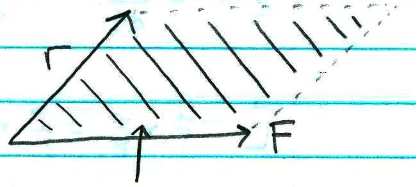
AP Physics C – Video Lecture Notes
Chapter 11-12: Thank You, Emily Rencsok, for these notes.
Video Lecture #1 - Introduction to Torque with a review of the Cross Product

$\vec{\tau} = \vec{r} \times \vec{F} \neq \vec{F} \times \vec{r}$

$\tau = rF\sin\theta$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$



area is cross product

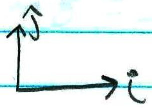
Ex| $\vec{A} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$
$\vec{B} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -2 \\ 2 & -3 & 4 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ -3 & 4 \end{vmatrix}\hat{\imath} - \begin{vmatrix} -1 & -2 \\ 2 & 4 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix}\hat{k}$$

$$= [(1)(4)-(-2)(-3)]\hat{\imath} - [(-1)(4)-(-2)(2)]\hat{\jmath} + [(-1)(-3)-(1)(2)]\hat{k}$$

$$= \boxed{-2\hat{\imath} - 0\hat{\jmath} + \hat{k}}$$

Ex| $\vec{A} = \hat{\imath}$
$\vec{B} = \hat{\jmath}$



$A \times B$ = out of board $(+\hat{k})$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 1\hat{k} \qquad \vec{B} \times \vec{A} = -\hat{k}$$

Ex|

$\tau = \vec{r} \times \vec{F} = 0$

$\vec{r} = \hat{\imath}\,m$

$\vec{F} = -\hat{\imath}\,N$

$\tau = rF\sin\theta$
$= rF\sin 0$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{vmatrix} = 0$$

Video Lecture #2 - Introduction to the Angular Momentum of a System of Particles

$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$ $\Sigma \vec{F} = m\vec{a}$

$\Sigma \tau = \frac{dL}{dt}$ $\Sigma \tau = I\alpha$ L=angular momentum

$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = rmv\sin\theta$ PARTICLES

Video Lecture #3 - Introduction to the Angular Momentum of a Rigid Object with Shape with Example Problem

$\Sigma \tau_{ext} = I\alpha$ RIGID OBJECTS w/ SHAPE

$\int \Sigma \tau_{ext}\, dt = \int I\alpha\, dt$

$\int \frac{dL}{\cancel{dt}} \cancel{dt} = I \int \alpha\, dt$

$\int dL = I\omega$

$\boxed{L = I\omega}$

Ex) L of a solid sphere

$r = 0.25$ m

$m = 1.0$ kg

$V_t = 2.0$ m/s

$I = \frac{2}{5} mR^2$

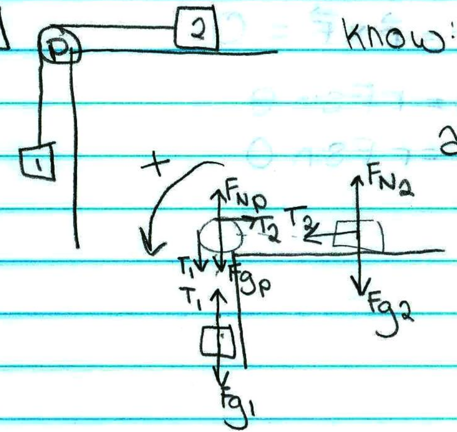
$V_t = r\omega$

$\omega = \frac{V_t}{r}$

$$L = I\omega = \left(\frac{2}{5} mR^2\right)\omega$$
$$= \left(\frac{2}{5} mR^{\cancel{2}}\right)\left(\frac{V_t}{\cancel{R}}\right) = \frac{2}{5} mRV_t$$
$$= \frac{2}{5}(1)(0.25)(2) = 0.20\ \frac{kg\,m^2}{s}$$

Video Lecture #4 - Using the Derivative of Angular Momentum with respect to Time on a Pulley Mass System

