AP Physics C – Video Lecture Notes

Chapter 13 & 15: Thank You, Emily Rencsok, for these notes.

Video Lecture # 1 – Introduction to Newton's Universal Law of Gravitation and a Derivation of Freefall Acceleration

$$F_g = \frac{G m_1 m_2}{r^2} \qquad G = 6.67\times10^{-11}\ \frac{N\cdot m^2}{kg^2}$$

$$\vec{F}_{12} = -\frac{G m_1 m_2}{r^2}\,\hat{r}_{12}$$

Free Fall

$$\Sigma F_y = F_g = m a_y$$

$$\cancel{m} g = \cancel{m} a_y$$

$$-g = a_y$$

$$\cancel{m} g = \frac{G \cancel{m_1} m_2}{r^2}$$

$$g = \frac{G m_2}{r^2}$$

$$g = \frac{G m_E}{(R_E + Alt)^2}$$

Video Lecture # 2 – Introduction to Kepler's 1st and 2nd Laws

Kepler's 1st Law:

ORBITS ARE Very nearly CIRCLES

Major axis → $2a$

Minor axis → $2b$

eccentricity $= \frac{c}{a}$

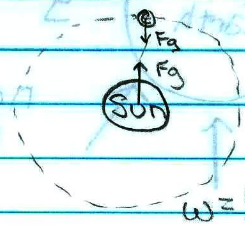
$e_e = 0.017$

$e_{circle} = 0$

Kepler's 2nd Law: Areas

Video Lecture # 3 – Derivation of Kepler's 3rd Law

Kepler's 3rd Law:



$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$

$\Sigma F_{in} = F_g = m a_c$

$\frac{G \cancel{m_E} m_s}{r^2} = \cancel{m_E} r \omega^2$

$G m_s = r^3 \omega^2$

$\omega^2 = \frac{G m_s}{r^3} = \left(\frac{2\pi}{T}\right)^2$

$\frac{G m_s}{r^3} = \frac{4\pi^2}{T^2}$

$T^2 = \frac{4\pi^2 r^3}{G m_s}$

$T^2 = \left(\frac{4\pi^2}{G m_s}\right) a^3$

constants

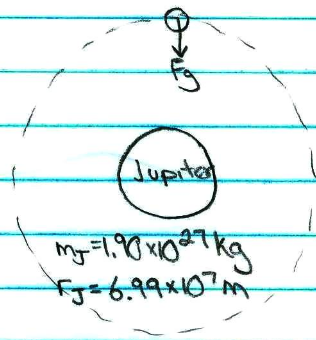
Not on equation sheet – Don't Memorize!

Video Lecture # 4 – Chapter 13 #19 - A Synchronous Orbit Problem - Don't Memorize Kepler's 3rd Law - Derive It

13-19 Alt of satellite over Jupiter

Synchronous orbit

Circles Once every 9.84 hrs $\left(\frac{3600\ s}{hr}\right) = 35424 s$



$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$

$\Sigma F_{in} = F_g = m_s a_c$

$\frac{G \cancel{m_s} m_J}{r^2} = \cancel{m_s} r \omega^2$

$\frac{G m_J}{r^2} = r\omega^2$

$G m_J = r^3 \omega^2$

$r = \sqrt[3]{\frac{G m_J}{\omega^2}}$

$r = \sqrt[3]{\frac{G m_J}{\left(\frac{2\pi}{T}\right)^2}}$

$r = \left(\frac{T^2 G m_J}{4\pi^2}\right)^{\frac{1}{3}}$

$r = \left(\frac{(35424)^2 (6.67 \times 10^{-11})(1.90 \times 10^{27})}{4\pi^2}\right)^{\frac{1}{3}}$

