Thank You, Emily Rencsok, for these notes.

Video Lecture #1 – Introduction to Momentum and Derivation of Conservation of Momentum

$\vec{p} = m\vec{v}$ $\quad \frac{kg \cdot m}{s}$

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(mv)$$
$$= \frac{dm}{dt}v + m\frac{dv}{dt}$$
$$= \frac{dm}{dt}v + ma$$

$\boxed{\Sigma \vec{F} = \frac{d\vec{p}}{dt}}$ mass can change

Conservation of Momentum

$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

when $\Sigma \vec{F} = \frac{d\vec{p}}{dt} = 0$

Forces are internal

Video Lecture #2 – Introductory Conservation of Momentum Problem using Unit Vectors

Ex) $m_{ball} = 15\ kg$
$v_{i\,ball} = (7.5\hat{i} + 4.7\hat{j})\ m/s$
on ice $\mu = 0$
$m_{you} = 75\ kg$
$v_{f\,you} = ?$

$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$
$$\cancel{m_b v_{ib}} + \cancel{m_y v_{iy}} = m_b v_{fb} + m_y v_{fy}$$
$$0 = m_b v_{fb} + m_y v_{fy}$$
$$-m_b v_{fb} = m_y v_{fy}$$
$$-(15)(7.5\hat{i} + 4.7\hat{j}) = (75)(v_{fy})$$
$$\boxed{v_{fy} = (-1.5\hat{i} - 0.94\hat{j})\ m/s}$$

$$\frac{7.5}{4.7} = \frac{-1.5}{-0.94}$$

Video Lecture #3 – Derivation of Impulse using an Integral and Impulse Approximation

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

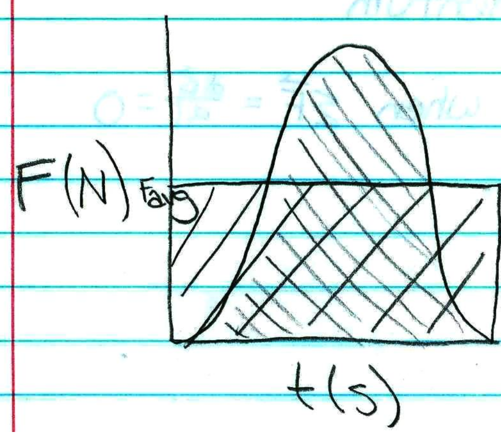
$$\int_{t_i}^{t_f} \sum F\,dt = \int_{p_i}^{p_f} dp \qquad \Delta p = \int_{t_i}^{t_f} \sum F\,dt = \underset{\substack{\uparrow \\ \text{impulse}}}{I} = J$$

Impulse approximation

$F_{collision} >>$ all other forces

$$\sum F = F_{collision} \qquad \Delta p = \int_{t_i}^{t_f} F\,dt = J$$

$$W = \int_{x_i}^{x_f} F\,dx$$



$$I = \underset{\substack{\uparrow \\ F_{avg}}}{\overline{F}}\,\Delta t$$

$a = \frac{dv}{dt}$

$\int dv = \int a\,dt$

indefinite

$V(t) = at + C$ ← C is an unknown (initial condition)

let $t = 0$

$V(0) = a(0) + C$

$V_i = C$

$V(t) = at + V_i$

$V_f = V_i + a\Delta t$

$V = \frac{dx}{dt}$

$\int dx = \int V\,dt$

$x(t) = \int (v_i + at)\,dt$

$x(t) = V_i t + \frac{at^2}{2} + C$

$x(0) = V_i(0) + \frac{a}{2}(0)^2 + C$

$x_i = C$

$x(t) = V_i t + \frac{1}{2}at^2 + x_i$

$x_f = x_i + V_i t + \frac{1}{2}at^2$

Video Lecture #5 – (Part 1 of 3) Dropping a Racket Ball on a Force Sensor - Demonstrating the Power of Calculus

