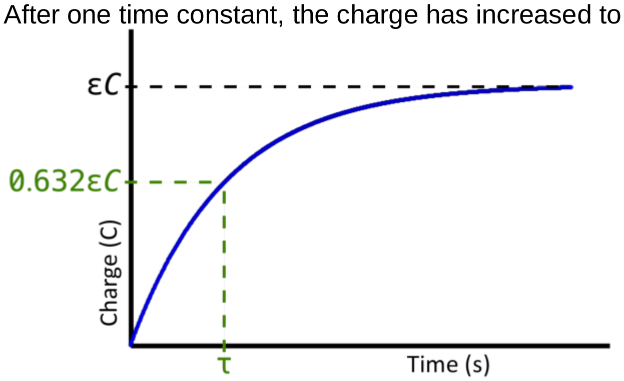
Let's replace RC with the time constant in our charge equation:

$$q(t) = q_{max}\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow q(t) = q_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$$

And determine the charge on the capacitor after one time constant:

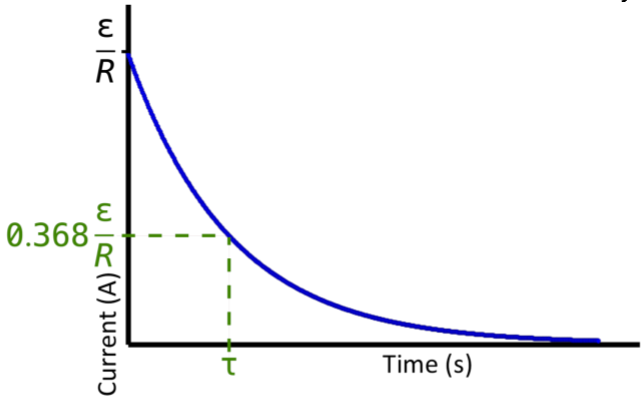
$$q(\tau) = q_{max}\left(1 - e^{-\frac{\tau}{\tau}}\right) = q_{max}\left(1 - e^{-1}\right) = q_{max}(1 - 0.368) \Rightarrow q(\tau) = 0.632q_{max}$$

After one time constant, the charge has increased to 63.2% of its maximum value.



$$i(t) = i_{max}e^{-\frac{t}{\tau}} \Rightarrow i(\tau) = i_{max}e^{-\frac{\tau}{\tau}} = i_{max}e^{-1} \Rightarrow i(\tau) = 0.368 i_{max}$$

After one time constant, the current has decreased by 63.2% from its maximum value.



The time constant is the time it takes for a change of 63.2%. If you want to know more about the time constant, I talk about it in more detail in my video *Time Constant and the Drag Force*: https://www.flippingphysics.com/drag-force-time-constant.html

There are similar equations for discharging a capacitor through a resistor which we are not going to derive today.

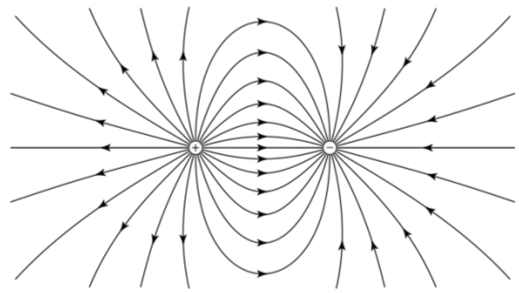
Please realize the following two calculus equations are on the AP Equation Sheet:

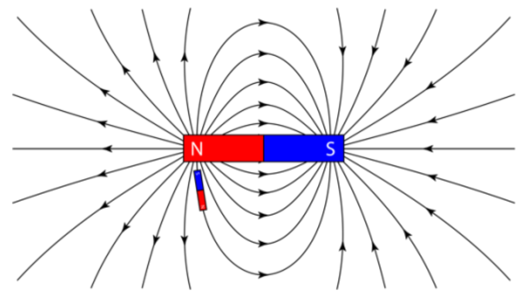
$$\int \frac{dx}{x+a} = \ln|x+a| \quad \& \quad \frac{d}{dx}(e^{ax}) = ae^{ax}$$

Flipping Physics Lecture Notes:
Magnetic Fields and Magnetic Forces on Moving Charges
http://www.flippingphysics.com/magnetic-field-force.html

Magnetic fields (or "B" fields) are be created by magnetic dipoles:

- Just like electric charges are described as positive and negative charges, magnetic poles are described as north and south poles.
  - A magnetic monopole has never been found.
    - This does not mean magnetic monopoles do not exist.
      - We cannot prove magnetic monopoles do not exist.
        - We can only say we have no evidence that they exist.
  - If a magnetic dipole is broken in half, it becomes two new magnetic dipoles.
- Like poles repel and unlike poles attract.
  - Just like the Law of Charges
- The magnetic field caused by a magnetic dipole looks remarkably like the electric field caused by an electric dipole.
  - B field lines external to the magnet, point from north pole to south pole.
    - Just like E field points from positive charge to negative charge.
  - Magnetic field lines must be closed loops.
    - Due to Gauss' law for magnetism which we will get to, eventually.
    - This means B fields inside the magnet point from the south pole to the north pole, to complete the closed loop.
  - A magnetic dipole placed in a magnetic field will align itself with the magnetic field.
    - Think *compass!*





- For planet Earth:
  - The location of the geographic north pole is close to that of the magnetic south pole.
  - The location of the geographic south pole is close to that of the magnetic north pole.
    - The north pole of a compass points north because it is attracted to the magnetic south pole of the Earth.
      - (unlike poles attract)
  - The magnetic field of the Earth can be approximated as a magnetic dipole.

Magnetic dipoles are the result of electric charges moving in circles.

- We will cover electric charges moving in circles creating magnetic fields extensively later. At this point, just know that electric charges moving in circles create magnetic fields.
- The magnetism of magnets is most often the motion of electrons moving in circles inside them.
- Permanent magnetic dipoles and induced (temporary) magnetic dipoles are a property of the object which results from the alignment of magnetic dipoles within the object.

The material composition of a magnet affects its magnetic behavior when it is placed in an external magnetic field:

- *Ferromagnetic* materials can be *permanently* magnetized by an external magnetic field.
  - The alignment of the magnetic domains or atomic magnetic dipoles is *permanent*.
  - Example materials: nickel, iron, cobalt
- *Paramagnetic* materials are only *temporarily* magnetized by an external magnetic field.
  - The alignment of the magnetic domains or atomic magnetic dipoles is *temporary*.
  - Example materials: aluminum, magnesium, titanium

Just like materials have an electric permittivity, ε, materials also have a magnetic permeability, μ:

- Magnetic permeability: the measurement of the amount of magnetization a material has in response to an external magnetic field.
  - Ferromagnetic materials have high magnetic permeabilities that increase in the presence of an external magnetic field.
  - Paramagnetic materials have low magnetic permeabilities.
  - The magnetic permeability of materials is not constant. It changes depending on various factors such as temperature, orientation, and the strength of the external magnetic field.
- The magnetic permeability of free space has a constant value, μ_0: $\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$

A magnetic field is defined by the fact that a moving electric charge in a B field can experience a magnetic force, F_B.

- $\vec{F}_B = q\vec{v} \times \vec{B} \Rightarrow \|F_B\| = qvB\sin\theta$
  - This equation is an experimentally determined equation. In other words, there is no way to mathematically derive it! We know it is true because we have repeatedly measured it.
  - Notice the similarities to the torque equations: $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \|\tau\| = rF\sin\theta$
- $\Rightarrow B = \frac{F_B}{qv\sin\theta} \Rightarrow \frac{N}{C\left(\frac{m}{s}\right)} = \frac{N}{\left(\frac{C}{s}\right)m} = \frac{N}{A \cdot m} = \text{tesla}, T$
  - $1 \text{ tesla}, T = 10{,}000 \text{ gauss}, G$

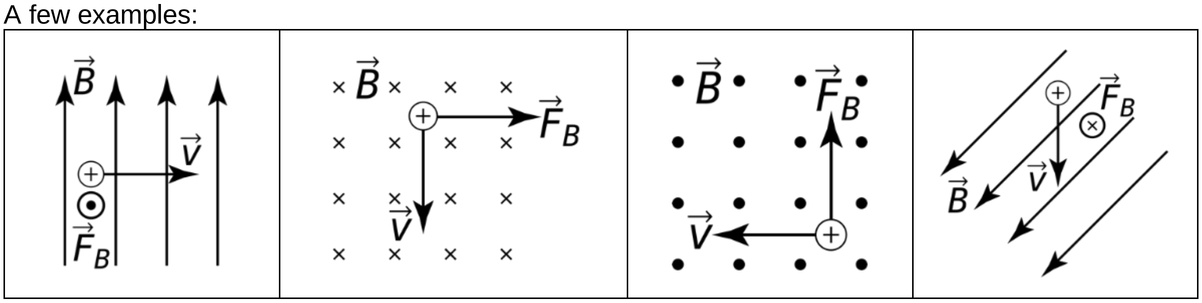
Flipping Physics Lecture Notes:
Magnetic Force Direction (Right-Hand Rule)
http://www.flippingphysics.com/magnetic-force-direction.html

Please recognize that the magnetic field is a vector. To that end, we need to know the direction of the magnetic force acting on an electric charge moving in a magnetic field. For that we use …

The Right-Hand Rule: [Don't be too cool. Limber up. Find your right hand.]

- Fingers point in the direction of the electric charge velocity.
- Fingers curl in the direction of the magnetic field.
  - It's a good rule of thumb[1] to start at 90°.
- Thumb points in the direction of the magnetic force on a positive charge.
  - For a negative charge, the thumb points 180° from the direction of the magnetic force.
  - Make sure your thumb points normal to the plane created by the velocity of the electric charge and the magnetic field.
    - In other words, realize the direction of the magnetic force is always normal to the plane created by the velocity of the electric charge and the magnetic field.
- Realize, the cross-product version of the magnetic force equation also gives you the direction of the magnetic force in terms of unit vectors.
- Since examples of this concept require vectors in all three dimensions, we introduce two symbols to indicate direction perpendicular to the page. A dot for out of the page, and an X for into the page, like the pointed tip and fletching (feathers) of a flying arrow respectively.

A few examples:



[1] Ha ha ha!

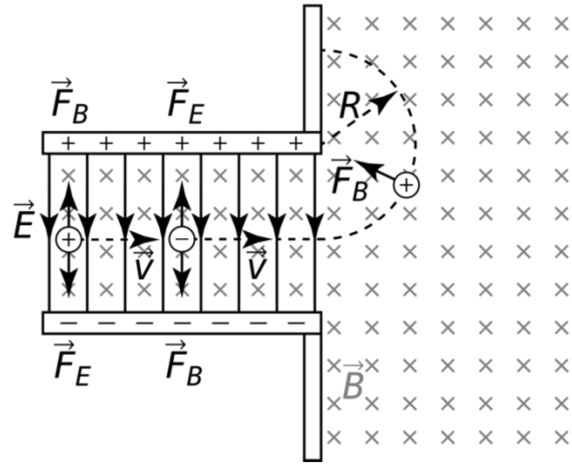
Flipping Physics Lecture Notes:
Magnetic Force on Current
http://www.flippingphysics.com/magnetic-force-current.html

What if we have a series of charges all moving in the same direction? Like a current carrying wire?