$r = 1.5911 \times 10^8$ m

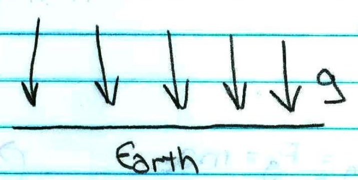
$r = R_J + Alt$

$Alt = r - R_J$

$Alt = 1.5911 \times 10^8\ m - 6.99 \times 10^7\ m$

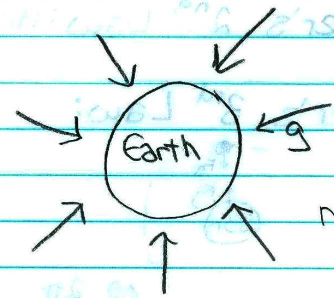
$Alt = 8.9213 \times 10^7$ m

$\boxed{Alt = 8.92 \times 10^7\ m = 89{,}200\ km}$



(almost) constant

$PE_g = mgh$ $g = const$



not constant

$F_g = -\frac{dU_g}{dr}$

$\int dU_g = \int F_g dr$

$\Delta U_g = -\int_{r_i}^{r_f} F_g dr = -W_{F_g}$

r^{-2}

$\Delta U_g = -\int_{r_i}^{r_f}\left(-\frac{Gm_1m_2}{r^2}\right)dr$

$\Delta U_g = Gm_1m_2\int_{r_i}^{r_f}\frac{1}{r^2}dr$

$= Gm_1m_2\int_{r_i}^{r_f} r^{-2}dr$

$= Gm_1m_2\left[\frac{r^{-1}}{-1}\right]_{r_i}^{r_f}$

$= Gm_1m_2\left[-\frac{1}{r}\right]_{r_i}^{r_f}$

Universal gravitational potential energy

$= Gm_1m_2\left[-\frac{1}{r_f} - \left(-\frac{1}{r_i}\right)\right]$

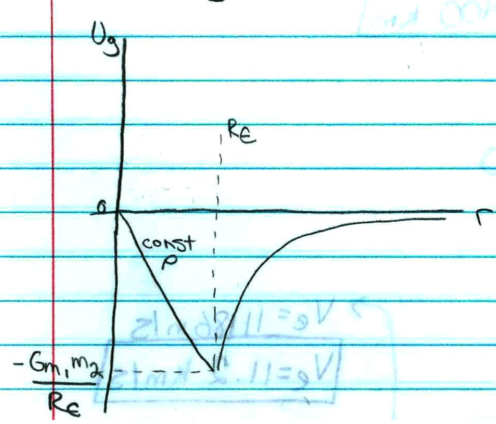
Take $U_{g_i} = 0 \rightarrow r_i = \infty$

$\Delta U_g = Gm_1m_2\left[\cancel{\frac{1}{r_i}} - \frac{1}{r_f}\right]$

$\Delta U_g = -\frac{Gm_1m_2}{r_f}$

*Need 2 objects

$U_g = -\frac{Gm_1m_2}{r}$ CAN NEVER BE POSITIVE



Video Lecture # 6 – Derivation of the Binding Energy of a Planet

Binding Energy — Work needed to remove an object from the planet

$$W_{Fa} = \Delta U_g = U_{gf} - U_{gi}$$

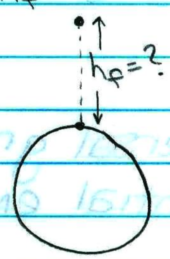
$$= 0 - \left(-\frac{G m_o m_e}{r_e}\right)$$

$$\boxed{W_{Fa} = \frac{G m_o m_e}{r_e}}$$

Video Lecture # 7 – Chapter 13 #28 - An Object Launched in to Space - Find (a) Max Altitude (b) Escape Velocity

13-28 $V_i = 10$ kg/sec

$h_f = ?$



$$ME_i = ME_f$$

$$PE_{gi} + KE_i = PE_{gf}$$

$$-\frac{G \cancel{m_t} m_e}{r_e} + \frac{1}{2}\cancel{m_t} v_{ti}^2 = -\frac{G \cancel{m_t} m_e}{r_f}$$

$$-\frac{G m_e}{r_e} + \frac{v_{ti}^2}{2} = -\frac{G m_e}{r_f}$$

$$\frac{-(6.67\times10^{-11})(5.98\times10^{24})}{6.375\times10^{6}} + \frac{10000^2}{2} = \frac{-(-6.67\times10^{-11})(5.98\times10^{24})}{r_f}$$