Demo $m_b = 40.5\,g$

$$F = \left[-3.034\times10^{6}t^{2} + 1.887\times10^{4}t + 0.7165\right] N$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-1.887\times10^{4} \pm \sqrt{(1.887\times10^{4})^{2} - 4(-3.034\times10^{6})(0.7165)}}{2(-3.034\times10^{6})}$$

$$t = -0.00003774\,s \quad \text{or} \quad 0.006257\,s$$

$$F(t) = \left(-3.034\times10^{6}t^{2} + 1.910\times10^{4}t\right) N$$

Video Lecture #6 – (Part 2 of 3) Dropping a Racket Ball on a Force Sensor - Demonstrating the Power of Calculus

$$F(t) = \left(-3.034\times10^{6}t^{2} + 1.910\times10^{4}t\right) N$$

$$0 = -3.034\times10^{6}t^{2} + 1.910\times10^{4}t$$

$$3.034\times10^{6}t = 1.910\times10^{4}$$

$$t = 0.006295\,s$$

$$J = \int_0^{0.006295} F\,dt = \int_0^{0.006295} -3.034\times10^{6}t^{2} + 1.910\times10^{4}t$$

$$= \left.\frac{-3.034\times10^{6}t^{3}}{3} + \frac{1.91\times10^{4}t^{2}}{2}\right]_0^{0.006295}$$

$$\boxed{J = 0.1262\,N\cdot s}$$

$$J = F_{avg}\Delta t$$

$$F_{avg} = \frac{J}{\Delta t}$$

$$F_{avg} = \frac{0.1261588}{0.006295}$$

$$F_{avg} = 20.04\ N$$

much bigger than F_g

$$m = 0.0405\ kg$$

$$F_g = mg = (0.0405)(9.8) = 0.3969\ N$$

$$\Sigma\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

$$\vec{a} = \frac{-3.034\times10^6 t^2 + 1.910\times10^4 t}{0.0405}$$

$$a(t) = (-7.506\times10^7 t^2 + 4.716\times10^5 t)\ m/s^2$$

$$\frac{da}{dt} = jerk = 0 = \frac{d}{dt}(-7.506\times10^7 t^2 + 4.716\times10^5 t)$$

$$0 = -1.5012\times10^8 t + 4.716\times10^5$$

$$t = 0.003141\ s \qquad t @ F_{max}$$

Video Lecture #7 – (Part 3 of 3) Dropping a Racket Ball on a Force Sensor - Demonstrating the Power of Calculus

$$F_{max} = -3.034\times10^6 t^2 + 1.91\times10^4 t$$

$$F(0.003141) = -3.034\times10^6(0.003141)^2 + 1.91\times10^4(0.003141)$$

$$\boxed{F_{max} = 29.85\ N}$$

$$V(t) = \int a\,dt$$

$$v(t) = \int(-7.506\times10^7 t^2 + 4.716\times10^5 t)\,dt$$

$$V(t) = \frac{-7.506\times10^{7}t^{3}}{3} + \frac{4.716\times10^{5}t^{2}}{2} + C$$

$$V(t) = -2.502\times10^{7}t^{3} + 2.358\times10^{5}t^{2} + C$$

$$V(0.003141) = 0 = -2.502\times10^{7}(0.003141)^{3} + 2.358\times10^{5}(0.003141)^{2} + C$$

$$V_i = C = -1.551 \text{ m/s}$$

$$V(t) = -2.502\times10^{7}t^{3} + 2.358\times10^{5}t^{2} - 1.551$$

$$x(t) = \int V\,dt$$

$$= \int 2.502\times10^{7}t^{3} + 2.358\times10^{5}t^{2} - 1.551$$

$$= -\frac{2.502\times10^{7}t^{4}}{4} + \frac{2.358\times10^{5}t^{3}}{3} - 1.551t + C$$

let $x_i = 0$

$$= -6.255\times10^{6}t^{4} + 7.86\times10^{4}t^{3} - 1.551t + 0$$

Can find spring constant of raquetball

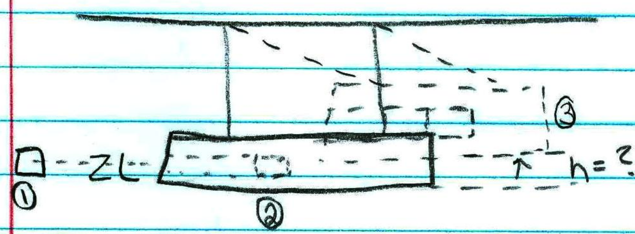
Video Lecture #8 – Introduction to Elastic, Inelastic and Perfectly Inelastic Collisions