- We already derived the equation for the current in a wire: $I = nAv_dq$
- And we know the magnetic force acting on *each individual charge* moving in the wire:
$$\vec{F}_B = q\vec{v} \times \vec{B}$$
- However, what we want to know is the net magnetic force acting on all the charges moving in the wire. So, we need to use charge carrier density, n:
    - $$n = \frac{\#\text{ of charges}}{V} \Rightarrow \#\text{ of charges} = nV = nAL$$
- Which we can use to get the magnetic force acting on *all the charges* moving in the wire:
$$\vec{F}_B = \left(q\vec{v} \times \vec{B}\right) nAL = nAvq\vec{L} \times \vec{B}$$
- And we have derived the general equations for the magnetic force on a current carrying wire both for a straight wire and, using an integral, a wire that does not follow a straight path.
$$\Rightarrow \vec{F}_B = I\vec{L} \times \vec{B} \Rightarrow \vec{F}_B = \int I\left(d\vec{L} \times \vec{B}\right) \quad \& \quad \|F_B\| = ILB\sin\theta$$
- You use the same wonderful right-hand rule to determine the direction of this force.

Flipping Physics Lecture Notes:
Mass Spectrometer
http://www.flippingphysics.com/mass-spectrometer.html

Because it involves many concepts that are likely to come up on the AP exam, let's take a moment to analyze a mass spectrometer:



The magnetic field is uniform into the page throughout, and in the velocity selector, the electric field uniform and down.

Velocity Selector:

- For a positive charge the magnetic force is up and the Coulomb force is down.
- For a negative charge the magnetic force is down and the Coulomb force is up.
- Regardless of whether the charge is positive or negative, the free body diagrams result in the same Newton's Second Law equation:

$$\sum F_y = F_B - F_E = ma_y = 0 \Rightarrow F_B = F_E$$

$$\Rightarrow qvB\sin\theta = qE \Rightarrow vB\sin(90°) = E \Rightarrow v = \frac{E}{B}$$

- So, all charged objects with the same constant velocity will all move in a straight horizontal line in the velocity selector. Regardless of mass, charge sign, and charge magnitude.

Deflection Chamber:

- The uniform magnetic field is the only field present in the deflection chamber.
- The only force acting on the charged particle is the magnetic force which acts inward.
  - The charged particles will move along a circular path with radius, R.
  - Positive charges will be deflected upward.
  - Negative charges will be deflected downward.
- Again, we use Newton's Second Law:

$$\sum F_{\text{in}} = F_B = ma_c \Rightarrow qvB\sin\theta = qvB\sin(90°) = qvB = m\left(\frac{v_t^2}{R}\right)$$

$$\Rightarrow qB = \frac{mv_t}{R} \Rightarrow qB = \frac{m\left(\frac{E}{B}\right)}{R} \Rightarrow mE = qRB^2 \Rightarrow \frac{m}{q} = \frac{RB^2}{E}$$

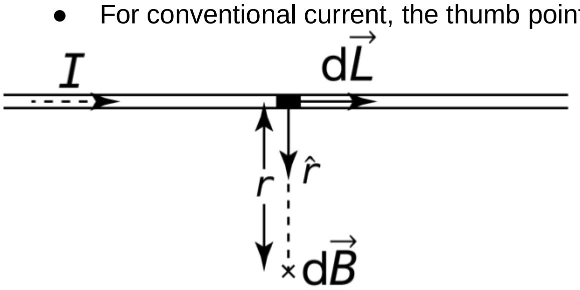
The mass spectrometer is a tool for determining velocities and mass-to-charge ratios of electric charges. Imagine how useful this could be for learning information about new particles!

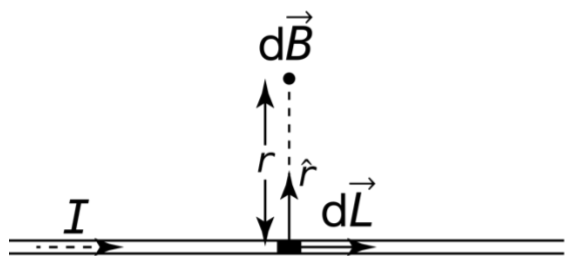
The Biot-Savart law:

- $$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{r}}{r^2}$$
- This is an experimentally determined equation; you cannot derive it.
- Unit vector r is a position vector which points from the location of the infinitesimally small length of the wire, dL, to the location of the infinitesimally small magnetic field, dB.
    - r is the magnitude of the distance between those two points
- Magnetic permeability, μ, is the measurement of the amount of magnetization of a material in response to an external magnetic field. μ_0 is the magnetic permeability of free space:
  $$\mu_0 = 4\pi \times 10^{-7}\frac{T \cdot m}{A}$$
- This equation shows that a current carrying wire creates a magnetic field. In fact, because current is composed of individually moving electric charges, even a single moving electric charge causes a magnetic field.

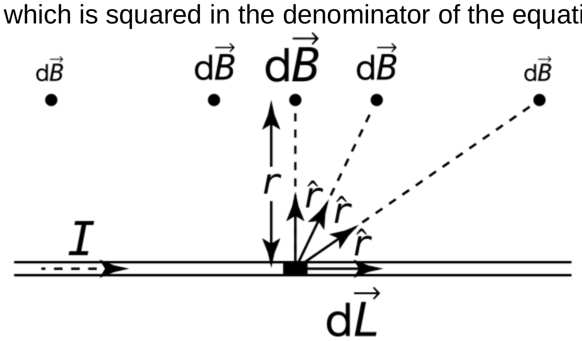
The direction of the magnetic field created by a current carrying wire can be seen using the Biot-Savart law. It is the cross product, so again, we use the right-hand rule!

- Fingers point in the direction of current/wire.
- Fingers curl in the direction of unit vector r.
- For conventional current, the thumb points in the direction of the magnetic field.

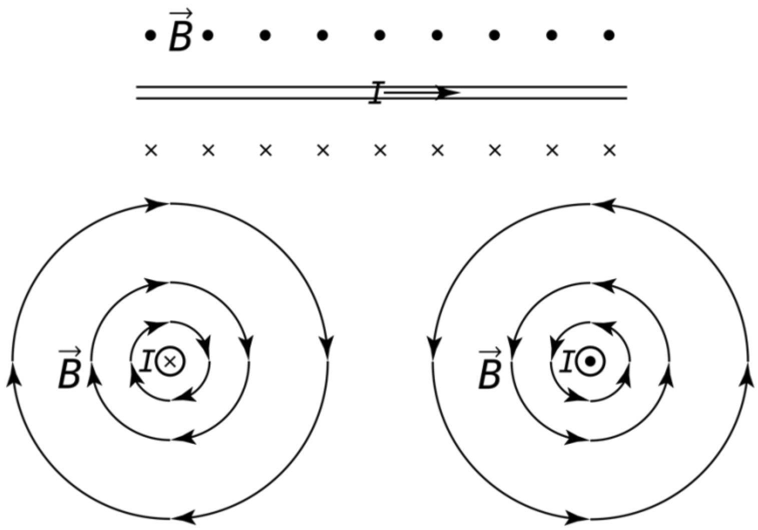




Notice the direction of the magnetic field caused by an infinitesimally small portion of the current carrying wire, dL, is the same along a line parallel to the straight wire, however, the magnitude of the magnetic field decreases as you get farther from a line perpendicular to the straight wire. The direction remains the same because the cross product of dL and unit vector r always gives the same direction. The magnitude decreases as the value of r, which is squared in the denominator of the equation, increases.



However, now realize that there are, for an infinitely long, straight, current carrying wire, an infinite number of dL's and all of their magnetic fields add up to cause the magnetic field to have a uniform value at a distance r straight out from the wire. And, the magnitude of the magnetic field decreases as r, the distance from the wire, increases.

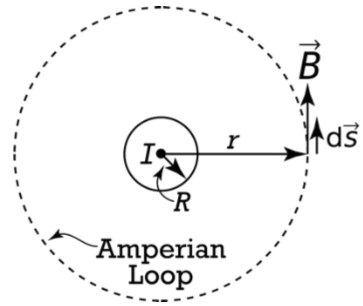


- An alternate right-hand rule exclusively for the magnetic field which surrounds a current carrying wire is:
  - Point thumb in direction of current.
    - Fingers curl in the direction of the magnetic field.
- The Biot-Savart law can also be used to determine the magnitude of the magnetic field a distance r from an infinitely long, straight, current carrying wire. That equation is: $B = \frac{\mu_0 I}{2\pi r}$
  - We now know the magnetic field magnitude is inversely proportional to distance from the wire.

Flipping Physics Lecture Notes:
Ampère's Law
Review for AP Physics C: Electricity and Magnetism
http://www.flippingphysics.com/ampere.html

Ampère’s law is the magnetic field equivalent to Gauss’ law:

- Gauss’ law: $$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$
    - Closed surface integral and charge inside a Gaussian surface.
- Ampère’s law: $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$
    - Closed loop integral and current inside an Amperian loop.



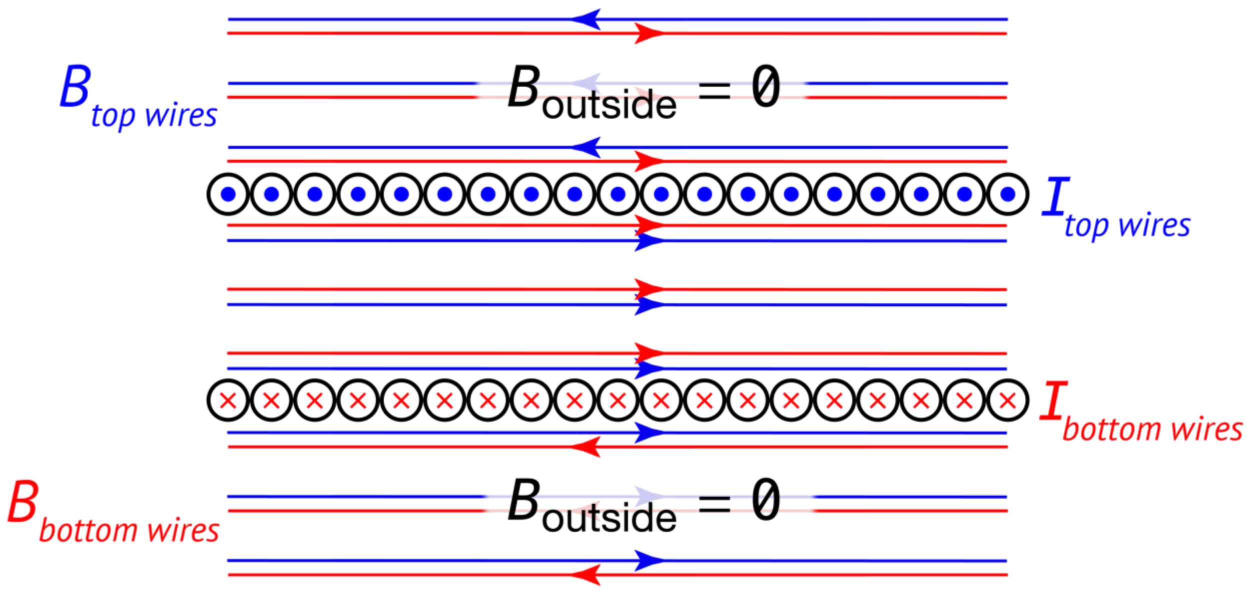
Example: Determine the magnitude of the magnetic field outside an infinitely long, straight, wire with radius R and current I.

Start by drawing an Amperian loop in the shape of a circuit of radius $r \geq R$ which is concentric with the wire. And let’s use Ampère’s law.