$$r_f = 3.173\times10^{7}\ \text{m}$$

$$Alt = r_f - R_e$$

$$Alt = 3.173\times10^{7} - 6.375\times10^{6}$$

$$Alt = 2.535\times10^{7}\ \text{m}$$

$$\boxed{Alt = 25400\ \text{km}}$$

Escape Velocity

$$-\frac{G \cancel{m_t} m_e}{r_e} + \frac{1}{2}\cancel{m_t} v_{ti}^2 = 0$$

$$-\frac{G m_e}{r_e} + \frac{v_{ti}^2}{2} = 0$$

$$\frac{v_{ti}^2}{2} = \frac{G m_e}{R_e}$$

$$V_e = \sqrt{\frac{2 G m_e}{R_e}}$$

$$V_e = \sqrt{\frac{2(6.67\times10^{-11})(5.98\times10^{24})}{6.375\times10^{6}}}$$

$$V_e = 11186\ \text{m/s}$$

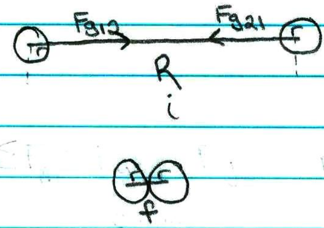
$$\boxed{V_e = 11.2\ \text{km/s}}$$

13-49

$m + m$

$r + r$

$v_i = 0$



$I = \Delta p = J$

$I = p_f - p_i$

$I = mv_f - \cancel{mv_i}$

$I = mv_f$

$ME_i = ME_f$

$PE_{g_i} = \frac{1}{2} m_1 v_f^2 + PE_{gf} + \frac{1}{2} m_2 v_f^2$

$-\frac{Gm_1m_2}{r_i} = \frac{1}{2} m_1 v_f^2 - \frac{Gm_1m_2}{r_f} + \frac{1}{2} m_2 v_f^2$

$-\frac{G\cancel{m}m}{R} = \frac{1}{2}\cancel{m} v_f^2 - \frac{G\cancel{m}m}{2r} + \frac{1}{2}\cancel{m} v_f^2$

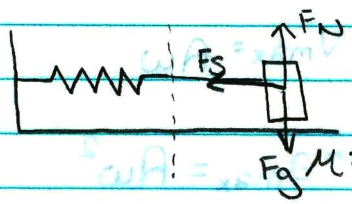
$-\frac{Gm}{R} = \frac{v_f^2}{2} - \frac{Gm}{2r} + \frac{v_f^2}{2}$

$-\frac{Gm}{R} = v_f^2 - \frac{Gm}{2r}$

$v_f = \sqrt{\frac{Gm}{2r} - \frac{Gm}{r}} = \sqrt{Gm\left(\frac{1}{2r} - \frac{1}{R}\right)}$

$\Delta p = mv_f = m\sqrt{Gm\left(\frac{1}{2r} - \frac{1}{R}\right)}$

$= \sqrt{Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)}$



$a \propto \Delta x$ + opposite displacement from equilibrium position

$F_s = -kx$

$\Sigma F_x = -F_s = ma_x$

$-kx = ma_x$

$a_x = -\frac{k}{m}x$

$a_{max} = \frac{k}{m}A$

Amplitude, A, is maximum distance from equilibrium position

$a = \frac{dV}{dt}$

$V = \frac{dx}{dt}$

$$a = \frac{d^2x}{dt^2} = \frac{-k}{m}x$$

Let $\frac{k}{m} = \omega^2$

ω = angular frequency

$$\underbrace{\frac{d^2x}{dt^2} = -\omega^2 x}_{\text{Condition for SHM}} \longleftarrow \text{MEMORIZE}$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$\sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$T = \frac{1}{f}$

$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$ mass-spring

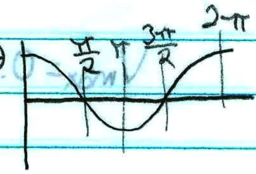
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$x(t) = A\cos(\omega t + \phi)$$

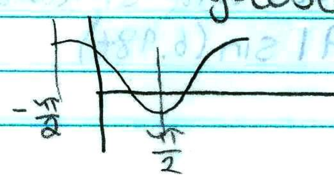
amplitude, ang frequency, phi, phase shift