	Types of Collisions	p cons?	KE cons?
	Elastic (bounce)	yes	yes
(Pefectly)	Inelastic (stick)	yes	no

Video Lecture #9 – A Ballistic Pendulum Problem - A Classic Example

Ex) Ballistic pendulum

Knowns: $V_{1b}, m_b, m_w, V_{1w}=0$



$1 \rightarrow 2$

$\Sigma F = \frac{dp}{dt} = 0$

$\Sigma \vec{p}_i = \Sigma \vec{p}_f$

$\Sigma \vec{p}_1 = \Sigma \vec{p}_2$

$V_{2w} = V_{2b} = V_2$

$m_b v_{1b} + \cancel{m_w v_{1w}} = m_b v_{2b} + m_w v_{2w}$

$m_b v_{1B} = (m_b + m_w)(v_2)$

$2 \rightarrow 3$

$ME_2 = ME_3$

$\frac{1}{2} m_t v_2^2 = U_{g+3}$

$\frac{1}{2} \cancel{m_t} v_2^2 = \cancel{m_t} g h_3$

$\frac{v_2^2}{2} = g h_3$

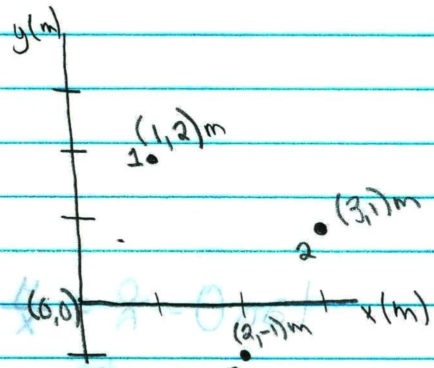
$h_3 = \frac{v_2^2}{2g}$

$h_3 = \frac{\left(\frac{m_b v_{1B}}{(m_b + m_w)}\right)^2}{2g}$

Video Lecture #10 – Introduction to Center of Mass of a System of Particles with Example Problem

Center of mass

$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ particles



$m_1 = 1\,kg$

$m_2 = 2\,kg$

$m_3 = 3\,kg$

$x_{cm} = \frac{(1)(1)+(2)(3)+(3)(2)}{1+2+3}$

$x_{cm} = 2.1667$

$x_{cm} = 2.2\,m$

$$x_{cm} = \lim_{\Delta m_i \to 0} \frac{\sum x_i \Delta m_i}{M_{total}}$$

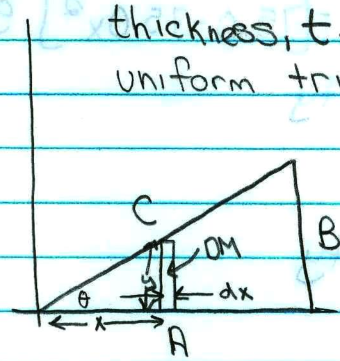
$$x_{cm} = \frac{1}{M_{total}} \int x \, dm$$