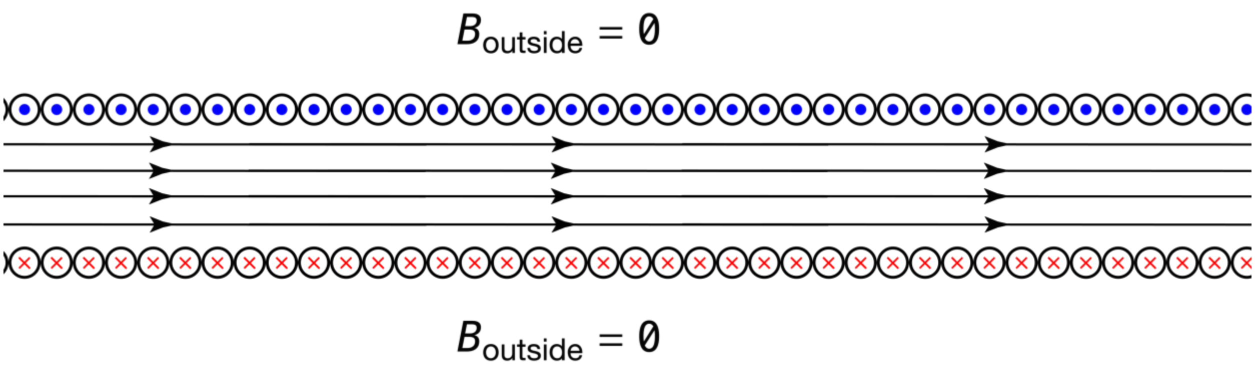
$$\Rightarrow \oint B ds \cos\theta = \oint B ds \cos 0^\circ = B \oint ds = B(2\pi r) = \mu_0 I_{in}$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

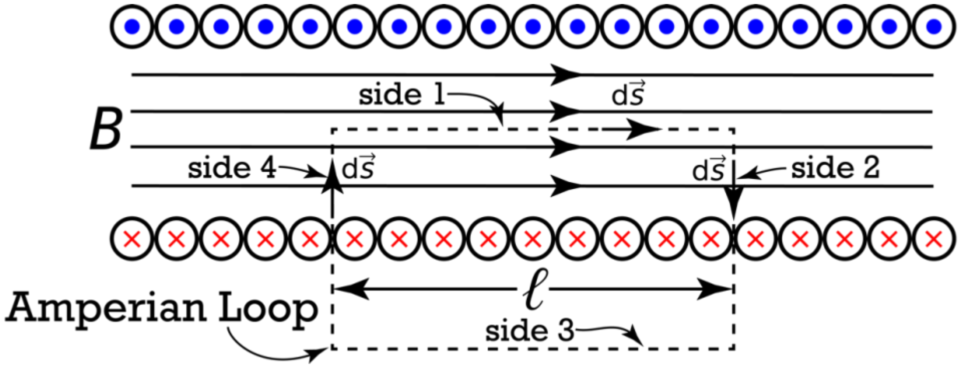
A solenoid is a very common tool for creating a uniform magnetic field. A typical solenoid is a single, very long, current carrying, insulated wire wrapped to form a hollow cylinder. An ideal solenoid has a length which is much, much larger than its diameter. The cross section of a solenoid looks like this.



Outside the solenoid, the magnetic field caused by the current in the top wires completely cancels out the magnetic field caused by the bottom wires. In other words, an ideal solenoid has zero magnetic field outside the cylinder of the solenoid. (ideal solenoid below)



Now let's derive the equation for the magnetic field inside an ideal solenoid. In order to do so, we begin with Ampère's law and draw an Amperian loop. Just like Gaussian surfaces, we want to pick Amperian loops to have sides which are at integer multiples of 90° relative to the magnetic field, and such that the magnetic field is uniform on the sides of the Amperian loop. For an ideal solenoid, we pick an Amperian loop shape of a rectangle with one side inside the solenoid and parallel to the magnetic field inside the solenoid and the opposite side of the Amperian loop is completely outside the solenoid.



And now we can begin using Ampère's law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \Rightarrow \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s} = \mu_0 I_{in}$$

For side 3, the magnetic field is zero outside the solenoid, so that integral equals zero. For sides 2 and 4, the magnetic field and ds are 90° to one another and the cosine of 90° is zero, so both of those integrals equal zero. That means, the only integral which remains is the integral for side 1.

$$\Rightarrow \int_1 \vec{B}\cdot d\vec{s} = B\int_1 ds\cos 0° = B\ell = \mu_0 N I \;\;\&\;\; I_{in} = NI$$

$$\Rightarrow B = \frac{\mu_0 N I}{\ell} \;\;\&\;\; n = \frac{N}{\ell} \Rightarrow B_{solenoid} = \mu_0 n I$$

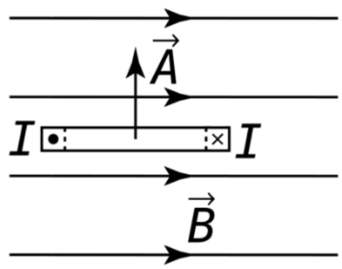
Where "n" is the turn density of the solenoid.

Flipping Physics Lecture Notes:
Magnetic Flux
http://www.flippingphysics.com/magnetic-flux.html

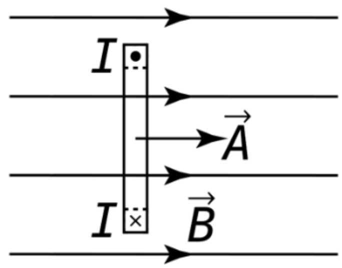
Before we learn about electromagnetic induction, we need to learn about magnetic flux. Before we do that, let's review electric flux:

- Electric flux is the measure of the number of electric field lines which pass through a surface.
- When the electric field is uniform, and the surface is a two-dimensional plane:
$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$
- The general equation for electric flux: $\Phi_E = \int \vec{E} \cdot d\vec{A}$

Magnetic flux:



- Magnetic flux is the measure of the number of magnetic field lines which pass through a surface.
- When the magnetic field is uniform, and the surface is a two-dimensional plane: $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$
- The general equation for magnetic flux:
$$\Phi_B = \int \vec{B} \cdot d\vec{A} \Rightarrow T \cdot m^2 = \text{webbers}, Wb$$
- Example #1: Current through a wire loop. Use the right-hand rule to determine the direction of the area vector. (Similar to the right-hand rule for angular velocity direction.) Fingers curl in the direction of the current, thumb points in the direction of the area vector.
$$\Phi_B = BA\cos\theta = BA\cos 90° = 0$$
- Example #2: $\Phi_B = BA\cos\theta = BA\cos 0° = \Phi_{B_{max}}$



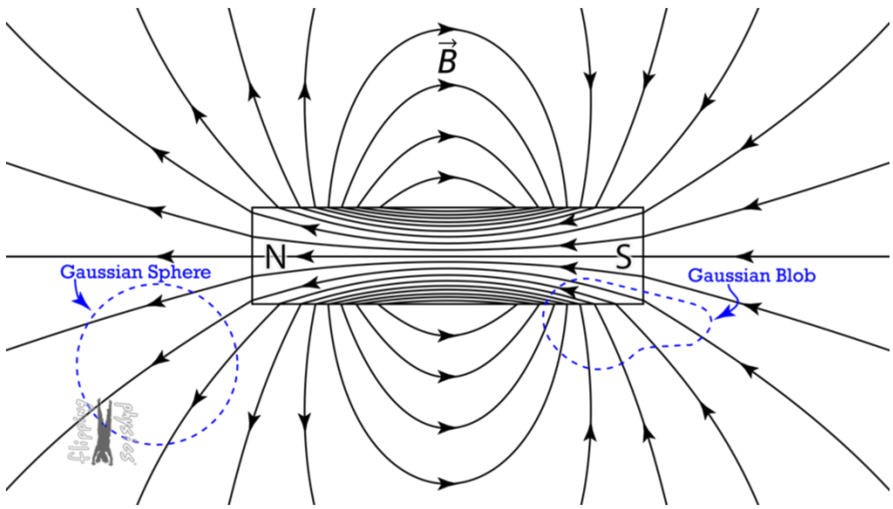
Gauss's law has to do with electric flux: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$

Gauss's law for magnetism has to do with magnetic flux:

- Because a magnetic monopole has never been found in nature or in a manmade experiment, every magnetic field line is a closed loop.
- Therefore, no matter what shape the gaussian surface has, every magnetic field line which enters the gaussian surface will also leave the gaussian surface:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

- The above equation, Gauss's law for magnetism, is the second of Maxwell's equations which are a collection of equations which fully describe electromagnetism.



Flipping Physics Lecture Notes:
Electromagnetic Induction
http://www.flippingphysics.com/electromagnetic-induction.html

Electromagnetic Induction:

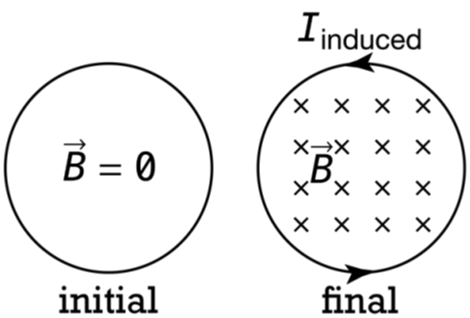
- We have already discussed that moving electric charges create magnetic fields.
- It should be no surprise that moving magnetic poles create electric fields.
  - Notice how these interact with one another!
- When a magnetic field changes over time, this can induce an electric potential difference called an induced emf, this causes charge to flow in a closed loop of wire which is called an induced current. More specifically, the relationship is between a changing magnetic flux and the resulting induced emf in a single closed loop of wire and is described by Faraday's law of electromagnetic induction: $$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right|$$
  - Induced emf = the derivative of magnetic flux with respect to time. (magnitudes)
- Substitute in the equation for magnetic flux: $$|\varepsilon| = N\left|\frac{d\Phi_B}{dt}\right| = N\left|\frac{d\left(\vec{B}\cdot\vec{A}\right)}{dt}\right| = N\left|\frac{d\left(BA\cos\theta\right)}{dt}\right|$$
  - N is the number of loops
  - An emf can be induced by changing:
    - Magnitude of the magnetic field.
    - Area enclosed by the loop.
    - Angle between magnetic field and loop area. (θ between $\vec{B}$ and $\vec{A}$)
    - Or any combination of the three.
      - In other words, if the only one of those three ($\vec{B}$, $\vec{A}$, and θ) which is changing is the magnitude of the magnetic field, then the magnitude of the induced emf through one loop of wire is: $$|\varepsilon| = \left|A\cos\theta\left(\frac{dB}{dt}\right)\right|$$
- Electromagnetic induction is the process of inducing an electromotive force by a change in magnetic flux.
- Faraday's law is the third of Maxwell's equations which are a collection of equations which fully describe electromagnetism.

Flipping Physics Lecture Notes:
Lenz's Law
http://www.flippingphysics.com/lenz-law.html

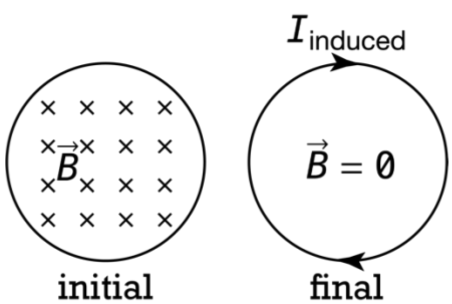
We need to determine the direction of the induced emf caused by a changing magnetic flux. That is shown by removing the absolute value from the equation, which gives us, assuming only one loop:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- The negative in this equation means the induced emf is opposite the direction of the change in magnetic flux.
- The direction of the induced emf is called Lenz' law.
  - Yes, the negative added to Faraday's law is called Lenz' law.
  - Lenz' law: The current induced in a circuit due to a change in a magnetic field is directed to oppose the change in magnetic flux and to exert a mechanical force which opposes the motion.
- We use the right-hand rule[1] to determine the direction of the induced emf. Examples below:

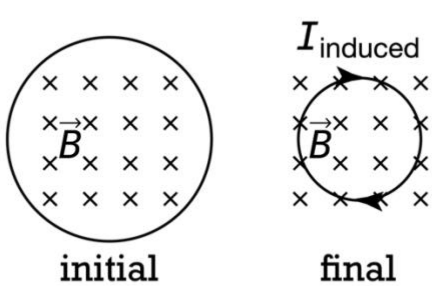


- Zero initial magnetic flux inside the loop.
- Original B field is into the screen and increasing, therefore the original magnetic flux is increasing.
- Induced magnetic field opposes the change in the original magnetic flux and therefore is induced out of the screen to counteract the change in original magnetic flux.
- According to the right-hand rule, fingers curl out of the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is counterclockwise.



