Thank You, Emily Rencsok, for these notes.

Video Lecture #1 – Defining Work with the Dot Product and a Review of the Dot Product - Constant Force

Work w/ constant force $\quad W = F\Delta r\cos\theta$

Joule = N · m

Dot product/scalar product

$W = F \cdot r$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A} = 4\hat{i} + 3\hat{j} + 0\hat{k}$

$\vec{B} = -2\hat{i} + 0\hat{j} + 4\hat{k}$

$\vec{A}\cdot\vec{B} = (4)(-2) + (3)(0) + (0)(4)$

$= -8$

Video Lecture #2 – Some Introductory Example Problems of Work using the Dot Product - Constant Force

Ex) $\vec{F} = 35N$ @ $25°$ below horiz

$\Delta r = [50.0\,\hat{i}]\,m$

$W = \vec{F}\cdot\vec{\Delta r}$ $\qquad \vec{F} = [35\cos 25\,\hat{i} - 35\sin 25\,\hat{j}]\,N$

$W = (50)(35\cos 25) + 0(-35\sin 25)$

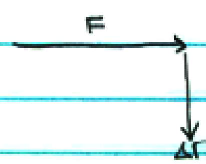
$W = 1586$

$W = 1600\ J$ or $1.6\ kJ$

$W = F\cdot\Delta r = F\Delta r\cos\theta$

$= (35)(50)\cos 0$

$= 1750\ J$



$W = (35)(50)\cos 90$

$W = 0\ J$

Video Lecture #3 – Introduction to Work with a Non-Constant Force using an Integral

Non-constant force

$$W = \int_{x_i}^{x_f} F_x \, dx$$

Video Lecture #4 – Introduction to the Integral or Anti-Derivative for use in Work from a Non-Constant Force

Integral/Anti-derivative

$y = x^2 \quad \frac{dy}{dx} = 2x \qquad y = 3x^3 \quad \frac{dy}{dx} = 9x^2$

$y = \int 2x\,dx = \frac{2x^2}{2} = x^2 \qquad y = \int 9x^2\,dx = \frac{9x^3}{3} = 3x^3$

$y = \int 4x^4\,dx = \frac{4x^5}{5}$

$y = \int (6x^2 + x)\,dx = \frac{6x^3}{3} + \frac{x^2}{2} = 2x^3 + \frac{1}{2}x^2$

derivative = slope of line

Integral = area "under" curve

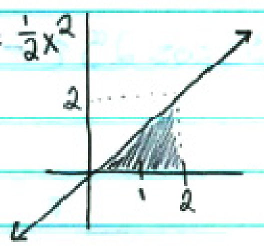
$\int_0^2 x\,dx = \frac{1}{2}x^2$

$\frac{1}{2}x^2 \Big]_0^2$

$= \frac{x_f^2}{2} - \frac{x_i^2}{2}$

$= \frac{2^2}{2} - \frac{0^2}{2}$

$= 2$



$A = \frac{1}{2}bh$

$= \frac{1}{2}(2)(2)$

$= 2$

$$\int_{-1}^{2} 3x^2\,dx$$

$$= \frac{3x^3}{3}\Big]_{-1}^{2}$$

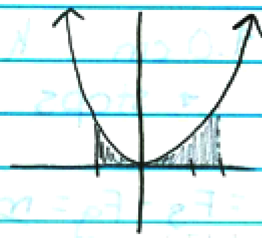
$$= x^3\Big]_{-1}^{2}$$

$$= x_f^{\,3} - x_i^{\,3}\Big]_{-1}^{2}$$

$$= (2)^3 - (-1)^3$$

$$= 8 + 1$$

$$= 9$$



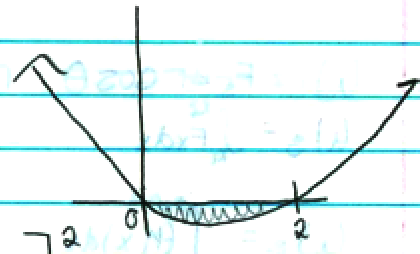
$$\int_{0}^{2} (x^2 - 2x)\,dx = \frac{x^3}{3} - \frac{2x^2}{2}\Big]_0^2$$

$$= \frac{x^3}{3} - x^2\Big]_0^2$$

$$= \left[\frac{x_f^{\,3}}{3} - x_f^{\,2}\right] - \left[\frac{x_i^{\,3}}{3} - x_i^{\,2}\right]_0^2$$