$$r_{cm} = \frac{1}{m_t} \int r \, dm$$ objects w/ shape

Ex1 CM of a △

thickness, t, constant density

uniform triangle



$$x_{cm} = \frac{1}{m_t} \int x \, dm$$

$$\rho = \frac{m}{V} = \frac{m_t}{V_t} = \frac{dm}{dV}$$

$dm = \rho \, dv$

$dV = y \, dx \, t$

$dV = y \, t \, dx$

$dm = \rho (y)(t)(dx)$

$$\rho = \frac{m_t}{V_t} = \frac{m_t}{\frac{1}{2}ABt}$$

$$dm = \frac{m_t}{(\frac{1}{2}AB\cancel{t})}(y\cancel{t}dx)$$

$$dm = \frac{2m_t y}{ab}dx$$

$$x_{cm} = \frac{1}{m_t}\int x\,dm$$

$$x_{cm} = \frac{1}{m_t}\int x\left(\frac{2m_t y}{AB}\right)dx$$

$$x_{cm} = \frac{2\cancel{m_t}}{\cancel{m_t}ab}\int xy\,dx$$

$$x_{cm} = \frac{2}{ab}\int xy\,dx$$

$$x_{cm} = \frac{2}{ab}\int x\left(\frac{b}{a}x\right)dx$$

$$x_{cm} = \frac{2}{a\cancel{b}}\frac{\cancel{b}}{a}\int x^2\,dx$$

$$x_{cm} = \frac{2}{a^2}\int_0^a x^2\,dx$$

$$x_{cm} = \frac{2}{a^2}\left[\frac{x^3}{3}\right]_0^a$$

$$x_{cm} = \frac{2a^3}{3a^2} = \frac{2}{3}a$$

$$\tan\theta = \frac{O}{A} = \frac{y}{x} = \frac{b}{a}$$

$$y = \frac{bx}{a}$$

$$y = mx + b$$

$$y = \frac{b}{a}x + 0$$

$$y = \frac{b}{a}x$$

Ex) Rod length = 90 cm $\lambda = [75.0 - 75.0x^2]$ g/m

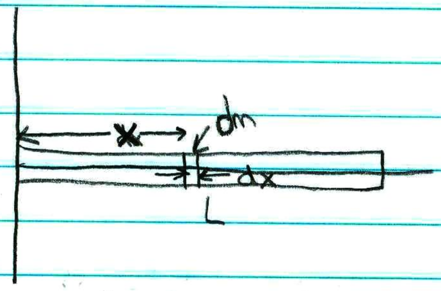
$\rho = \frac{m}{V}$ volumetric mass density

Lowercase sigma $\sigma = \frac{m}{A}$ surface mass density

$\lambda = \frac{m}{L}$ linear mass density

$x_{cm} = ? = \frac{1}{m_t}\int x\,dm$

$m_t = \int dm = \int \lambda\,dx$



$\lambda = \frac{m}{L} = \frac{dm}{dx}$

$dm = \lambda\,dx$

$$m_t = \int_0^{0.90} (75 - 75x^2)\,dx$$

$$= 75x - \frac{75x^3}{3}\Big]_0^{0.9}$$

$$= 75x - 25x^3\Big]_0^{0.9}$$

$$= 75(0.9) - 25(0.9)^3$$

$m_t = 49.275$

$\boxed{m_t = 49.3\ g}$

$$x_{cm} = \frac{1}{m_t}\int x\,dm$$

$$x_{cm} = \frac{1}{m_t}\int x\,\lambda\,dx$$

$$x_{cm} = \frac{1}{m_t}\int x(75 - 75x^2)\,dx$$

$$x_{cm} = \frac{1}{m_t}\int_0^{0.9} (75x - 75x^3)\,dx$$

$$x_{cm} = \frac{1}{m_t}\left[\frac{75x^2}{2} - \frac{75x^4}{4}\right]_0^{0.9}$$

$$x_{cm} = \frac{1}{49.275}\left(\frac{75(0.9)^2}{2} - \frac{75(0.9)^4}{4}\right)$$

$\boxed{x_{cm} = 0.367\ m}$

Video Lecture #13 – Introduction to Velocity and Acceleration of a System of Particles

$V = \frac{dx}{dt}$ $\quad r_{cm}$ $\quad a = \frac{dV}{dt}$

$V_{cm} = \frac{dr_{cm}}{dt}$ $\quad a_{cm} = \frac{dV_{cm}}{dt}$

Video Lecture #14 – An Introductory Review of Circular Motion and Deriving Tangential Velocity and Acceleration

$s = r\theta$ $\quad C = 2\pi r$

use radians

$1\,\text{rev} = 360° = 2\pi\,\text{rad}$

$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$ $\quad \omega_{inst} = \frac{d\theta}{dt}$

$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$ $\quad \alpha_{inst} = \frac{d\omega}{dt}$

$\omega_f = \omega_i + \alpha\Delta t$

$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha\Delta t^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