$\phi \Rightarrow$ radians

$y = \cos\theta$



$y = \cos(\theta + \frac{\pi}{2})$



$$V(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi) \rightarrow V_{max} = A\omega$$

$$a(t) = \frac{dV}{dt} = -A\omega^2\cos(\omega t + \phi) \rightarrow a_{max} = A\omega^2$$

$$a(t) = -\omega^2(A\cos(\omega t + \phi))$$

$$a(t) = -\omega^2 x(t)$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Ex) $m = 0.5\ kg$

$t_i = 1.9820$

$t_f = 2.8820$

$T = t_f - t_i = 2.8820 - 1.9820 = \boxed{0.900\ s}$

$y_i = 0.589\ m$

$y_f = 0.701$

$A = \frac{y_f - y_i}{2} = \frac{0.701 - 0.589}{2} = \boxed{0.056\ m}$

$T = 2\pi\sqrt{\frac{m}{K}}$

$T/2\pi = \sqrt{\frac{m}{K}}$

$\frac{T^2}{4\pi^2} = \frac{m}{K}$

$K = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.5)}{0.9^2} = \boxed{21.9\ N/m}$

$\omega = \sqrt{\frac{K}{m}} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$

$\omega = \frac{2\pi}{0.9} = \boxed{6.981\ rad/s}$

$x(t) = A\cos(\omega t + \phi)$

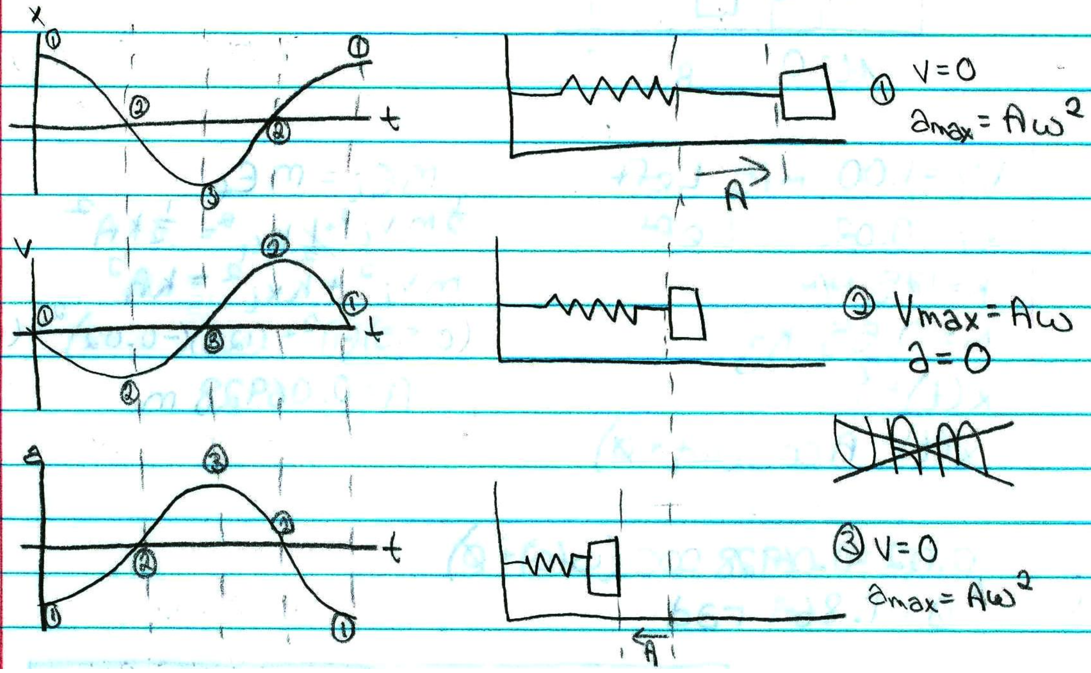
$x(t) = \boxed{0.056\cos(6.98t)}\ m$

$V(t) = \frac{dx}{dt} = 0.056 \cdot 6.98(-\sin(6.98t))$

$V(t) = -0.391\sin(6.98t)$ $\qquad V_{max} = 0.391\ m/s$

$$a(t) = \frac{dv}{dt} = -0.39088(6.98)\cos(6.98t)$$

$$a(t) = -2.73\cos(6.98t) \qquad a_{max} = 2.73\ m/s^2$$



Video Lecture # 11 – Comparing Graphs in Simple Harmonic Motion - Position, Velocity and Acceleration vs. Time
(no lecture notes)