$$= \left(\frac{2^3}{3} - 2^2\right) - 0$$

$$= \frac{8}{3} - 4 = \boxed{-\tfrac{4}{3}}$$



Video Lecture #5 – Introductory Example Problem: Work with a Non-Constant Force using an Integral

ex) $F_x = (4x - 8)\ N$

$$W_{(0\text{-}5m)} = \int_0^5 (4x-8)\,dx = \frac{4x^2}{2} - \frac{8x}{1}\Big]_0^5$$

$$= 2x^2 - 8x\Big]_0^5$$

$$= \left(2(5)^2 - 8(5)\right) - 0$$

$$= 50 - 40 = \boxed{10\ J}$$

$F_s = -kx$

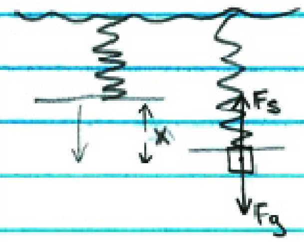
k = spring constant $\left(\frac{N}{m}\right)$

x = displacement from equilibrium position (rest position)

F_s is opposite direction of displacement

Ex| $m = 1.0\ kg$ $x = 4.0\ cm$ $k = ?$

& stops



$$\Sigma F_y = F_s - F_g = \cancel{ma_y} = 0$$
$$F_s = F_g$$
$$kx = mg$$
$$k(0.04) = (1)(9.8)$$
$$k = 245\ N/m$$
$$\boxed{k = 240\ N/m}$$

$\cancel{W_s = F_s \Delta r \cos\theta}$ not constant

$$W_s = \int_{x_i}^{x_f} F_x\,dx$$

$$W_s = \int_0^{-0.04} (-kx)\,dx$$
$$= \frac{-kx^2}{2}\Big]_0^{-0.04} = \frac{-kx_f^2}{2} - \left(\frac{-kx_i^2}{2}\right)\Big]_0^{-0.04} = \frac{-245(-0.04)^2}{2} - 0 = \boxed{-0.196\ J}$$

$$PE_e = \frac{1}{2}kx^2$$

Video Lecture #7 – Deriving Kinetic Energy and the Net Work Equals Change in Kinetic Energy Equation

Thank You, Amanda Ciccarelli, for these notes.

$$\Sigma W = \int_{x_i}^{x_f} \Sigma F\,dx = \int_{x_i}^{x_f} ma\,dx = \int_{x_i}^{x_f} m\frac{dv}{dt}\,dx \qquad \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$= \int_{x_i}^{x_f} m\frac{dv}{dx}\frac{dx}{dt}\,dx = \int_{x_i}^{x_f} m\frac{dx}{dt}\,dv = \int_{v_i}^{v_f} mv\,dv = \frac{mv^2}{2}\Big]_{v_i}^{v_f}$$

$$\Sigma W = \frac{1}{2}mv^2\Big]_{v_i}^{v_f}$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$$

$$\boxed{\Sigma W = \Delta KE}$$

not on equation sheet

Video Lecture #8 – An Introductory Example Problem using The Net Work equals The Change In Kinetic Energy

EX| $m = 5.0\ kg$ $v_i = [4.0\hat{i} - 3.0\hat{j}]\ m/s$ $v_f = [2.0\hat{i} + 6.0\hat{j}]\ m/s$

$\Sigma W = ? = \Delta KE = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$= \frac{1}{2}m(v_f^2 - v_i^2)$

$= \frac{1}{2}(5)[(2\hat{i}+6\hat{j})^2 - (4\hat{i}-3\hat{j})^2)$

$= \frac{1}{2}(5)(2^2+6^2-(4^2+3^2)) = 37.5\ J$

Video Lecture #9 – Chapter 07 #34 - A Problem - Work and Energy on a Block with a Force Applied and Unit Vectors

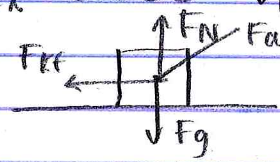
7-34| $m = 15\ kg$ $F_a = 70.0\ N$ @ $20.0°$ above horiz