$s = r\theta$

$\frac{d}{dt}(s = r\theta)$

$\frac{ds}{dt} = r\frac{d\theta}{dt}$

$V_t = r\omega$

$\frac{d}{dt}(V_t = r\omega)$

$\frac{dV_t}{dt} = r\frac{d\omega}{dt}$

$a_t = r\alpha$

$s = r\theta$

$V_t = r\omega$

$a_t = r\alpha$

$a_c = \frac{V_t^2}{r} = \frac{(r\omega)^2}{r}$

$a_c = \frac{r^{\cancel{2}}\omega^2}{\cancel{r}}$

$a_c = r\omega^2$

$$KE_t = \sum_i KE_i$$
$$= \sum_i \tfrac{1}{2} m_i (v_i)^2$$
$$= \sum_i \tfrac{1}{2} m_i (r_i \omega_i)^2$$
$$= \sum_i \tfrac{1}{2} m_i r_i^2 \omega_i^2$$

$$KE_t = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega_i^2$$

dist from axis of rotation

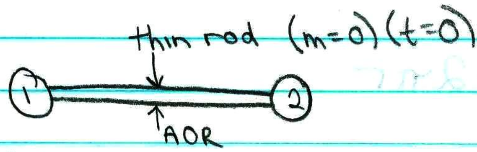
$I = \sum_i m_i r_i^2$ (moment of inertia)

"Rotational mass"

$$KE_t = \tfrac{1}{2} I \omega^2$$

rotational kinetic energy

Ex]



$m = 2.0$ kg $\quad L = 1.0$ m

$\omega = 15 \text{ rpm} \left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right) = 0.157079$ rad/s

$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 = (2)(0.5)^2 + (2)(0.5)^2$

$I_{cm} = 1.0 \text{ kg}\cdot\text{m}^2$

$KE_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2}(1)(0.157079)^2$

$KE_{rot} = 0.012337$ $\qquad (\text{kg m}^2)\left(\frac{\text{rad}}{\text{s}}\right)^2 = \frac{\text{kg m}^2 \cancel{\text{rad}^2}}{\text{s}^2} = \left(\frac{\text{kg m}}{\text{s}^2}\right)\text{m} = \text{N}\cdot\text{m} = \text{J}$

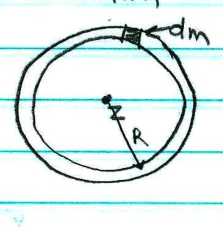
$KE_{rot} = 12.3$ mJ

Video Lecture #16 – Deriving the Moment of Inertia of a Uniform Thin Hoop about its Center of Mass

$$I = \lim_{\Delta m \to 0} \sum r_i^2 \Delta m_i$$

$I = \int r^2\, dm$ rigid object w/ shape

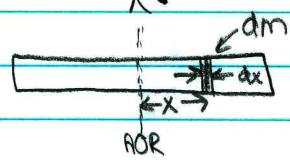
Ex] Thin, uniform hoop



$$I_z = \int r^2\, dm$$
$$= r^2 \int dm$$
$$= r^2 m$$
$$I_z = mR^2$$

Ex) Uniform rigid rod

λ.



$$\lambda = \frac{m}{L} = \frac{dm}{dx}$$

$$dm = \lambda dx$$

$$dm = \frac{m}{L} dx$$

$$I_y = \int r^2 dm$$

$$I_y = \int r^2 \frac{m}{L} dx$$

$$I_y = \frac{m}{L} \int_{-L/2}^{L/2} x^2 dx$$

$$I_y = \frac{m}{L} \left(\frac{x^3}{3} \right)_{-L/2}^{L/2}$$

$$I_y = \frac{m}{L} \left(\frac{(L/2)^3}{3} - \frac{(-L/2)^3}{3} \right)$$

$$I_y = \frac{m}{\cancel{L}} \left(\frac{L^{\cancel{3}}}{24} + \frac{L^{\cancel{3}}}{24} \right)$$

$$I_y = m \left(\frac{L^2}{24} + \frac{L^2}{24} \right)$$

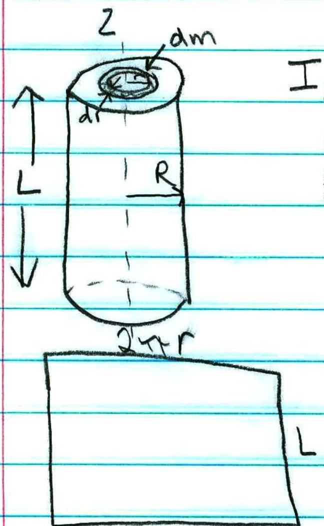
$$\boxed{I_y = \frac{1}{12} mL^2}$$

Ex) $I_{end} = ?$

$$I_{end} = \frac{m}{L} \int_0^L x^2 dx$$

$$= \frac{m}{L} \left[\frac{x^3}{3} \right]_0^L = \frac{mL^3}{3L} = \boxed{\frac{1}{3} mL^2}$$