Ex1

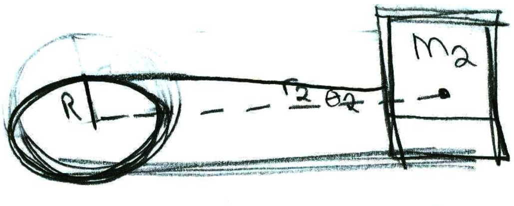


know: m_p, m_1, m_2, r_p, P is solid disk

$\mu_k = 0$

$a = ?$

Important



$\sin\theta = \frac{O}{H} = \frac{R}{r_2}$

$R = r_2 \sin\theta_2$

$$L_p = I\omega$$
$$= \frac{1}{2} m_p R^2 \omega$$
$$= \frac{1}{2} m_p R^2 \left(\frac{V_t}{R}\right)$$
$$= \frac{1}{2} m_p R V_t$$

$$L_2 = r_2 \times p_2 = r_2 m_2 v_2 \sin\theta$$
$$L_2 = R m_2 V_2$$

$$L_1 = r_1 \times p_1 = r_1 m_1 v_1 \sin\theta$$
$$= R m_1 v_1$$

$$L_t = \frac{1}{2} m_p R V_t + R m_1 v_1 + R m_2 v_2$$
$$L_t = RV(m_1 + m_2 + \frac{1}{2} m_p)$$

$$\Sigma \tau = +\cancel{\tau_{T_2}} + \cancel{\tau_{T_1}} \quad \cancel{\tau_{F_{Np}} \quad \tau_{F_{gp}}} \quad \cancel{\tau_{F_{N2}}} \quad \cancel{\tau_{F_{g2}}} + \tau_{F_{g1}} - \cancel{\tau_{T_2}} - \cancel{\tau_{T_1}}$$

AOR pulley whole

$r = 0$ no lever arm

= but opp

$$\Sigma \tau = \tau_{F_{g1}} = \frac{dL_t}{dt}$$
$$= r_1 F_{g1} \sin\theta_1 = \frac{d}{dt}\left(RV\left(m_1 + m_2 + \frac{m_p}{2}\right)\right)$$
$$= \cancel{R} m_1 g \cancel{\sin 90} = \cancel{R}\left(m_1 + m_2 + \frac{m_p}{2}\right)\left(\frac{d}{dt}(v)\right)$$
$$= m_1 g = \left(m_1 + m_2 + \frac{m_p}{2}\right)(a)$$

$$\boxed{a = \frac{m_1 g}{\left(m_1 + m_2 + \frac{m_p}{2}\right)}}$$

$\Sigma F_{2\parallel} = T_2 = m_2 a_\parallel$

$\Sigma F_{1\parallel} = F_{g_1} - T_1 = m_1 a_\parallel$

$\underset{\text{pulley}}{\Sigma \tau} = \cancel{\tau_{F_{Np}}}\ \cancel{\tau_{F_{gp}}} - \tau_{T_2} + \tau_{T_1} = \frac{dL_p}{dt}$

OR

COE

Video Lecture #5 - Introduction to Conservation of Angular Momentum with Demonstrations

$\Sigma \vec{F} = \frac{d\vec{p}}{dt} = 0$

$\rightarrow \Sigma \vec{p}_i = \Sigma \vec{p}_f$

$\underset{ext}{\Sigma \vec{\tau}} = \frac{d\vec{L}}{dt} = 0$

AOR $\rightarrow \Sigma \vec{L}_i = \Sigma \vec{L}_f$

Video Lecture #6 - A Merry-Go-Round Demonstration - Conservation of Angular Momentum and Centripetal Force
(no lecture notes)

Video Lecture #7 - Example - Conservation of Angular Momentum on a Merry-Go-Round