- Original B field in the loop is into the screen and decreasing, which means the original magnetic flux is decreasing.
- $B_{induced}$ opposes this change in magnetic flux and attempts to maintain the original magnetic flux. Therefore, $B_{induced}$ is into the screen.
- According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is clockwise.

Note: Magnetic flux is a dot product, so magnetic flux is a scalar. So, the induced magnetic flux does not have a direction, however, the induced magnetic field does have a direction and the direction of the induced magnetic field in the plane of the loop is always normal to the loop in which the induced current is created.

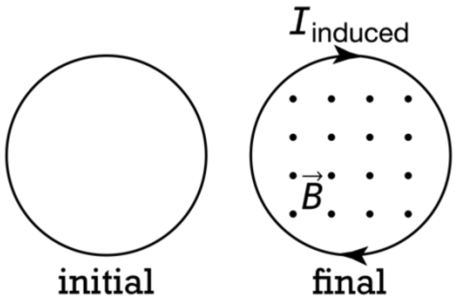


- Original B field inside the loop is into the screen and the area is decreasing which means the original magnetic flux is decreasing.
- $B_{induced}$ opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, $B_{induced}$ is into the screen.
- According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is clockwise.

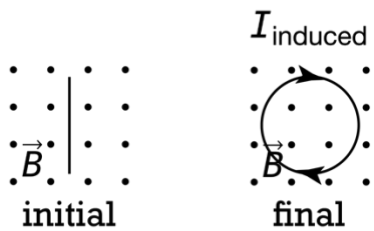
---

[1] This is the "alternate" right-hand rule with the thumb pointing in the direction of the current in the wire and fingers curling in the direction of the magnetic field created by the current in the wire.

(The next example was cut out of the video, however, y'all still get to enjoy it here!)

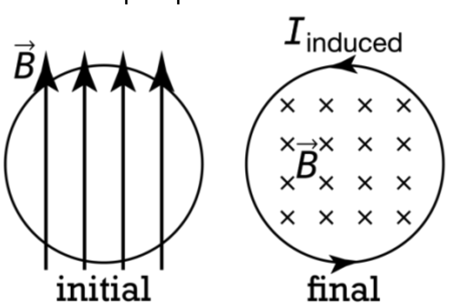


- There is no original B field so no original magnetic flux. The B field is increasing out of the screen so the original magnetic flux is increasing.
- $B_{induced}$ opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, $B_{induced}$ is into the screen.
- According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is counterclockwise.

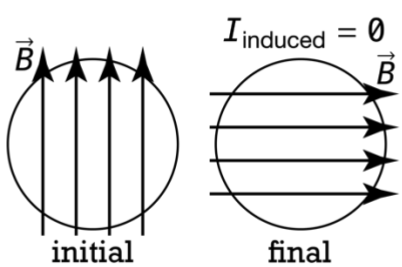


- B field is originally parallel to the loop, so there is zero original magnetic flux through the loop. Loop turns to cause the area of the loop to now be normal to the B field which is out of the screen. So, the original magnetic flux is increasing.
- $B_{induced}$ opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, $B_{induced}$ is into the screen.
- According to the right-hand rule, fingers curl into the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is clockwise.

Note: No matter which way the loop is turned, the change in the magnetic flux through the loop is the same and the induced magnetic field is into the screen caused by the induced current which is clockwise from this perspective.



- B field is originally parallel to the loop, so there is no original magnetic flux. B field turns to now be into the screen. So, the original magnetic flux is increasing.
- $B_{induced}$ opposes this change in magnetic flux and produces a magnetic field to maintain the original magnetic flux. Therefore, $B_{induced}$ is out of the screen.
- According to the right-hand rule, fingers curl out of the screen in the direction of the induced magnetic field, thumb points in the direction of the induced current which is counterclockwise.



- B field is originally parallel to the loop, so there is no original magnetic flux. B field turns to now be … still parallel to the loop. So, the magnetic flux through the loop is still zero.
- No change in the magnetic flux means there is no induced current.

We now have covered all four of Maxwell's equations which are a collection of equations which fully describe electromagnetism:

1) Gauss' law: $\Phi_E = \oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$
2) Gauss' law in magnetism: $\Phi_B = \oint_{\text{surface}} \vec{B} \cdot d\vec{A} = 0$
3) General form of Maxwell-Faraday's law of induction: $\varepsilon = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
4) The Ampère-Maxwell law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{in}} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

Maxwell's third equation is:

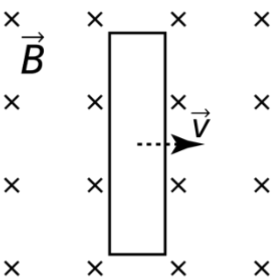
- The Faraday's law of induction we previously learned: $\varepsilon = -\frac{d\Phi_B}{dt}$
  - Which shows that changing magnetic fields create an electric potential difference
- plus the more general addition: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$
  - Which shows that a changing magnetic field must also create a nonconservative electric field.

Maxwell's fourth equation is:

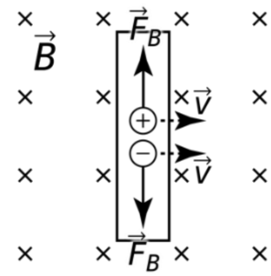
- Ampère's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{in}}$
  - Which shows that magnetic fields can be generated by electric currents
- plus Maxwell's addition of $\varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$
  - Which shows that a changing electric field creates a magnetic field.
    - In a similar manner to how a moving charge creates a magnetic field.

Flipping Physics Lecture Notes:
Motional EMF via Newton's Second Law
http://www.flippingphysics.com/motional-emf-newton.html

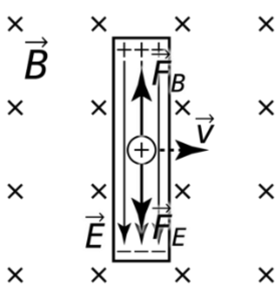
Motional emf is the idea that the motion of a conductor moving in a magnetic field can cause charges to move in the conductor creating a voltage across the conductor. In other words, a conductor moving in a magnetic field can acquire an induced emf across it.



- The conductor is moving to the right with a constant velocity at a right angle to a magnetic field which is into the page.
- According to the right-hand rule, positive charges will experience an upward magnetic force, and negative charges will experience a downward magnetic force.
- This will result in the movement of charges with the final result being that there will be a net positive charge on the top end of the conductor and a net negative charge on the bottom end of the conductor. This arrangement of charges creates a uniform, downward electric field in the moving conductor.



- As a result of the downward electric field in the conductor, positive charges will experience a downward electrostatic force, and negative charges will experience an upward electrostatic force.



- Because the conductor is moving at a constant velocity, the charges will arrange themselves such that equilibrium is reached between the magnetic and electric forces acting on the charges such that the electric field has a constant magnitude and the charges in the conductor are moving with a constant velocity to the right; there is no vertical motion of the electric charges.
- We can now sum the forces on a positive charge.
    - The same final equation is derived when using a negative charge.

$$\sum \vec{F}_y = F_B - F_E = ma_y = m(0) = 0 \Rightarrow F_B = F_E \Rightarrow qvB\sin\theta = qE$$

$$\Rightarrow vB\sin 90^\circ = E \Rightarrow vB = E$$

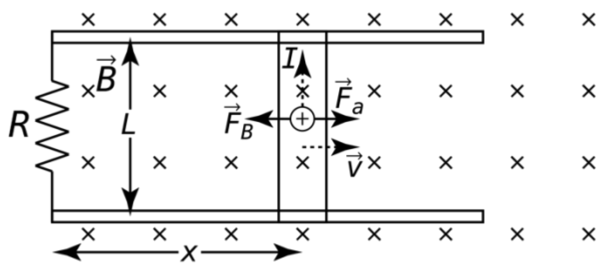
Previously we derived the equation relating voltage and a uniform electric field. We have already identified the direction of the electric field, so we only need the absolute value of the voltage.

$$\& \ \Delta V = -Ed \Rightarrow |\Delta V| = EL \Rightarrow E = \frac{\Delta V}{L} = vB \Rightarrow \Delta V = vBL \Rightarrow \varepsilon = vBL$$

L is the length of the conductor. We have derived the voltage or the induced emf across the conductor moving at a right angle to a uniform magnetic field. This is called *motional emf*.

Flipping Physics Lecture Notes:
Motional EMF via Faraday's Law
http://www.flippingphysics.com/motional-emf-faraday.html

Previously we derived the motional emf equation using Newton's Second Law. There is actually an entirely different approach to deriving the same motional emf equation. This approach starts with a conductor moving to the right while in contact with two parallel, metal rails connected by a wire at the left end with a uniform magnetic field going into the page. The resistance of the circuit is represented by the resistor shown in the wire on the left. A force is applied to the conductor to cause it to move to the right. We can use Lenz' law to determine the direction of the induced current in the loop.



- The magnetic field is into the screen and the magnetic flux is increasing because the area of the loop is increasing which increases the number of field lines passing through the loop.
- The induced magnetic field opposes this change in flux and is directed out of the page.
- Using the alternate right-hand rule, our fingers curl in the direction of the induced magnetic field which is out of the page inside the loop and our thumb points in the counterclockwise direction which is in the direction of the induced current in the loop.
- Notice this means that, because positive charges are moving in the direction of conventional current in the conductor, we can use the right-hand rule to show that the fingers point in the direction of the motion of the positive charges which is up, fingers curl in the direction of the magnetic field, which is into the page, and our thumb points in the direction of the magnetic force, which is to the left. In other words, there is a magnetic force which opposes the motion of the conductor in the magnetic field. If the applied force is constant, the magnetic force will also be constant to keep the conductor moving at a constant velocity.
- Now we can use Faraday's law to determine the magnitude of the induced emf in the conductor.

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{dBA\cos\theta}{dt} = -(1)B\cos(180^\circ)\frac{d(Lx)}{dt} = BL\frac{dx}{dt}$$

$$\Rightarrow \varepsilon = vBL$$

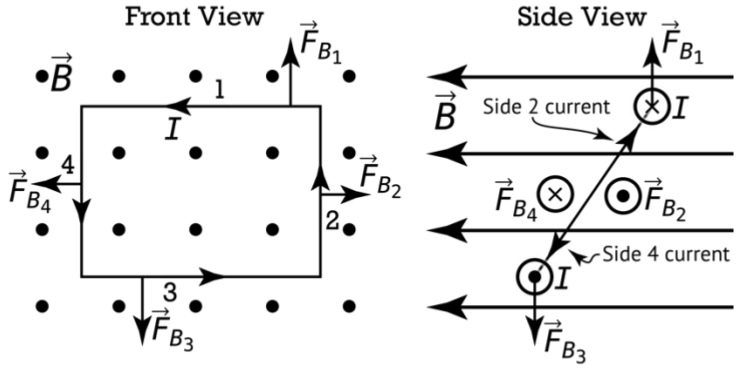
Flipping Physics Lecture Notes:
Induced Forces on Current Carrying Loops
http://www.flippingphysics.com/induced-force-loops.html

Next, let's look at a conductive loop which has a current induced in it, something we talked about previously[1], that induced current is now a bunch of charge carriers which are moving in a magnetic field. Those moving charges now have induced forces acting on them, again this is something we talked about quite a before now[2]. The following equations determine that magnetic force:

$$\vec{F}_B = I\vec{L} \times \vec{B} \Rightarrow \|F_B\| = ILB\sin\theta$$

Let's walk through an example.
Below is a front view and a side view of a conducting loop in the shape of a rectangle. Let's start by only looking at the front view. Again, all directions for now refer to the *front view only*.