Ex) Uniform solid cylinder



$$I_z = \int r^2\,dm$$

r = variable

R = constant

$$\rho = \frac{m}{V} = \frac{dm}{dV}$$

$$dm = \rho\,dV$$

$$dV = (2\pi r)L\,dr$$

$$dm = \rho\,2\pi r L\,dr$$

$$\rho = \frac{m_t}{V_t} = \frac{m_t}{\pi R^2 L}$$

$$I_z = \int r^2 \rho\,2\pi r L\,dr$$

$$= \rho\,2\pi L \int_0^R r^3\,dr$$

$$= \rho\,2\pi L \left(\frac{r^4}{4}\right)_0^R$$

$$= \rho\,2\pi L \frac{R^4}{4}$$

$$= \frac{m_t}{\pi R^2 L}\,2\pi L\,\frac{R^4}{4} = \frac{1}{2} m_t R^2$$

Video Lecture #19 – A Statement about Moment of Inertia Memorization for the AP Physics C Test (no lecture notes)

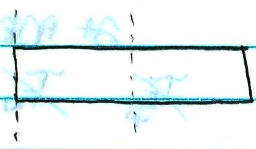
Video Lecture #20 – Introduction to the Parallel Axis Theorem with an Example Problem

Parallel-axis Theorem

$$I = I_{cm} + mD^2$$ *Memorize

D is the distance from center of mass to new AOR

*Only true with constant densities



uniform rod

$$I_{cm} = \frac{1}{12} mL^2$$

$$I_{end} = I_{cm} + mD^2$$

$$I_{end} = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2$$

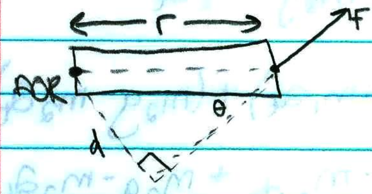
$$= \frac{1}{12} mL^2 + \frac{mL^2}{4}$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right) mL^2$$

$$= \frac{1}{3} mL^2$$

Torque $\tau = rF\sin\theta$

↑ lever arm



$\sin\theta = \frac{O}{H} = \frac{d}{r}$

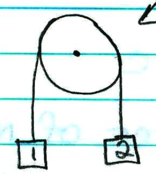
$d = r\sin\theta$

$\sum \vec{F} = m\vec{a}$

$\sum \tau = I\alpha$

AOR

Ex1



Solid disk as our pulley

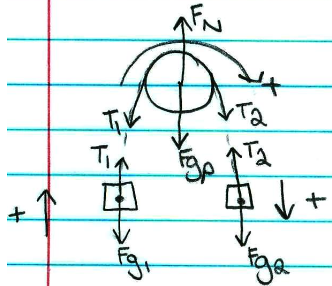
frictionless

$m_1 = 1\,kg$ $\quad m_2 = 2\,kg$

$m_p = 0.5\,kg$ $\quad R = 25cm$

$T = ?$ $\quad a = ?$

$I_{cm} = \frac{1}{2} m R^2$



at AOR $d=0$

$$\sum_{\text{pulley}} \tau = -\tau_{T_1} + \tau_{T_2} \;\; \cancel{\tau_{Fg_p}} \;\; \cancel{\tau_{F_N}} = I_p \alpha$$