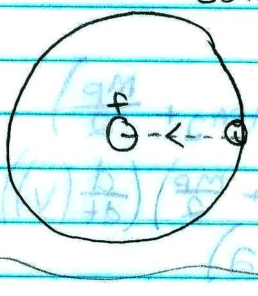
Merry go round

$M_d = 15\ kg$ $\omega_i = 2\ rad/s$

$m_\omega = 235\ kg$ $\omega_f = ?$

outside → inside

solid disk $= \frac{1}{2} mR^2$



$\Sigma \vec{L}_i = \Sigma \vec{L}_f$

$r_{di} m_d v_{di} \sin\theta_i + I\omega_i = I\omega_f$

$\cancel{R} m_d V_t \sin 90 + \frac{1}{2} m_\omega R^{\cancel{2}} \omega_i = \frac{1}{2} m_\omega R^{\cancel{2}} \omega_f$

$m_d V_t + \frac{m_\omega R \omega_i}{2} = \frac{m_\omega R \omega_f}{2}$

$2 m_d \cancel{R} \omega_i + m_\omega \cancel{R} \omega_i = m_\omega \cancel{R} \omega_f$

$2 m_d \omega_i + m_\omega \omega_i = m_\omega \omega_f$

$2(15)(2) + (235)(2) = (235)\omega_f$

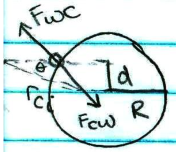
$\omega_f = 2.2553$

$\omega_f = 2.3\ rad/s$

Video Lecture #8 - Chapter 11 #37 - A Conservation of Angular Momentum Problem - A Piece of Clay hits a Cylinder

11-37

clay	cylinder
m_c	m_w
v_{ic}	R
	$I = \frac{1}{2} m_w R^2$
	$v_{iw} = 0$
	ω_f



$$\Sigma L_i = \Sigma L_f$$

$$\overbrace{r_{ci} m_{ci} v_i \sin\theta_i}^{\text{linear}} + 0 = \overbrace{r_{cf} m_c v_f \sin\theta_f}^{\text{tangential}} + I\omega_f$$

$$d m_c v_i = R m_c v_f \cancel{\sin 90} + \frac{1}{2} m_w R^2 \omega_f$$

$$d m_c v_i = R m_c R \omega_f + \frac{1}{2} m_w R^2 \omega_f$$

$$\boxed{\omega_f = \frac{d m_c v_i}{R^2 \left(m_c + \frac{m_w}{2}\right)}}$$

$V = r\omega$

$\omega = \frac{v}{r}$

$\Sigma F \neq 0$

$r_{ci} \sin\theta_i = d$

Video Lecture #9 - Introduction to Rotational Equilibrium

Translational Equilibrium

$\underset{\text{Direction}}{\Sigma F} = 0 = ma$ not accelerating (v=0 or CV)

Rotational Equilibrium

$\underset{\text{AOR}}{\Sigma \tau} = 0 = I\alpha$

If $\Sigma F = 0$ + $\underset{\text{1 AOR}}{\Sigma \tau = 0}$, $\Sigma \tau = 0$ for any AOR

Video Lecture #10 - Comparing Center of Mass to Center of Gravity in a Constant Gravitational Field

Center of Gravity

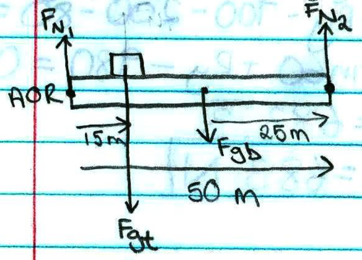
If g is const, then C of mass = C of gravity

Video Lecture #11 - Chapter 12 #39 - Vehicles on a Bridge - An Introductory Static Equilibrium Problem using Net Torque