$\Delta x = 5.00\ m$ $\mu_k = 0.300$ $v_i = 0$ $v_f = ?$

a) $W = ?$ (all forces)

d) $\Delta E_{int} = ?$

e) $\Delta KE = ?$



$F_a = F_a\cos\theta\,\hat{i} + F_a\sin\theta\,\hat{j}$

$= 70\cos 20\,\hat{i} + 70\sin 20\,\hat{j}$

$F_a = 65.779\hat{i} + 23.941\hat{j}\ N$

$\Sigma\vec{F} = F_a + F_g + F_N + F_{kf} = m\vec{a}$

$[65.778\hat{i} + 23.941\hat{j}] + (-mg\hat{j}) + F_N\hat{j} + (-\mu_k F_N\hat{i}) = m(a_x\hat{i} + 0\hat{j})$

$(\hat{j})$ $23.941 - mg + F_N = 0$ $F_N = mg - 23.941$

$= 15(9.8) - 23.941$ $F_N = 123.059\ N$

$(\hat{i})$ $65.778 - \mu_k F_N = ma_x$ $a_x = \frac{65.778 - \mu_k F_N}{m}$

$a_x = \frac{65.778 - (0.3)(123.059)}{15}$

$a_x = 1.9240\ m/s^2$

$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$

$= \sqrt{0 + 2(1.9247)(5)}$ $v_{fx} = 4.3864 \approx 4.39\ m/s$

$W_{F_N} = F_N \cdot \Delta r = 123.059\hat{j} \cdot 5\hat{i} = 0$

$W_{F_a} = F_a \cdot \Delta r = (65.778\hat{i} + 23.941\hat{j}) \cdot (5\hat{i}) = 328.89 \approx 329\ J$

$W_{F_g} = F_g \cdot \Delta r = 0$ $W_{F_{kf}} = F_f \cdot \Delta r = (-\mu_k F_N\hat{i}) \cdot (5\hat{i}) = -185\ J$

$\Delta E_{int} = -W_{F_f} = -(-184.588) = 185\ J$

$\Sigma W = \Delta KE = 0 + 328.89 + 0 - 184.588 = 144.305 \approx 144\ J$

$\Delta E_{sys} = \Sigma T$ $\Delta ME + \Delta E_{int} = \Sigma T$

↑ transferred $144 + 185 = 329$

$P = \frac{\Delta W}{\Delta t}$ $P = \frac{dW}{dt}$ $\frac{J}{s} = \text{watts}$

Average Instantaneous

746 watts = 1 hp (will be given)

Rate at which work is done

$P = \frac{dW}{dt} = \frac{d}{dt}(F\Delta r\cos\theta)$

$= F\frac{dr}{dt}\cos\theta = Fv\cos\theta$

Ex) $m = 0.280\ kg$ Net power @ 2.00 s = ?

$x(t) = (5t^3 - 8t^2 - 30t)\ m$ ↖instantaneous

$v(t) = \frac{dx}{dt} = (15t^2 - 16t - 30)\ m/s$

$\Sigma\vec{F} = m\vec{a} = m(30t - 16)$

$a(t) = \frac{dv}{dt}(30t - 16)\ m/s^2$

$P_{net} = F_{net}v\cos\theta$

$= (m(30t-16))(15t^2 - 16t - 30)\cos\theta$

$= (0.28)(30(2)-16)(15(2)^2 - 16(2) - 30)\cos\theta$

$= -24.64\cos\theta$

$= -24.64\cos(180)$

$= \boxed{24.6\ \text{watts}}$

$v(2) = 15(2)^2 - 16(2) - 30$

$v(2) = -2\ m/s$

$a(2) = 30(2) - 16$

$a(2) = 44\ m/s^2$

Video Lecture #11 – Mechanical Power Example Problem using an Integral - Power as a function of Time

Ex) $P_{net} = (4.00t^2 + t)$ W $\quad W_{(0-4s)} = ?$

$W = \int_{x_i}^{x_f} F_{net}\, dr \qquad (dt)\, P = \frac{dW}{\cancel{dt}} \cancel{(dt)}$

$$dW = P\,dt$$

$$\int_{W_i}^{W_f} dW = \int_{t_i}^{t_f} P\,dt$$

$$W_f - W_i = \int_{t_i}^{t_f} P\,dt$$

$$\Delta W = \int_{t_i}^{t_f} P\,dt$$

$$W_{(0-4)} = \int_0^4 (4t^2 + t)\,dt$$

$$= \left.\frac{4t^3}{3} + \frac{t^2}{2}\right]_0^4$$

$$= \left(\frac{4t_f^3}{3} + \frac{t_f^2}{2}\right) - \left.\left(\frac{4t_i^3}{3} + \frac{t_i^2}{2}\right)\right]_0^4$$

$$= \left(\frac{4(4)^3}{3} + \frac{4^2}{2}\right) - 0$$

$$= \frac{280}{3}$$

$$= 93.\overline{3}$$

$$= \boxed{93.3\ J}$$

Video Lecture #12 – Chapter 07 #54 - A Problem where using Net Work equals Change in Kinetic Energy makes it Easier