AOR→cm pulley

$$-\tau_{T_1} + \tau_{T_2} = I_p \alpha$$

$$-T_1 r_1 \sin\theta_1 + T_2 r_2 \sin\theta_2 = \frac{1}{2} m_p R_p^2 \alpha$$

$$-T_1 \cancel{R} \cancel{\sin 90} + T_2 \cancel{R} \sin 90 = \frac{1}{2} m_p R^{\cancel{2}} \alpha$$

$$-T_1 + T_2 = \frac{m_p R \alpha}{2}$$

$$\sum_{m_1} F_t = T_1 - Fg_1 = m_1 a_t$$

$$T_1 - m_1 g = m_1 a_t$$

$$T_1 = m_1 g + m_1 a_t$$

$$\sum_{m_2} F_t = Fg_2 - T_2 = m_2 a_t$$

$$m_2 g - T_2 = m_2 a_t$$

$$T_2 = m_2 g - m_2 a_t$$

$$-(m_1 g + m_1 a_t) + (m_2 g - m_2 a_t) = \frac{m_p R \alpha}{2}$$

$a_t = r\alpha$

$$-m_1 g - m_1 a_t + m_2 g - m_2 a_t = \frac{1}{2} m_p a_t$$

$$-m_1 g + m_2 g = m_1 a_t + m_2 a_t + \frac{1}{2} m_p a_t$$

$$-m_1 g + m_2 g = a_t\left(m_1 + m_2 + \frac{m_p}{2}\right)$$

$$a_t = \frac{-(1)(9.8) + (2)(9.8)}{1 + 2 + \frac{0.5}{2}}$$

$$\boxed{a_t = 3.02\ m/s^2}$$

$$\sum_{\text{all}} \tau = -\tau_{Fg_1} + \tau_{T_1} - \tau_{T_1} + \tau_{T_2} - \tau_{T_2} + \tau_{Fg_2} \neq I\alpha$$

$m_1 + m_2$ no α

Don't work

$$\sum_{\text{all}} F = -Fg_1 + T_1 - T_1 + T_2 - T_2 + Fg_2 = (m_1 + m_2 + m_p) a_t$$

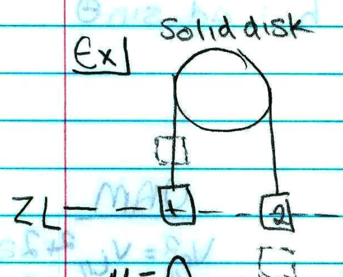
pulley no constant a_t

Video Lecture #23 – Comparing Linear and Rotational Variables

Linear	Rotational
x	θ
V	ω
a	α
UAM	UαM
mass	I
$KE = \frac{1}{2}mv^2$	$KE_{rot} = \frac{1}{2}I\omega^2$
~	$\tau = rF\sin\theta$
$\Sigma\vec{F} = m\vec{a}$	$\Sigma\vec{\tau} = I\vec{\alpha}$
$P = F\cdot V$	$P_{rot} = \tau\cdot\omega$

Video Lecture #24 – Problem - Finding Acceleration of Two Masses on a Pulley using Conservation of Energy

Ex



$\mu = 0$

$m_1 = 1\,kg$

$m_2 = 2\,kg$

$m_p = 0.5\,kg$

$R = 0.25\,m$

$v_i = 0$

dist $= 0.50\,m$

$V_f = ?$ $a = ?$

$ME_i = ME_f$

$0 = U_{g1f} + U_{g2f} + KE_{1f} + KE_{2f} + KE_{pf}$

$0 = m_1gh_{1f} + m_2gh_{2f} + \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 + \frac{1}{2}I_p\omega^2$

$0 = m_1gh_{1f} + m_2gh_{2f} + \frac{1}{2}m_1V_f^2 + \frac{1}{2}m_2V_f^2 + \frac{1}{4}m_pV_f^2$

$0 = m_1gh_{1f} + m_2gh_{2f} + V_f^2\left(\frac{m_1}{2} + \frac{m_2}{2} + \frac{m_p}{4}\right)$

$V_f^2 = \dfrac{-m_1gh_{1f} - m_2gh_{2f}}{\frac{m_1}{2} + \frac{m_2}{2} + \frac{m_p}{4}}$

$V_t = r\omega$

$V_t^2 = r^2\omega^2$

$KE_{rot} = \frac{1}{2}I\omega^2$

$= \frac{1}{2}\left(\frac{1}{2}m_pR^2\right)\omega^2$

$= \frac{1}{4}m_pR^2\omega^2$

$= \frac{1}{4}m_pV_f^2$

$V_f^2 = \dfrac{-(1)(9.8)(0.5) - 2(9.8)(-0.5)}{\frac{1}{2} + \frac{2}{2} + \frac{0.5}{4}}$