12-39

$m_b = 80000$ Kg

$m_t = 30000$ kg



$$\underset{\text{(left end)}}{\Sigma \tau} = \cancel{\tau_{F_{N1}}} - \tau_{F_{gt}} - \tau_{F_{gb}} + \tau_{F_{N2}} = I\alpha = 0$$

$$-r_t F_{g_t} \sin\theta_t - r_b F_{g_b} \sin\theta_b + r_2 F_{N_2} \sin\theta_2 = 0$$

$$-r_t m_t g \cancel{\sin 90} - r_b m_b g \cancel{\sin 90} + r_2 F_{N_2} \cancel{\sin 90} = 0$$

$$-(15)(30000)(9.8) - (25)(80000)(9.8) + (50)F_{N_2} = 0$$

$$F_{N_2} = 480200 = \boxed{480 \text{ kN}}$$

$$\Sigma F_y = F_{N_1} + F_{N2} - F_{g_t} - F_{g_b} = \cancel{ma_y} = 0$$

$$F_{N_1} + 480200 - (30000)(9.8) - (80000)(9.8) = 0$$

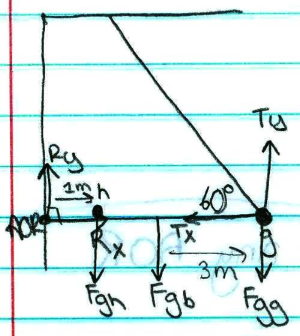
$$F_{N_1} = 597800 = \boxed{598 \text{ kN}}$$

Video Lecture #12 - Chapter 12 #42 - A Hungry Bear - Another Introductory Static Equilibrium Problem using Net Torque

43) $F_{gh} = 700\text{ N}$

$F_{gb} = 200\text{ N}$ $\quad \ell_b = 6\text{ m}$

$F_{gg} = 80\text{ N}$



$$\sum_{\substack{\text{Left}\\\text{end}}} \tau = +\tau_{T_y} - \tau_{F_{gh}} - \tau_{F_{gb}} - \tau_{F_{gg}} = \cancel{I\alpha} = 0$$

$$T r \sin\theta - F_{gh} r \sin\theta - F_{gb} r \sin\theta - F_{gg} r \sin\theta = 0$$

$$T(6)\sin 60 - 700(1)\cancel{\sin 90} - (200)(3)\cancel{\sin 90} - (80)(6)\cancel{\sin 90} = 0$$

$$T = 342.56116\text{ N}$$

$$\boxed{T = 343\text{ N}}$$

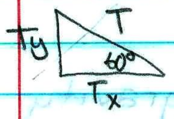
$$\sum F_x = R_x - T_x = \cancel{ma_x} = 0$$

$$R_x = T\cos 60$$

$$R_x = 342.56116(\cos 60)$$

$$R_x = 171.28$$

$$\boxed{R_x = 171\text{ N}}$$



$T_y = T\sin 60$

$T_x = T\cos 60$

$$\sum F_y = T_y + R_y - F_{gh} - F_{gb} - F_{gg} = \cancel{ma_y} = 0$$

$$T\sin\theta + R_y - 700 - 200 - 80 = 0$$

$$(342.56)\sin 60 + R_y - 980 = 0$$

$$R_y = 683.\overline{3}$$

$$\boxed{R_y = 683\text{ N}}$$

$$T r \sin\theta - F_{gh} r \sin\theta - F_{gb} r \sin\theta - F_{gg} r \sin\theta = 0$$

$$(900)(6)\sin 60 - (700)x\cancel{\sin 90} - (200)(3)\cancel{\sin 90} - (80)(6)\cancel{\sin 90} = 0$$

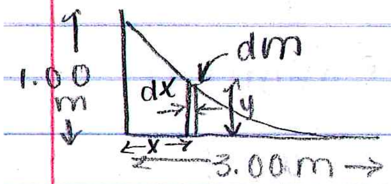
$$x = 5.1379$$

$$\boxed{x = 5.14\text{ m}}$$

Video Lecture #13 - Chapter 12 #6 - Problem - Finding the Center of Gravity of a Rigid Object with Shape

Thank You, Amanda Ciccarelli, for these notes.