- A uniform magnetic field is directed out of the page and is decreasing.
- That means the magnetic flux through the loop is decreasing.
- Lenz' law states the induced B field is out of the page to counteract the decreasing magnetic flux.
- Using the alternate right-hand rule
  - Fingers curl with the induced B field, out of the page.
  - Thumb points counterclockwise with induced current.
- The right-hand rule on the induced current in side 1 of the loop:
  - Fingers point to the left in the direction of the induced current.
  - Fingers curl out of the page in the direction of the original magnetic field.
  - Thumb points up in the direction of the induced magnetic force on side 1.

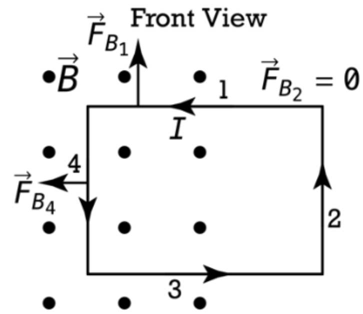


- For the remaining sides the right-hand rule shows the induced magnetic forces are:
  - To the right on side 2.
  - Down on side 3.
  - To the left on side 4.
- Notice that the net induced magnetic force on the loop equals zero!
  - The induced magnetic forces on sides 1 and 3 are equal and opposite.
  - The induced magnetic forces on sides 2 and 4 are equal and opposite.

[1] Electromagnetic Induction
[2] Magnetic Force on Current

Now let's switch to the side view. Again, all directions for now refer to the *side view only*.

- Side 1: The induced current is into the page, and the induced magnetic force is up.
- Side 2: The induced current is up and to the right, and the induced B force is out of the page.
- Side 3: The induced current is out of the page, and the induced magnetic force is down.
- Side 4: The induced current is down and to the left, and the induced B force is into the page.
- You can see the net induced magnetic force on the loop is still zero, however, …
- The induced magnetic forces cause a net torque on the loop! Net torque is not zero!
    - Assuming the loop is not attached to anything, the net torque on the loop would cause an angular acceleration around its center of mass which is counterclockwise at this specific moment in time.



Now let's change the example by making it so the magnetic field abruptly ends partway through the loop.[3]

- Again, the magnetic field is uniform, directed out of the page, and is decreasing in magnitude.
- Lenz' law gives us the same direction for the induced current in the loop; counterclockwise.
- Using the right-hand rule to determine the directions of the induced magnetic force:
    - Side 2: This entire side of the wire is not in the magnetic field, so there is *no induced magnetic force on side 2!*
    - Side 4: Everything is the same here. Induced magnetic force is to the left.
    - Sides 1 and 3: The directions are the same as before (1 is up, 3 is down), however, only the part of each side which is in the magnetic field will experience an induced magnetic force, therefore, the magnitudes of these induced magnetic forces are smaller than in the previous example.
- The net induced magnetic force on this loop *does not equal zero.*
    - The induced magnetic forces on sides 1 and 3 are equal and opposite.
    - The net force would accelerate the loop to the left.

In other words, the net induced magnetic force on a current carrying loop:

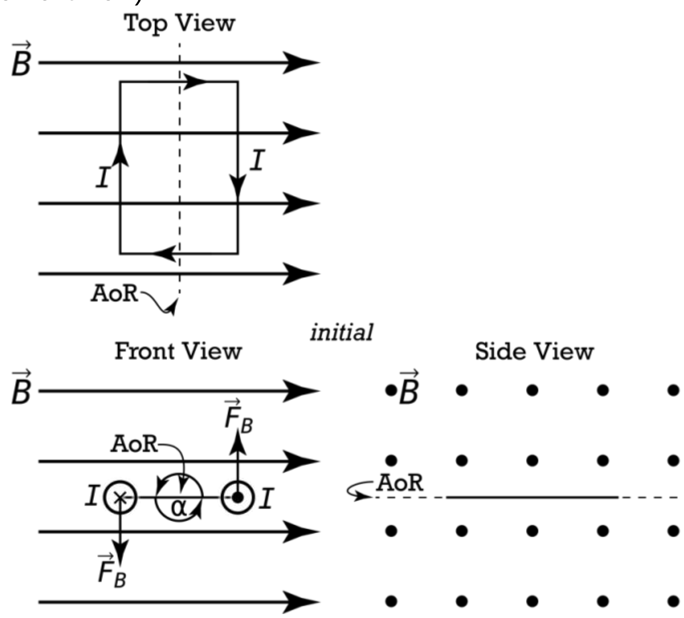
- Which is entirely in a uniform external magnetic field always equals zero.
    - (The induced magnetic forces can cause a net torque on the loop.)
- Which is only partially in a uniform magnetic field is nonzero.
    - This can cause a translational acceleration of the loop.

[3] I would argue that creating a magnetic field which looks like this is impossible, however, it is helpful for learning. So, step off!

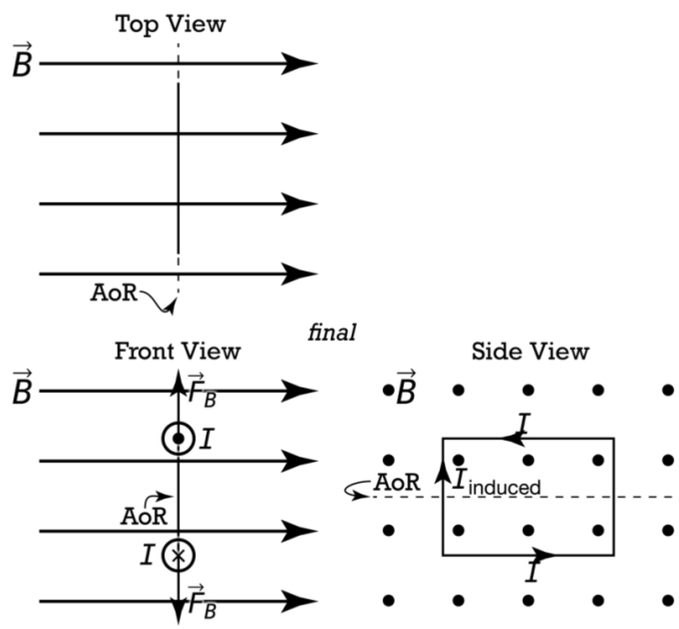
Let's look at a rectangular conducting loop in a uniform magnetic field oriented as shown below. We place an emf across the loop to cause current I in the loop.

In the *front view* you can see that, according to the right-hand rule, a net torque acts on the loop causing it to angularly accelerate in a clockwise direction (in the front view).

Everything we have been referring to is the initial position of the loop.



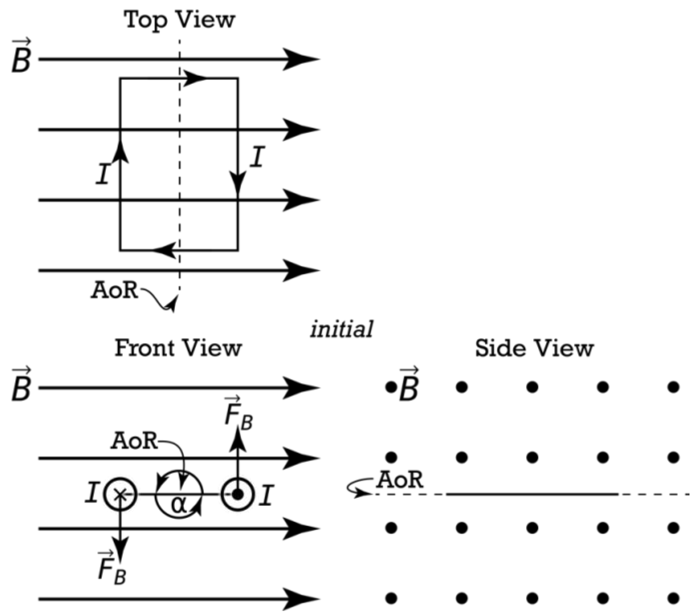
After the loop has turned 90 degrees, we are now at the final position of the loop.



This is a very basic illustration of how an electric motor works. Current is placed through wire loops in magnetic fields which causes the loops to rotate converting electric potential energy to mechanical energy.

Flipping Physics Lecture Notes:
Back emf (electromotive force)
http://www.flippingphysics.com/back-emf.html

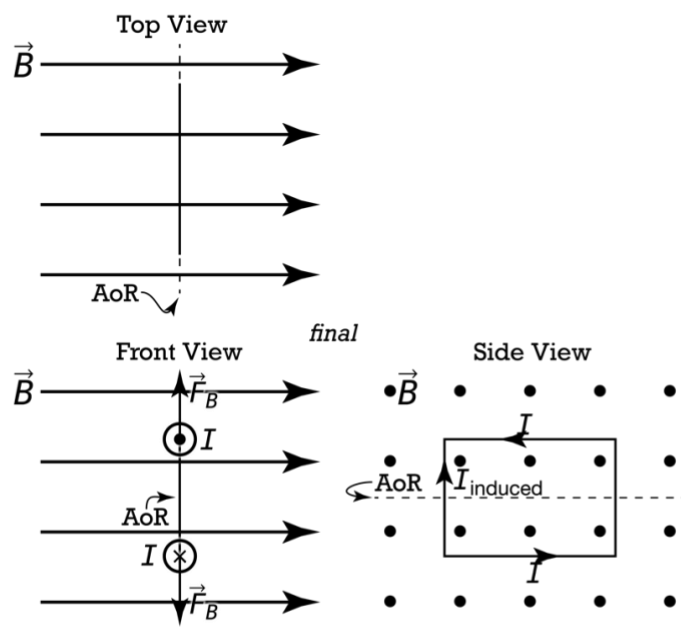
We already walked through how this is an example of a basic electric motor. Now let's look at how the magnetic flux changes from the initial to final positions.



The initial magnetic flux through the loop is zero. The final magnetic flux through the loop is nonzero. The magnetic flux through the loop changes, which means there is an induced magnetic field, an induced emf, and an induced current in the loop. We need to use Lenz' law to determine the direction of the induced current.

In the side view, the magnetic flux is out of the page and increasing. In order to resist this change in magnetic flux, the induced magnetic field is into the screen (in the side view). According to the alternate right-hand rule, the fingers curl into the screen in the direction of the induced magnetic field inside the loop, thumb points clockwise (in the side view) in the direction of the induced current in the loop.

In other words, in electric motors, there is an induced emf and an induced current caused by the change in the magnetic flux in the loops of the motor, and that induced current is opposite the direction of the current placed in the loops to cause the loops to rotate. This induced current decreases the current in a turning electric motor. This concept is called *back emf* and is present in all electric motors when they are rotating.