$V_f = 1.736$

$\boxed{V_f = 1.7\ m/s}$

$V_f^2 = V_i^2 + 2a_y$

$\dfrac{V_f^2 - V_i^2}{2\Delta y} = a$

$\dfrac{(1.736)^2 - 0^2}{2(0.5)} = \boxed{3.02\ m/s^2}$

Video Lecture #25 – Introduction to Rolling Without Slipping

Rolling without Slipping

$V_{cm} = r\omega$

$KE_t = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

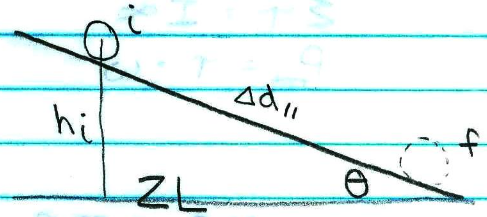
$a_{cm} = r\alpha$

Video Lecture #26 – Finding the Acceleration of a Solid Sphere Rolling Without Slipping Down An Incline

Ex) Solid sphere rolling down incline

$V_i = 0 \quad I_{cm} = \frac{2}{5}mR^2 \quad V_{cm}^2 = r^2\omega^2 = V_f^2$

$a_{cm} = ?$



$\sin\theta = \frac{O}{H} = \frac{h_i}{\Delta d_{\parallel}}$

$h_i = \Delta d_{\parallel}\sin\theta$

(Cons of E)

$ME_i = ME_f$

$mgh_i = \frac{1}{2}mV_f^2 + \frac{1}{2}I\omega_f^2$

$\cancel{m}gh_i = \frac{1}{2}\cancel{m}V_f^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}R^2\right)\omega^2$

$gh_i = \frac{V_f^2}{2} + \frac{1}{5}R^2\omega^2$

$gh_i = \frac{V_f^2}{2} + \frac{1}{5}V_f^2$

$gh_i = \frac{7}{10}V_f^2$

$g(\Delta d_{\parallel}\sin\theta) = \frac{7}{10}V_f^2$

$V_f^2 = \frac{10}{7}g\Delta d_{\parallel}\sin\theta$

UAM

$V_{f\parallel}^2 = V_{i\parallel}^2 + 2a_{\parallel}\Delta d_{\parallel}$

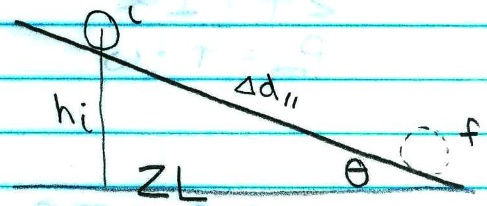
$\frac{10}{7}g\cancel{\Delta d_{\parallel}}\sin\theta = 0^2 + 2a_{\parallel}\cancel{\Delta d_{\parallel}}$

$\boxed{a = \frac{5}{7}g\sin\theta}$

Ex) Solid sphere
rolling down incline

$V_i = 0$ $\quad I_{cm} = \frac{2}{5} mR^2$ $\quad V_{cm}^2 = r^2\omega^2 = V_f^2$

$a_{cm} = ?$



$\sin\theta = \frac{O}{H} = \frac{h_i}{\Delta d_{//}}$

$h_i = \Delta d_{//} \sin\theta$

(Cons of E)

$ME_i = ME_f$

$mgh_i = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2$

$\cancel{m}gh_i = \frac{1}{2}\cancel{m}v_f^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}R^2\right)\omega^2$

$gh_i = \frac{V_f^2}{2} + \frac{1}{5}R^2\omega^2$

$gh_i = \frac{V_f^2}{2} + \frac{1}{5}V_f^2$

$gh_i = \frac{7}{10}V_f^2$

$g(\Delta d_{//} \sin\theta) = \frac{7}{10}V_f^2$

$V_f^2 = \frac{10}{7} g \Delta d_{//} \sin\theta$

UAM

$V_{f//}^2 = V_{i//}^2 + 2a_{//}\Delta d_{//}$

$\frac{10}{7} g \cancel{\Delta d_{//}} \sin\theta = 0^2 + 2a_{//}\cancel{\Delta d_{//}}$

$\boxed{a = \frac{5}{7} g \sin\theta}$