12-6) $t = 0.0500\ m \quad y = \frac{(x-3)^2}{9} \quad m = \int dm$



$$x_{cm} = \frac{1}{m}\int x\,dm$$

$$\rho = \frac{m}{V} = \frac{dm}{dV}$$

$$dm = \rho\,dV = \rho\,y\,dx\,t$$

$$m = \int_0^3 \rho y t\,dx = \rho t \int_0^3 y\,dx$$

$$= \rho t \int_0^3 \frac{(x-3)^2}{9}\,dx = \frac{\rho t}{9}\int_0^3 (x^2 - 6x + 9)\,dx$$

$$m = \frac{\rho t}{9}\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^3$$

$$m = \frac{\rho t}{9}(9) = \rho t$$

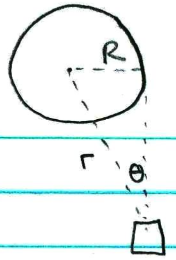
$$x_{cm} = \frac{1}{m}\int x\,dm = \frac{1}{\rho t}\int x \rho y t\,dx = \frac{\cancel{\rho t}}{\cancel{\rho t}}\int x y\,dx$$

$$= \int_0^3 x\,\frac{(x-3)^2}{9}\,dx = \int_0^3 \frac{x}{9}(x^2 - 6x + 9)\,dx$$

$$x_{cm} = \frac{1}{9}\left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2}\right]_0^3 = 0.75\ m$$

Video Lecture #14 - Chapter 11 #16 - Finding the Angular Momentum and Acceleration of a Block Hanging off a Cylinder

Thank You, Emily Rencsok, for these notes.



$\sin\theta = \frac{O}{H} = \frac{R}{r}$

$R = r\sin\theta$

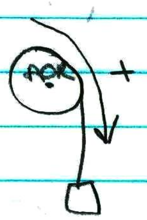
11-16

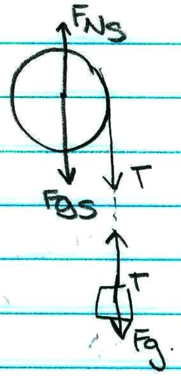
$m = 4\text{ kg}$

$R = 0.08\text{ m}$

$m_s = 2\text{ kg}$

$I_{cm} = \frac{1}{2}m_s R^2$





= but opp

$\sum_{AOR}\tau = \cancel{\tau_{F_{NS}}}\ \cancel{\tau_{F_{gs}}}\ \tau_T\ \ \tau_T + \tau_{F_g}$

$r=0$

$= rF_g\sin\theta = rmg\sin\theta = Rmg$

$\sum\tau = (0.08)(4)(9.8)$

$\sum\tau = 3.136\text{ N}\cdot\text{m}$

$\boxed{\sum\tau = 3.14\text{ N}\cdot\text{m}}$

$L_t = L_m + L_s$

$L_t = rmv\sin\theta + I\omega$

$= Rmv + \frac{1}{2}m_s R^2\omega$

$= Rmv + \frac{1}{2}m_s R^{\cancel{2}}\left(\frac{V}{\cancel{R}}\right)$

$= Rmv + \frac{1}{2}m_s RV$

$= VR\left(m + \frac{m_s}{2}\right)$

$= (V)(0.08)\left(4 + \frac{2}{2}\right)$

$\boxed{L_t = 0.400V}$

$\sum\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$

$3.136 = \frac{d}{dt}(0.400v)$

$3.136 = 0.400a$

$a = \frac{3.136}{0.4}$

$\boxed{a = 7.84\text{ m/s}^2}$

Video Lecture #15 - Review: Torque as the Cross Product, Angular Momentum, Center of Gravity and Rotational Equilibrium

$\sum_{AOR} \tau = \frac{dL}{dt}$

$\tau = rF\sin\theta$

$\tau = r \times F$ $(\hat{i}, \hat{j}, \hat{k})$

$L = r \times p$ (particles)

$L = I\omega$ (rigid objects w/ shape)

$\sum \tau_{ext} = 0 = \frac{dL}{dt}$ $\underbrace{\sum \vec{L}_i = \sum \vec{L}_f}_{AOR}$

$\sin\theta = \frac{O}{H} = \frac{R}{r}$

$r\sin\theta = R$

Translational equil $\sum F = 0$

Rotational equil $\sum \tau = 0$

$x_{cm} = x_{CG} = ?$