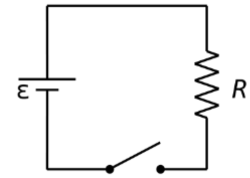


Realize this back emf is not present when the electric motor is not rotating. In other words, when an electric motor is first starting up, the current through the electric motor is larger than when the electric motor is running at a constant angular velocity. This lack of back emf when an electric motor is not moving can cause lights which are on the same circuit to dim when an electric motor is first starting up and can even cause a circuit breaker to trip if something suddenly binds the electric motor causing it to stop rotating which brings the back emf to down zero and suddenly increases the current in the circuit above the maximum current allowed through the circuit breaker.[1]

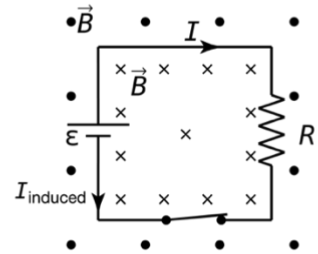
[1] Yes, I have done this. ▯

Flipping Physics Lecture Notes:
Inductance
http://www.flippingphysics.com/inductance.html

Let's look at a basic circuit. Before time t = 0, the switch in the circuit is open and zero current flows through the open loop. At time t = 0, the switch is closed and remains closed. From this perspective, a clockwise current, I, is now in the circuit. Up to this point we have assumed the current appears instantaneously in the circuit. You should realize that, in the real world, nothing changes instantaneously. So, let's look at what really happens when the switch closes.



According to the alternate right-hand rule, the clockwise current, I, in the circuit causes a magnetic field which is out of the page outside the loop and a magnetic field which is into the page inside the loop. In other words, this circuit is a loop which initially, before time t = 0, has zero magnetic flux in it and, as soon as the switch is closed, the loop has magnetic flux in it. We know, according to Faraday's law, that a changing magnetic flux induces an emf and can induce a current. We can use Lenz' law to determine the direction the induced current would be in the loop:



$$\varepsilon_{induced} = -N\frac{d\Phi_B}{dt}$$

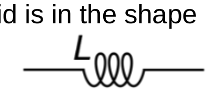
- Initially, there is zero magnetic flux.
- Finally, there is a B field which is into the page inside the loop.
  - Note: Only the magnetic field inside the loop causes a magnetic flux inside the loop.
- Therefore, the magnetic flux is increasing.
- Lenz's law states that an induced magnetic field is created to counteract the change in magnetic flux.
- Therefore, the induced magnetic field is out of the page.
- According to the alternate right-hand rule, an induced current would be counterclockwise in the loop from this perspective.
- This means the current in the circuit does not instantly change from 0 to I. The current in the circuit takes time to transition from 0 to I, because, the circuit itself opposes the change in current.
- This opposition of a circuit to a change in current in that same circuit is called *self-inductance*.
- In general, opposition to a change in current in a conductor is called *inductance*.

To get to the equation for inductance, we need to return to the simple circuit example and the basic concept of Faraday's law.

- Induced emf is proportional to change in magnetic flux with respect to time. $\varepsilon_{induced} \propto \frac{d\Phi_B}{dt}$
- The magnitude of magnetic flux equals the magnetic field times the area of the loop times the cosine of the angle between the direction of the magnetic field and the direction of the area. $\Phi_B = BA\cos\theta$
- Assuming the area and angle are not changing with respect to time, the induced emf is proportional to the change in the magnetic field with respect to time. $\varepsilon_{induced} \propto \frac{dB}{dt}$
- An example of a magnetic field around a current carrying wire is the one which surrounds an infinitely long current carrying wire which we have derived previously. $B = \frac{\mu_0 I}{2\pi a}$
  - "a" is the straight-line distance perpendicular out from the wire to the location of the B field.
- This means the induced emf in a conductor is proportional to change in current in the conductor with respect to time. $\varepsilon_{induced} \propto \frac{dI}{dt}$

An inductor is a circuit element with a known inductance.
The equation for the inductance of an inductor is: $\varepsilon_L = -L\frac{dI}{dt}$

- "L" is the inductance of the inductor.
- The simplest version of an inductor is a small, ideal solenoid. Because a solenoid is in the shape of a coil, the symbol for an inductor looks like the coils of a miniature solenoid.



- The units for inductance are henrys, H.
  - $\varepsilon_L = -L\frac{dI}{dt} \Rightarrow L = -\frac{\varepsilon_L}{dI/dt} \Rightarrow \frac{V}{A/s} \Rightarrow \text{henry}, H = \frac{V\cdot s}{A}$

Flipping Physics Lecture Notes:
Inductors vs. Resistors: Exploring the Fundamental Differences
http://www.flippingphysics.com/inductors-resistors.html

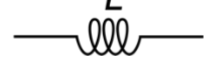
It is important to understand the difference between resistance, resistivity, resistors, inductance, self-inductance, and inductors.

$$R = \frac{\Delta V}{I}$$

$$\rho = \frac{RA}{L}$$

- *Resistance* is an opposition to current. *(concept)*
    - The units for resistance are ohms, Ω.
    - The resistance of a circuit is often assumed to be zero. (self-resistance?)
    - A *resistor* is a circuit element with a specific resistance. *(physical object)*
        - "R" is the resistance of a resistor.
        - A resistor is made of a material with a material property called *resistivity*, ρ.
            - The units for resistivity are ohm meters, Ω•m.
        - A resistor can be added to a circuit to change the resistance of the circuit.
        - A resistor can be added to a circuit diagram to model the resistance of the circuit itself.
- *Inductance* is an opposition to changes in current. *(concept)*
    - The units for inductance are henrys, H.
    - The opposition of a circuit to the change in the current in that circuit is called *self-inductance*. *(concept)*
        - The self-inductance of a circuit is often assumed to be zero.
    - An *inductor* is a circuit element with a specific inductance. *(physical object)*
        - "L" is the inductance of an inductor.
        - A typical shape for an inductor is a small, ideal solenoid.
            - There is no material property called "inductivity" because the inductance of an inductor is mostly caused by the shape, not the material, of the inductor. A magnetic material in its core can affect the inductance through its magnetic permeability, but not the material of the wire coil.
        - An inductor can be added to a circuit to change the inductance of the circuit.
        - An inductor can be added to a circuit diagram to model the self-inductance of the circuit itself.

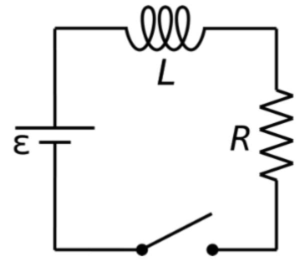
$$L = -\frac{\varepsilon_L}{dI/dt}$$



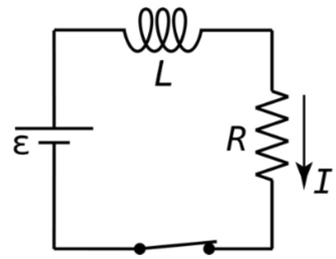
Flipping Physics Lecture Notes:
Inductance of an Ideal Solenoid
http://www.flippingphysics.com/inductance-solenoid.html

Considering the most common shape for an inductor is a small, ideal solenoid, let's look at that case specifically. We have two different equations for induced emf which we can set equal to one another:

- $$\varepsilon_{\text{induced}} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \Rightarrow N d\Phi_B = L dI$$
  - N is the total number of loops or coils in the solenoid shaped inductor.
  - We can cancel out *dt* on both sides of the equation
- $$\Rightarrow \int N d\Phi_B = \int L dI \Rightarrow N\int_0^{\Phi_B} d\Phi_B = L\int_0^{I} dI \Rightarrow N\Phi_B = LI \Rightarrow L_{\text{solenoid}} = \frac{N\Phi_B}{I} = \frac{N(BA\cos\theta)}{I}$$
  - Take the integral of the whole equation.
  - Both N and L are constants and can be taken out from their integrals.
  - Substitute in the equation for the magnitude of magnetic flux.
- $$\Rightarrow L_{\text{solenoid}} = \frac{NBA\cos(0°)}{I} = B\left(\frac{NA}{I}\right) \;\&\; B_{\text{solenoid}} = \mu_0 n I = \frac{\mu_0 N I}{\ell}$$
  - In an ideal solenoid, angle between magnetic field and loop area vector is always 0°.
  - We have the equation for an ideal solenoid which we derived earlier. $n = \frac{N}{\ell}$
    - n is the turn density of the solenoid.
    - We already defined N as the total number of loops in the solenoid,
    - Therefore, the curly ℓ, is the entire length of the ideal solenoid.
      - Note $L \neq \ell$. (Inductance does not equal solenoid length.)
      - (L for a resistor is its length not its inductance.)
- $$\Rightarrow L_{\text{solenoid}} = \left(\frac{\mu_0 N I}{\ell}\right)\left(\frac{NA}{I}\right) \Rightarrow L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{\ell}$$
  - The inductance of an ideal solenoid is determined by:
    - N, the number of turns: A, the cross-sectional area: ℓ, solenoid length.
    - μ, the magnetic permeability of the space inside the solenoid. For an ideal solenoid with nothing inside it, that equals the magnetic permeability of free space.
    - μ, the magnetic permeability of the core material, replaces μ_0 when the solenoid has a core material such as iron.
      - Inductance does *not* depend on current through the solenoid!
        - Resistance does *not* depend on current either!



Let's derive the equation for the energy stored in the magnetic field generated in an inductor as charges move through the inductor. To do that, we need to discuss an LR circuit. A circuit with an inductor and a resistor in it. Initially, at time t < 0, the switch is open. At time t = 0, the switch is closed. The current will increase from zero to some steady-state current, I. We are not going to derive the time-dependent equations for LR circuits today, we will do that in a future lesson.



Using Kirchhoff's Loop Rule, starting from the lower left-hand corner we get:

- $\Delta V = 0 = \varepsilon - \Delta V_L - \Delta V_R = \varepsilon - L\frac{dI}{dt} - IR \Rightarrow \varepsilon = L\frac{dI}{dt} + IR$
  - Electric potential across the battery goes up because the battery is adding electric potential energy to the circuit.
  - Electric potential across the inductor goes down because electric potential energy is being stored in the magnetic field of the inductor.
  - Electric potential across the resistor goes down because the resistor dissipates electric potential energy from the system.
  - We can now multiply this whole equation by the circuit current, I.
- $\Rightarrow P = I\varepsilon = LI\frac{dI}{dt} + I^2R$
  - We get the equation for power for each circuit element:
    - The rate at which energy is being added to the circuit by the battery.
    - The rate at which energy is being stored in the magnetic field of the inductor.
    - The rate at which energy is being dissipated by the resistor.
- We can now look specifically at the rate at which energy is being stored in the magnetic field of the inductor.
- $\Rightarrow P = \frac{dU}{dt} = LI\frac{dI}{dt} \Rightarrow dU = LI(dI) \Rightarrow \int_0^{U_L} dU = \int_0^I LI(dI) = L\int_0^I I(dI)$

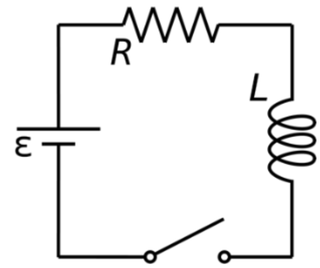
  $\Rightarrow U_L = \left[L\left(\frac{I^2}{2}\right)\right]_0^I \Rightarrow U_L = \frac{1}{2}LI^2$
- We now have an equation for the energy stored in the magnetic field generated in an inductor as charges move through the inductor.
  - That energy is only present when current is passing through the inductor. This is because the magnetic field generated in the inductor is due to the charges moving through the inductor. If the charges are not moving, there is no magnetic field in the inductor.

A capacitor functions differently:

- The energy stored in a capacitor is stored in the electric field of the capacitor.
  - The energy stored in a capacitor can remain when a capacitor is disconnected from a circuit because charges can remain separated on the plates of the capacitor which would maintain the electric field between the plates of the capacitor.