Video Lecture # 12 – Introduction to Energy in Simple Harmonic Motion

Energy

$$KE = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\left(-A\omega\sin(\omega t + \phi)\right)^2$$
$$= \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi)$$

$$PE_e = \frac{1}{2}kx^2$$
$$= \frac{1}{2}k\left(A\cos(\omega t + \phi)\right)^2$$
$$= \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

$$ME_t = KE + PE_e$$
$$= \frac{1}{2}\underbrace{mA^2\omega^2}\sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

$\omega^2 = \frac{k}{m}$

$k = \omega^2 m$

$$= \frac{1}{2}kA^2\sin^2(\omega t + \phi) + \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$
$$= \frac{1}{2}kA^2\underbrace{\left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right]}_{=1}$$

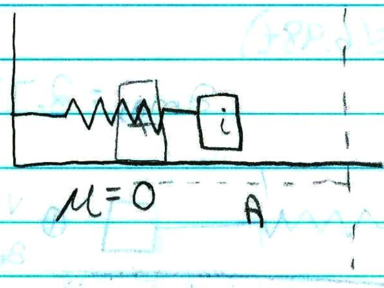
$$\boxed{ME_t = \frac{1}{2}kA^2} = \frac{1}{2}mv_{max}^2$$

Video Lecture # 13 – Comparing Simple Harmonic Motion to Circular Motion (no lecture notes)

Video Lecture # 14 – Finding Amplitude, Phase Constant and Position as a Function of Time in Simple Harmonic Motion

Ex

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{125}{0.55}} = 15.0755$ rad/s



$v_i = 1.00$ m/s Left

$x_i = 0.020$ m Left

$k = 125$ N/m

$m = 0.55$ kg

$x(t) = ?$

$x(t) = A\cos(\omega t + \phi)$

$ME_i = ME_f$

$\cancel{\tfrac{1}{2}} m v_i^2 + \cancel{\tfrac{1}{2}} k x_i^2 = \cancel{\tfrac{1}{2}} k A^2$

$m v_i^2 + k x_i^2 = kA^2$

$(0.55)(-1)^2 + (125)(-0.02)^2 = (125)A^2$

$A = 0.06928$ m

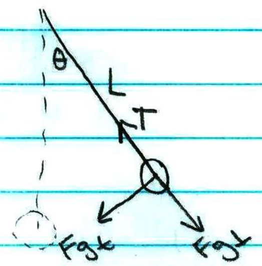
x_i $\quad -0.02 = 0.06928\cos(\cancel{\omega(0)} + \phi)$

$\phi = 1.864$ rad

$$\boxed{x(t) = [0.693\cos(15.1t + 1.86)]\ \text{m}}$$

Video Lecture # 15 – Derivation of Period and Position vs Time for a Pendulum in Simple Harmonic Motion

Pendulum:



$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$S = L\theta$$

$$\frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2}$$

$$\sum F_{tang} = -F_{g_t} = ma_t$$

$$-\cancel{m}g\sin\theta = \cancel{m}\frac{d^2s}{dt^2}$$

$$-g\sin\theta = \frac{d^2s}{dt^2}$$

$$-g\sin\theta = L\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

For "small" angles

$\sin\theta = \theta$
(in radians)

$< 15°$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\omega = \sqrt{\frac{g}{L}} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\theta = \theta_{max}\cos(\omega t + \phi)$$

$$x(t) = A\cos(\omega t + \phi)$$

Video Lecture # 16 – A Statement about Memorization of Angular Frequencies for the AP Physics C Test