VAM + free fall

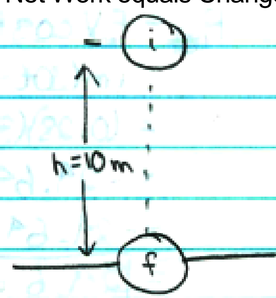
Blah blah blah

7-54) $m = 5$ kg

$h = 10$ m

dent $= 3.20$ mm

$F_{avg\ plate\ on\ ball} = ?$



$$W_{net} = \Delta KE = KE_f - KE_i$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$W_{net} = 0 - 0 = 0$$

$$W_{F_a} + W_{F_g} = 0$$

$$W_{F_a} = -W_{F_g}$$

only use magnitude

$$F_a \Delta r \cos\theta = -F_g \Delta r \cos\theta$$

$$F_a(0.0032)\cos 180 = -mg(10.0032)\cos(0)$$

$$F_a(0.0032)(-1) = -5(9.8)(10.0032)(1)$$

$$F_a = 153{,}174 = \boxed{153\ kN}$$

Video Lecture #13 – Introduction to Gravitational and Elastic Potential Energies

Potential energy, PE

$U_g = mgh$ $\qquad$ $U_s = \frac{1}{2}kx^2$

h = ht above zero-line

Video Lecture #14 – Derivation of Conservation of Mechanical Energy

$\Delta E_{sys} = \Sigma T$ ← energy transferred

$\Delta ME + \Delta E_{int} = W_{Fa}$

$\Delta ME = 0$

$ME_f - ME_i = 0$

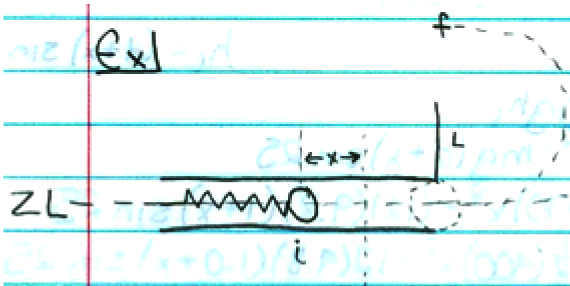
$\boxed{ME_f = ME_i}$ No Fa, No friction

no Fa → $W_{Fa} = 0$

no friction → $\Delta E_{int} = -W_f = 0$

Video Lecture #15 – Conservation of Mechanical Energy Problem involving a Spring and No Numbers

Ex1



Knowns: m, k, x, L

$\mu_k = 0$ $\qquad$ $V_{top} = ?$

$ME_i = ME_f$

$\frac{1}{2}kx_i^2 = mgh_f + \frac{1}{2}mv_f^2$

$\frac{1}{2}kx^2 = mg(2L) + \frac{1}{2}mv_f^2$

$kx^2 = 4mgL + mv_f^2$

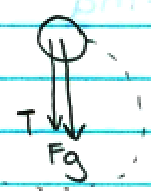
$kx^2 - 4mgL = mv_f^2$

$\boxed{V_f = \sqrt{\frac{kx^2 - 4mgL}{m}}}$

b) $x_{min} = ?$ sphere barely makes it to the top

$V_{top} \neq 0$

$T = 0$



$\Sigma F_{in} = \cancel{T} + F_g = ma_c$

$\cancel{m}g = \frac{\cancel{m}v_t^2}{r}$

$g = \frac{v_t^2}{r} = \frac{v_t^2}{L}$

$v_t = \sqrt{Lg}$

$\sqrt{Lg} = \sqrt{\frac{kx^2 - 4mgL}{m}}$

$mLg = kx^2 - 4mgL$

$5mLg = kx^2$ $\qquad$ $x = \sqrt{\frac{5mLg}{k}}$

Video Lecture #16 – Derivation of Work due to Friction Equation with Example Problem (uses Quadratic Formula)

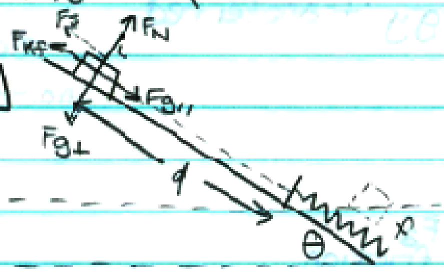
$\Delta E_{sys} = \Sigma T$

$\Delta ME + \Delta E_{int} = 0$ No F_a

$\Delta ME + (-W_f) = 0$ $\Delta E = -W_f$

$\Delta ME = W_f$

Ex)