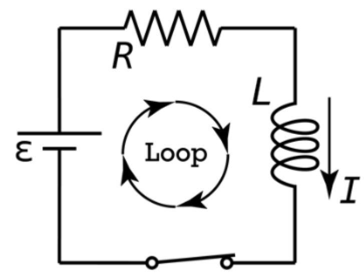
$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2 = \frac{1}{2}Q\Delta V$$

Flipping Physics Lecture Notes:
LR Circuit Basics
http://www.flippingphysics.com/lr-circuit.html



This LR circuit is a circuit with a battery, a resistor, an inductor, and a switch. Before time t = 0, the switch is open. At time t = 0, the switch is closed and remains closed. A few general things to realize:

- The initial current in the circuit, at time t = 0, is zero.
- The inductor opposes the change in current in the circuit which is what causes the current to change from its initial current of zero to its final steady state current.
- After a long time, the inductor behaves like any other ideal wire in a circuit and has zero resistance. In other words, after a long time the current has reached its maximum value and behaves as if the inductor is not there.



Let's determine equations for the limits. To do so, we use Kirchhoff's Loop Rule starting in the lower left-hand corner of the circuit:

$$\Delta V_{\text{Loop}} = 0 = \varepsilon - \Delta V_R - \Delta V_L = \varepsilon - IR - L\frac{dI}{dt}$$

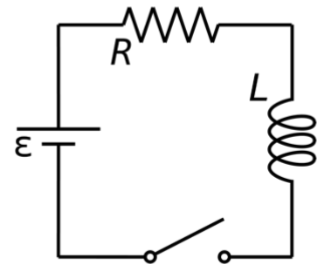
We can use this equation to determine the remaining limits.

$@\ t_i = 0; I_i = 0$

$$\Rightarrow 0 = \varepsilon - L\frac{dI}{dt} \Rightarrow \varepsilon = L\frac{dI}{dt} \Rightarrow \frac{dI}{dt} = \frac{\varepsilon}{L} \Rightarrow \left(\frac{dI}{dt}\right)_{\text{initial}} = \frac{\varepsilon}{L} \text{ [max value]}$$

$$@\ t_f \approx \infty; \left(\frac{dI}{dt}\right)_{\text{final}} = 0 \Rightarrow 0 = \varepsilon - IR \Rightarrow I_f = \frac{\varepsilon}{R} \text{ [max value]}$$

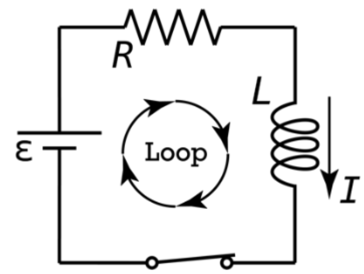
Previously we learned about these basics of an LR circuit:



LR Circuit Limits:

- At $t_{initial} = 0$; $I_{initial} = 0$ & $\left(\frac{dI}{dt}\right)_{initial} = \frac{\varepsilon}{L}$ [max value]
- At $t_{final} \approx \infty$; $I_f = \frac{\varepsilon}{R}$ [max value] & $\left(\frac{dI}{dt}\right)_{final} = 0$

Today we are going to derive the equations for current as a function of time and the time rate of change of current as a function of time. To do this we use with Kirchhoff's Loop Rule starting in the lower left-hand corner of the LR circuit.



$$\Delta V_{Loop} = 0 = \varepsilon - \Delta V_R - \Delta V_L = \varepsilon - IR - L\frac{dI}{dt}$$

$$\Rightarrow L\frac{dI}{dt} = \varepsilon - IR \Rightarrow \frac{L}{R}\frac{dI}{dt} = \frac{\varepsilon}{R} - I$$

$$\text{\& Let } u = \frac{\varepsilon}{R} - I \Rightarrow du = -dI \Rightarrow \frac{L}{R}\frac{-du}{dt} = u \Rightarrow \frac{du}{u} = -\frac{R}{L}dt$$

$$\Rightarrow \int \frac{du}{u} = \int -\frac{R}{L}dt \Rightarrow \int_{u_i}^{u_f} \frac{1}{u}du = -\frac{R}{L}\int_0^t dt \Rightarrow \ln u\Big|_{u_i}^{u_f} = -\frac{R}{L}t\Big|_0^t$$

$$\text{\& } \int \frac{dx}{x-a} = \ln|x-a| \Rightarrow \int \frac{du}{u} = \ln|u| = \ln u$$

- In this problem a = 0 and, because I varies from 0 to $\frac{\varepsilon}{R}$, u is always positive (or zero).

$$\Rightarrow \ln u_f - \ln u_i = \ln\left(\frac{u_f}{u_i}\right) = -\frac{R}{L}t \Rightarrow e^{\left(\ln\left(\frac{u_f}{u_i}\right)\right)} = e^{\left(-\frac{R}{L}t\right)} \Rightarrow \frac{u_f}{u_i} = e^{\left(-\frac{Rt}{L}\right)}$$

$$\Rightarrow u_f = u_i e^{\left(-\frac{Rt}{L}\right)} \text{ \& } u_f = \frac{\varepsilon}{R} - I_f \text{ \& } u_i = \frac{\varepsilon}{R} - I_i = \frac{\varepsilon}{R}$$

$$\Rightarrow \frac{\varepsilon}{R} - I_f = \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)} \Rightarrow -I_f = -\frac{\varepsilon}{R} + \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)} \Rightarrow I_f = \frac{\varepsilon}{R} - \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)}$$

$$\Rightarrow I(t) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{Rt}{L}\right)}\right) = I_{max}\left(1 - e^{\left(-\frac{Rt}{L}\right)}\right)$$

Note, this fits our limits because:

- $I(0) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{R(0)}{L}\right)}\right) = \frac{\varepsilon}{R}\left(1 - e^0\right) = \frac{\varepsilon}{R}(1-1) = 0$
- $I(\infty) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{R(\infty)}{L}\right)}\right) = \frac{\varepsilon}{R}\left(1 - e^{-\infty}\right) = \frac{\varepsilon}{R}(1-0) = \frac{\varepsilon}{R}$

We can also determine the time rate of change of current as a function of time:

$$I_f = \frac{\varepsilon}{R} - \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)} \Rightarrow \frac{dI}{dt} = \frac{d}{dt}\left(\frac{\varepsilon}{R} - \frac{\varepsilon}{R}e^{\left(-\frac{Rt}{L}\right)}\right) = \left(-\frac{\varepsilon}{R}\right)\frac{d}{dt}e^{\left(-\frac{Rt}{L}\right)} = -\left(\frac{\varepsilon}{R}\right)\left(\frac{R}{L}\right)e^{\left(-\frac{Rt}{L}\right)}$$

$$\Rightarrow \frac{dI}{dt}(t) = \frac{\varepsilon}{L}e^{\left(-\frac{Rt}{L}\right)} = \left(\frac{dI}{dt}\right)_{max} e^{\left(-\frac{Rt}{L}\right)} \ \& \ \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

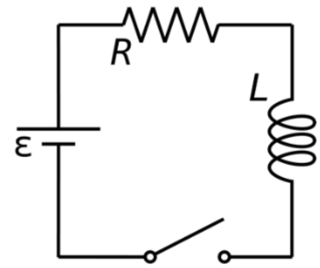
Again, this fits our limits because:

$$\frac{dI}{dt}(0) = \frac{\varepsilon}{L}e^{\left(-\frac{R(0)}{L}\right)} = \frac{\varepsilon}{L}e^{0} = \frac{\varepsilon}{L} \ \& \ \frac{dI}{dt}(\infty) = \frac{\varepsilon}{L}e^{\left(-\frac{R(\infty)}{L}\right)} = \frac{\varepsilon}{L}e^{-\infty} = 0$$

Flipping Physics Lecture Notes:
Time Constant - LR Circuit
http://www.flippingphysics.com/lr-circuit-time-constant.html

I'm not gonna lie, you really do need to have learned from these two previous lessons of mine in order to understand this:

- LR Circuit Basics
- LR Circuit Equation Derivations



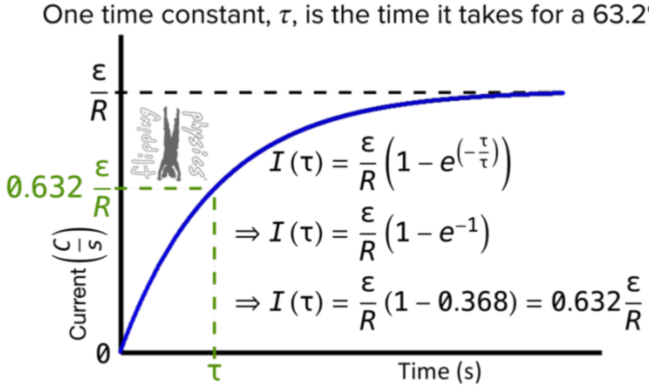
LR Circuit:

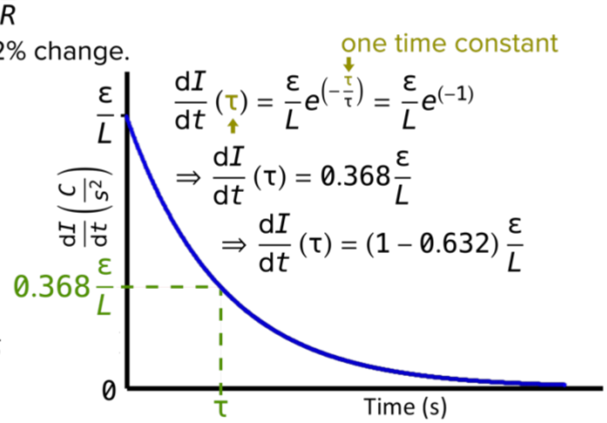
- At $t_{initial} = 0$; $I_{initial} = 0$ & $\left(\frac{dI}{dt}\right)_{initial} \overset{\text{[max value]}}{=} \frac{\varepsilon}{L}$
- At $t_{final} \approx \infty$; $I_f \overset{\text{[max value]}}{=} \frac{\varepsilon}{R}$ & $\left(\frac{dI}{dt}\right)_{final} = 0$

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{Rt}{L}\right)}\right) = I_{max}\left(1 - e^{\left(-\frac{Rt}{L}\right)}\right) \ \& \ \frac{dI}{dt}(t) = \frac{\varepsilon}{L}e^{\left(-\frac{Rt}{L}\right)} = \left(\frac{dI}{dt}\right)_{max} e^{\left(-\frac{Rt}{L}\right)}$$

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{t}{\tau}\right)}\right) \ \& \ \frac{dI}{dt}(t) = \frac{\varepsilon}{L}e^{\left(-\frac{t}{\tau}\right)} \Rightarrow \tau = \frac{L}{R}$$

One time constant, τ, is the time it takes for a 63.2% change.





Flipping Physics Lecture Notes:
Analogies Between LR Circuits and Falling Objects
http://www.flippingphysics.com/lr-circuit-falling-object.html

I'm not gonna lie, you really do need to have learned from these three previous lessons of mine in order to understand this:

- LR Circuit Basics
- LR Circuit Equation Derivations
- Time Constant - LR Circuit

We can consider derivative of current with respect to time to be like acceleration of moving objects.

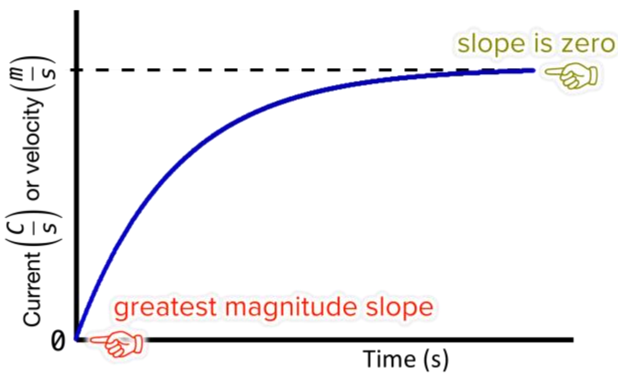
- I is in Amps or $\frac{C}{s}$ & v is in $\frac{m}{s}$
  - Current is like velocity.
- $\frac{dI}{dt}$ is in $\frac{C}{s^2}$ & $a = \frac{dv}{dt}$ in $\frac{m}{s^2}$
  - Derivative of current with respect to time is like acceleration.