$\omega = \sqrt{\frac{g}{L}}$ Pendulum

$\underbrace{\omega = \sqrt{\frac{k}{m}}}$ Mass-spring system

Just use these
+ memorize
...but still be
able to derive

15-52

$m_1 = 9.00$ kg

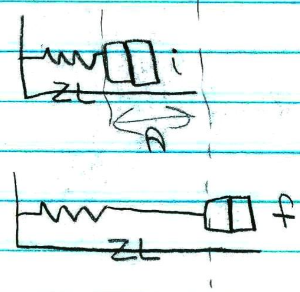
$k = 100.0$ N/m

$m_2 = 7.00$ kg

$A = 0.200$ m

a) $V_f = ?$

b) $D = ?$ @ blah, blah, blah



$$ME_i = ME_f$$

$$\cancel{\tfrac{1}{2}} k x_i^2 = \cancel{\tfrac{1}{2}} m v_f^2 \quad KE_t \rightarrow m_t$$

$$kA^2 = m v_f^2$$

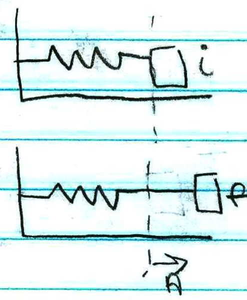
$$V_f = \sqrt{\frac{kA^2}{m}}$$

$$V_f = \sqrt{\frac{100(0.2)^2}{9+7}}$$

$$\boxed{V_f = 0.500 \text{ m/s}}$$

2nd block

$$V_x = \frac{\Delta x}{\Delta t}$$



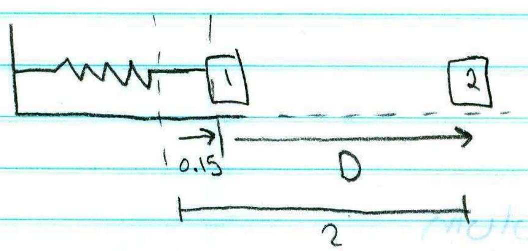
1st block

$$ME_i = ME_f$$

$$\cancel{\tfrac{1}{2}} m v_i^2 = \cancel{\tfrac{1}{2}} k x_f^2$$

$$m v_i^2 = kA^2$$

$$A = 0.15 \text{ m}$$



$$\Delta t = T/4 = \tfrac{1}{4} 2\pi \sqrt{\tfrac{m}{k}}$$

$$= \tfrac{\pi}{2} \sqrt{\tfrac{9}{100}}$$

$$\Delta t = 0.4712 \text{ s}$$

$$V_2 = \frac{\Delta x_2}{\Delta t}$$

$$\Delta x_2 = V_2 \Delta t$$

$$\Delta x_2 = (0.5)(0.4712)$$

$$\Delta x_2 = 0.2356 \text{ m}$$

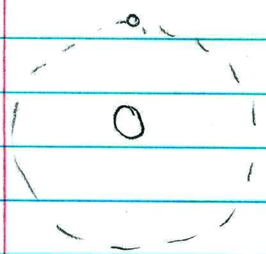
$$D = \Delta x_2 - A_1$$

$$= 0.2356 - 0.15$$

$$\boxed{D = 0.0856 \text{ m}}$$

$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r}_{12}$

Kepler's 3rd Law:



$\Sigma F_{in} = F_g = m a_c$

$\frac{Gm_1m_2}{r^2} = mr\omega^2 = m\frac{v_t^2}{r}$

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{T}$

$U_g = -\frac{Gm_1m_2}{r}$ ZL @ $r \to \infty$ $\underline{U_g = mgh}$ ZL + const. g

Escape Velocity
COE

$F_s = -kx$

$\underbrace{\frac{d^2x}{dt^2} = -\omega^2 x}_{\text{SHM}}$ MEMORIZE

$x(t) = A\cos(\omega t + \phi)$ mem + derive

$\omega \neq f$ $f = \frac{1}{T}$ $\omega = \sqrt{\frac{k}{m}}$ mass/spring

$\omega = 2\pi f$ $\omega = \sqrt{\frac{g}{L}}$ rad/s pendulum

$T = 2\pi\sqrt{\frac{m}{k}}$ mass-spring

$T = 2\pi\sqrt{\frac{L}{g}}$ pendulum

derive