$V_i = 0$
$K = 400.0\ N/m$
$\theta = 25°$
$\mu_K = 0.15$ $x_{max} = ?$
$m = 12\ kg$
$d = 1.0\ m$

$\Sigma F_\perp = F_N - F_{g\perp} = 0$

$F_N = F_{g\perp}$

$F_N = mg\cos\theta$ $W_f = \Delta ME = ME_f - ME_i$ $h_i = (d+x)\sin\theta$

$F_{kf}\Delta r\cos\theta = \frac{1}{2}Kx_f^2 - mgh_i$

$\mu_K F_N (d+x)\cos 180 = \frac{1}{2}Kx^2 - mg(d+x)\sin 25$

$(0.15)(mg\cos\theta)(1+x)(-1) = \frac{1}{2}(400)x^2 - (12)(9.8)(1+x)\sin 25$

$-(0.15)(12)(9.8)\cos 25(1+x) = \frac{1}{2}(400)x^2 - 12(9.8)(1.0+x)\sin 25$

$-15.987x - 15.987 = 200x^2 - 49.6999 - 49.6999x$

$x = 0.503$

$\boxed{x = 50.3\ cm}$

Video Lecture #17 – Introduction to the Conservative Force and its Potential Energy with Example Problem

memorize

$F_x = -\frac{dU}{dx}$ conservative forces

$F_s = \frac{-dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right)$

$F_s = -kx$

$F_g = \frac{-dU_g}{dy} = -\frac{d}{dy}(mgy)$

$F_g = -mg$

Ex) $F_x = [-4.0x + 8.0x^3]\hat{i}$ N

Find $\Delta U_{(x)}$ (2.0 - 4.0 m)

$F_x = -\frac{dU}{dx}$

$F_x\,dx = -dU$

$\int_{x_i}^{x_f} F_x\,dx = -\int_{U_i}^{U_f} dU$

$\Delta U = -\int_{x_i}^{x_f} F_x\,dx = -W_{F_x}$

$\Delta U = -\int_2^4 (-4x + 8x^3)\,dx$

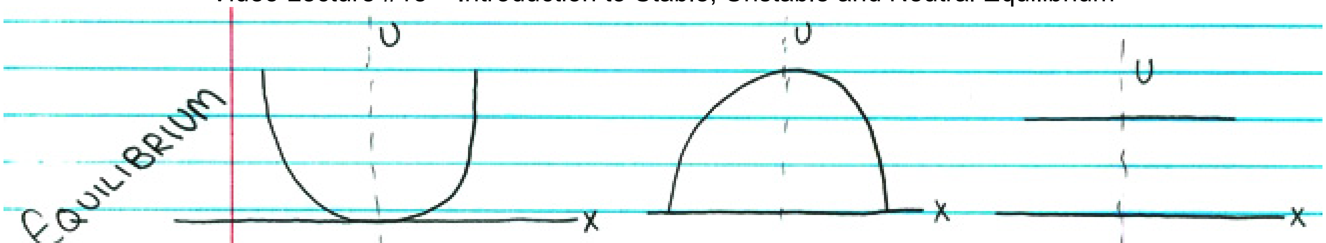
$\Delta U = -\left[-\frac{4x^2}{2} + \frac{8x^4}{4}\right]_2^4$

$= 2x^2 - 2x^4\Big]_2^4$

$= 2(4)^2 - 2(4)^4 - [2(2)^2 - 2(2)^4]$

$= \boxed{-456\text{ J}}$

Video Lecture #18 – Introduction to Stable, Unstable and Neutral Equilibrium



stable (pushing over water bottle)

unstable (pushing over highlighter)

neutral (ball rolling)

Ex $P = ?$ to go up

$V_{up} = 5\,m/s$

$V_{down} = 5\,m/s$

$\theta = 7.00°$

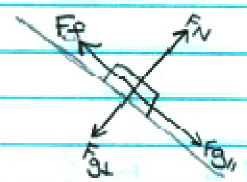
$m = 75\ kg$

$P = F_a V \cos\theta$

$P = (179.1479)(5)\cos\theta$

$P = 895.739674$

$\boxed{P = 9.0 \times 10^2\ W}$



Down

$\underset{down}{\Sigma F_\perp} = F_N - F_{g\perp} = \cancel{ma_\perp} = 0$

$F_N = mg\cos\theta = (75)(9.8)\cos 7 = 729.52N$

$\underset{down}{\Sigma F_\parallel} = F_f - F_{g\parallel} = \cancel{ma_\parallel} = 0$