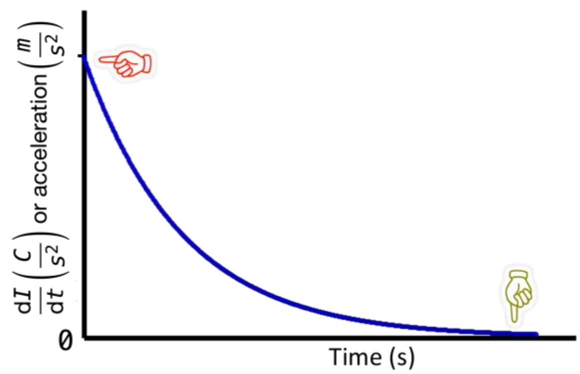
LR Circuit:

$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{\left(-\frac{t}{\tau}\right)}\right) \quad \& \quad \frac{dI}{dt}(t) = \frac{\varepsilon}{L}e^{\left(-\frac{t}{\tau}\right)}$$

Dropped Object with Drag Force:

$$v(t) = v_{\text{terminal}}\left(1 - e^{-\frac{t}{\tau}}\right) \quad \& \quad a(t) = ge^{-\frac{t}{\tau}}$$

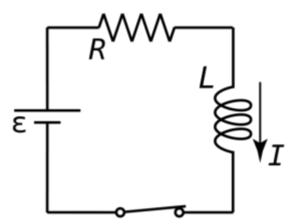


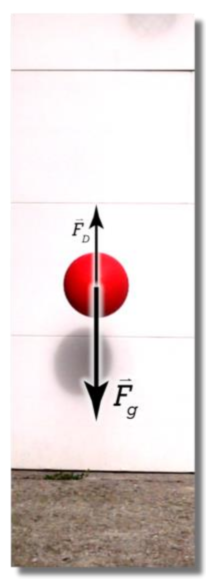


$$\Delta V_{\text{Loop}} = 0 = \varepsilon - \Delta V_R - \Delta V_L = \varepsilon - IR - L\frac{dI}{dt}$$

$$\sum F_y = F_g - F_D = ma_y \Rightarrow 0 = F_g - F_D - ma_y = F_g - F_D - m\frac{dv}{dt}$$

(down is positive)





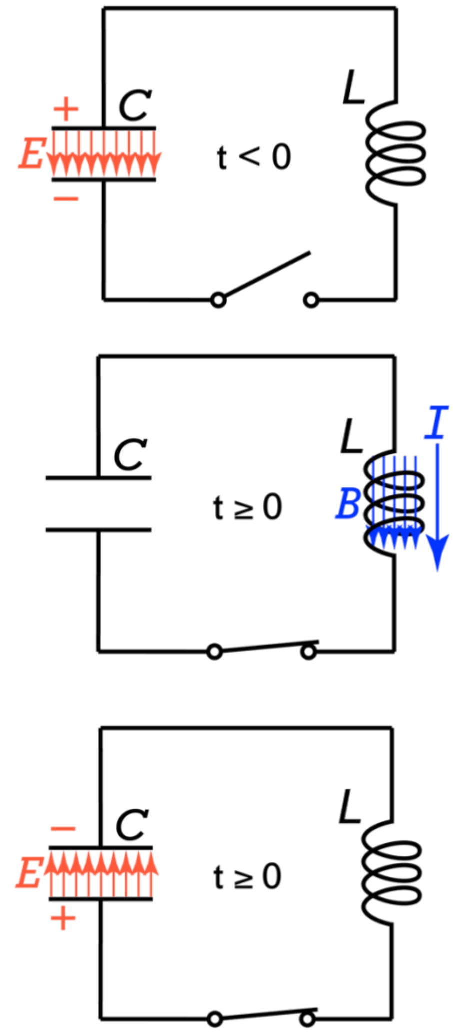
We can consider derivative of current with respect to time to be like acceleration of moving objects.

- ε is like F_g; constant values attempting to cause changes in their systems.
- ΔV_R is like F_D; dissipating energy from their systems.
- ΔV_L is like ma;
  - L is like m; L opposes changes in I and m opposes changes in v.
  - $\frac{dI}{dt}$ is like $\frac{dv}{dt}$

Units for ΔV and force don't match:

$$\Delta V = \frac{\Delta U_e}{q} \Rightarrow \Delta V \text{ in volts} = \frac{J}{C} = \frac{N \cdot m}{C} \quad \& \quad \text{Force in newtons}$$

Flipping Physics Lecture Notes:
LC Circuit Basics
http://www.flippingphysics.com/lc-circuit.html



This LC circuit is a circuit with a capacitor, an inductor, and a switch. Before time t = 0, the switch was open for a long time. At time t = 0, the switch is closed and remains closed. A few general things to realize:

- The initial charge on the capacitor must be nonzero, if it were zero, nothing would happen when the switch is closed.
- The initial current in the circuit must be zero because there was no current in the open circuit before the switch was closed.
- The inductor opposes the change in current in the circuit which is why it takes time for the current to change from zero.
- The current through the inductor is from the charges leaving the capacitor to flow through the circuit, therefore, as current through the inductor increases, charge on the capacitor decreases.
- The electric field in the capacitor is decreasing in magnitude and the magnetic field in the inductor is increasing in magnitude.
- Once the charge is completely discharged, $q = 0$, the inductor has its maximum magnitude magnetic field and the current through the inductor is at its maximum.
- Current will continue to flow and build up charges on the plates of the capacitor, however, the orientation of the positive and negative plates will be reversed, and the current is decreasing.
- Eventually the current through the inductor will reduce to zero and charge will be at a maximum on the plates of the capacitor.
- Repeat the whole cycle in reverse.
- This is *simple harmonic motion!*
    - A horizontal mass-spring system is a good analogous situation.

We have already learned the basics of how an LC circuit works. Now let's derive equations for the LC Circuit, starting with the total energy in the circuit:

$$U_t = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2 = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

- Typically, we use uppercase symbols for constants and lowercase symbols for variables.
- We know $I_{max} \to q = 0 \ \& \ Q_{max} \to i = 0$
- We can take the derivative with respect to time of the total energy equation. We know the derivative of total energy in the LC circuit equals zero because these are all ideal components with zero resistance. In other words, no energy is being dissipated from the system.

$$\Rightarrow \frac{dU_t}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right) = 0$$

- We need to use the chain rule for both energy expressions because time is not a variable in either energy expression, however, both charge, q, and current, i, are changing with respect to time.

$$\Rightarrow 0 = \frac{d}{dt}\left(\frac{q^2}{2C}\right) + \frac{d}{dt}\left(\frac{1}{2}Li^2\right) \Rightarrow 0 = \left(\frac{2q}{2C}\right)\frac{dq}{dt} + \left(\frac{2Li}{2}\right)\frac{di}{dt}$$

$$\Rightarrow 0 = \left(\frac{q}{C}\right)\frac{dq}{dt} + (Li)\frac{di}{dt} \ \& \ i = \frac{dq}{dt} \ \& \ \frac{di}{dt} = \frac{d^2q}{dt^2}$$

$$\Rightarrow 0 = \left(\frac{q}{C}\right)i + (Li)\frac{d^2q}{dt^2} = \frac{q}{C} + (L)\frac{d^2q}{dt^2} \Rightarrow -\frac{q}{C} = (L)\frac{d^2q}{dt^2} \Rightarrow \frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

- The equation definition for simple harmonic motion is: $\frac{d^2x}{dt^2} = -\omega^2 x$
- Therefore, we know the angular frequency of an LC circuit And we can determine the period of an LC Circuit:

$$\Rightarrow \omega_{LC}^2 = \frac{1}{LC} \Rightarrow \omega_{LC} = \frac{1}{\sqrt{LC}}$$

$$\& \ \omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow T_{LC} = \frac{2\pi}{1/\sqrt{LC}} \Rightarrow T_{LC} = 2\pi\sqrt{LC}$$

- And we know a general equation which satisfies the simple harmonic motion equation:

$$x(t) = A\cos(\omega t + \phi) \Rightarrow q(t) = Q_{max}\cos\left(\frac{t}{\sqrt{LC}} + \phi\right) \Rightarrow q(t) = Q_{max}\cos\left(\frac{t}{\sqrt{LC}}\right)$$

- For this specific LC circuit the initial charge on the capacitor is Q_{max}, therefore, the phase constant is zero.

$$\& \ i = \frac{dq}{dt} \Rightarrow i(t) = \frac{d}{dt}\left[Q_{max}\cos\left(\frac{t}{\sqrt{LC}}\right)\right] = -Q_{max}\sin\left(\frac{t}{\sqrt{LC}}\right)\frac{d}{dt}\left(\frac{t}{\sqrt{LC}}\right)$$

- We can also determine current in an LC circuit as a function of time and an equation relating current maximum to charge maximum.

$$\Rightarrow i(t) = -\frac{Q_{max}}{\sqrt{LC}} \sin\left(\frac{t}{\sqrt{LC}}\right) \Rightarrow I_{max} = \frac{Q_{max}}{\sqrt{LC}} \Rightarrow i(t) = -I_{max} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

- We can also derive the current maximum using the equation for total energy in the LC circuit.

$$U_t = U_C + U_L = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{Q_{max}^2}{2C} + 0 = 0 + \frac{1}{2}LI_{max}^2 \Rightarrow \frac{Q_{max}^2}{C} = LI_{max}^2$$

$$\Rightarrow I_{max}^2 = \frac{Q_{max}^2}{LC} \Rightarrow I_{max} = \frac{Q_{max}}{\sqrt{LC}}$$

- We can determine equations for energy as functions of time.

$$U_C = \frac{q^2}{2C} \Rightarrow U_C(t) = \frac{\left[Q_{max}\cos\left(\frac{t}{\sqrt{LC}}\right)\right]^2}{2C} \Rightarrow U_C(t) = \frac{Q_{max}^2}{2C}\cos^2\left(\frac{t}{\sqrt{LC}}\right)$$

$$U_L = \frac{1}{2}Li^2 \Rightarrow U_L(t) = \frac{1}{2}L\left[-\frac{Q_{max}}{\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right)\right]^2 = \left(\frac{1}{2}L\right)\left(\frac{Q_{max}^2}{LC}\right)\sin^2\left(\frac{t}{\sqrt{LC}}\right)$$

$$\Rightarrow U_L(t) = \frac{Q_{max}^2}{2C}\sin^2\left(\frac{t}{\sqrt{LC}}\right)$$

$$U_t(t) = U_C(t) + U_L(t) = \frac{Q_{max}^2}{2C}\cos^2\left(\frac{t}{\sqrt{LC}}\right) + \frac{Q_{max}^2}{2C}\sin^2\left(\frac{t}{\sqrt{LC}}\right)$$

$$\Rightarrow U_t(t) = \left(\frac{Q_{max}^2}{2C}\right)\left[\cos^2\left(\frac{t}{\sqrt{LC}}\right) + \sin^2\left(\frac{t}{\sqrt{LC}}\right)\right] \Rightarrow U_t(t) = \frac{Q_{max}^2}{2C}$$ & $\sin^2\theta + \cos^2\theta = 1$

Flipping Physics Lecture Notes:
Visualizing LC Circuits
http://www.flippingphysics.com/lc-circuit-visualizing.html

Below are two screenshots of the LC circuit animation. Honestly, you need to watch and hear the discussion of everything going on in the animation to understand it.