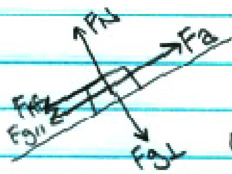
$F_f = F_{g\parallel}$

$\mu_k F_N = mg\sin\theta$

$\mu_k(\cancel{mg}\cos\theta) = \cancel{mg}\sin\theta$

$\mu_k \cos 7 = \sin 7$

$\mu_k = 0.122785$



up

$\underset{up}{\Sigma F_\parallel} = F_a - F_{kf} - F_{g\parallel} = \cancel{ma_\parallel} = 0$

$F_a = \mu_k F_N + mg\sin\theta$

$F_a = (0.122785)(729.52) + (75)(9.8)\sin 7$

$F_a = 179.1479\ N$

7-49

$m = 4\ kg$

$x(t) = (t + 2t^3)\,m$

(a) $KE_t = ?$

(b) $a(t) = ?$ $\quad \Sigma F(t) = ?$

(c) $P(t) = ?$

(d) $W_{(0-2s)} = ?$

$V = \frac{dx}{dt} = 1 + 6t^2 = v(t)$

$KE(t) = \frac{1}{2}(4)(1+6t^2)^2$

$\boxed{KE(t) = (2.00 + 24.0t^2 + 72.0t^4)\ J}$

$a(t) = \frac{dV}{dt} = 12t$

$\boxed{a(t) = (12.0t)\ m/s^2}$

$\Sigma \vec{F} = m\vec{a} = 4(12.0t)$

$\boxed{\Sigma F = (48.0t)\ N}$

$P = \frac{dW}{dt}$

$\int dW = \int P\,dt$

$\Delta W = \int_{t_i}^{t_f} P\,dt$

$\Delta W = \int_0^2 (48t + 288t^3)\,dt$

$\Delta W = 24t^2 + 72t^4 \Big]_0^2$

$\Delta W = 24(2)^2 + 72(2)^4 - 0$

$\Delta W = 1248$

$\boxed{\Delta W = 1250\ J}$

$\vec{P}(t) = F \cdot V = FV\cos\theta$

$P(t) = (48.0t)(1+6t^2)\cos 0$

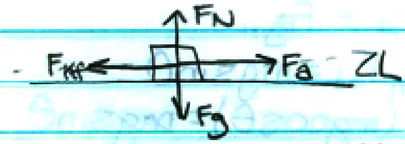
$\boxed{P(t) = (48.0t + 288t^3)\ W}$

$\cancel{W = \int_{x_i}^{x_f} F_x\,dx = \int_{x_i}^{x_f} (48t)\,dx}$

$W_{NET} = \Delta KE$

Video Lecture #20 – A Review of the Equations that relate Work and Mechanical Energy

Ex) $F_a = 45$ N $\mu_K = 0.15$ $CV = 4.5 \frac{m}{s}\,\hat{i}$ $m = ?$



$W_f = \Delta ME$ only when no F_a

$F_{kf} \Delta r \cos\theta = ME_f - ME_i$

$F_{kf} \Delta r \cos\theta = KE_f - KE_i = 0$

$F_{kf} \Delta r \cos 180 = 0$

$-F_{kf} \Delta r = 0$

$W_{net} = \Delta KE$ ALWAYS TRUE

$\Delta E_{sys} = \Sigma T$

$\Delta ME + \Delta E_{int} = W_{Fa}$

$\Delta ME - W_f = W_{Fa}$

$\Delta ME - W_f = 0$

$\Delta ME = W_f$ NO FORCE APPLIED

$0 = \Delta ME$

$0 = ME_f - ME_i$

$ME_f = ME_i$ NO F_A OR F_f

Video Lecture #21 – Chapter 07-08 Review: Work, Mechanical Energy and Power

$F_s = -kx$

$W_{net} = \Delta KE$ always

$W_f = \Delta ME$ No F_A

$ME_i = ME_f$ No F_f + no F_a

$F_x = -\frac{dU}{dx}$ Conservative forces

$\Delta U = -\int_{x_i}^{x_f} F_x\,dx$