The Law of Charges
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Chapter 23-24
Thank You, Emily Rencsok, for these notes.

Law of Charges
- Like charges repel
- Unlike charges attract
- Positive + negative charges

Charge via
1) Conduction (2 objects touching)
2) Induction (grounding) (never touch)
3) Polarization (alignment of charges)

Thank You, Emily Rencsok, for these notes.

Charge is quantized

$q = ne$ (n: integer) $\quad e = \pm 1.60 \times 10^{-19}$ C

Coulombs, C

Not a base SI unit

Point charge

$\vec{F}_{12} = \frac{k q_1 q_2}{r^2} \hat{r}$ Coulomb force

Coulomb constant, k

$k = 8.99 \times 10^9 \ N \cdot m^2 / C^2$

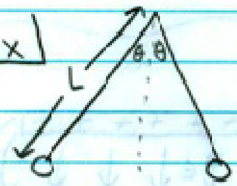
$k = \frac{1}{4\pi\epsilon_0}$

ϵ_0: permittivity of free space

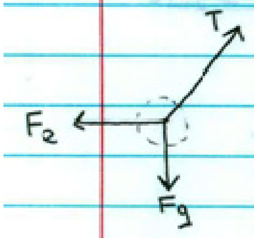
$\varepsilon_o = 8.85 \times 10^{-12} \ C^2/N \cdot m^2$ (also called vacuum permitivity)

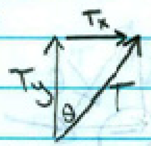
Thank You, Emily Rencsok, for these notes.

Ex



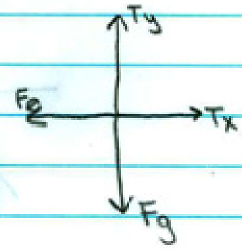
2 charged balloons
equal m + equal q
Know: m, L, θ
Find: q = ?





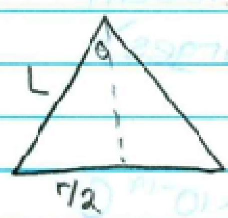
$T_x = T\sin\theta$

$T_y = T\cos\theta$



$$\Sigma F_y = -F_g + T_y = \cancel{ma_y} = 0$$
$$T_y = F_g$$
$$T\cos\theta = mg$$
$$T = \frac{mg}{\cos\theta}$$

$$\Sigma F_x = T_x - F_e = \cancel{ma_x} = 0$$
$$T_x = F_e$$
$$T\sin\theta = \frac{kq_1q_2}{r^2}$$
$$T = \frac{kq_1q_2}{r^2\sin\theta}$$
$$T = \frac{kq_1q_2}{r^2\sin\theta} = \frac{mg}{\cos\theta}$$
$$T = \frac{kq_1q_2}{(2L\sin\theta)^2\sin\theta} = \frac{mg}{\cos\theta}$$
$$\frac{kq^2}{4L^2\sin^3\theta} = \frac{mg}{\cos\theta}$$



$r = 2L\sin\theta$

$$\boxed{q = \sqrt{\frac{4L^2 mg\sin^3\theta}{k\cos\theta}}}$$

$F_g = mg$

$g = \frac{F_g}{m}$ (gravitational field)

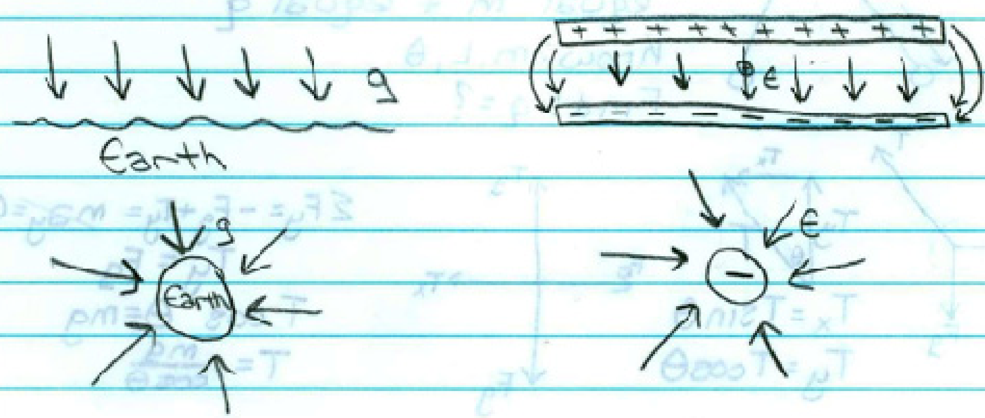
$F_E = qE$

$E = \frac{F_E}{q}$ $\left(\frac{N}{C}\right)$

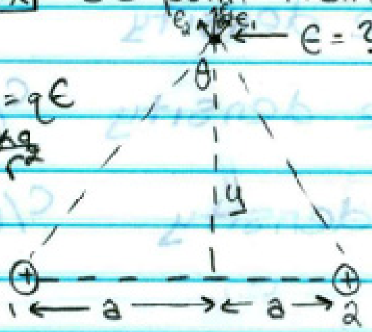
E field is defined by a small, positive test charge

$E = \frac{1}{\cancel{q}} \frac{kQ\cancel{q}}{r^2}$

$E = \frac{kQ}{r^2}$



Ex| E @ point from 2 equal positive charges



$F_E = qE$

$E = \frac{kq}{r^2}$

x-components of E_t cancel out due to symmetry

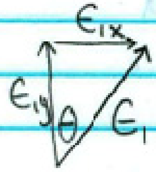
$E_t = E_{1y} + E_{2y} = 2E_y$

$E_t = 2E_1 \cos\theta$

$= 2\frac{kq}{r^2}\frac{y}{\sqrt{a^2+y^2}}$

$= 2\frac{kq}{a^2+y^2}\frac{y}{\sqrt{a^2+y^2}}$

$\boxed{E_t = \frac{2kqy}{(a^2+y^2)^{3/2}}}$



$\cos\theta = \frac{A}{H} = \frac{E_{1y}}{E_1}$

$E_{1y} = E_1 \cos\theta$

$\tan\theta = \frac{O}{A} = \frac{a}{y}$

$\theta = \tan^{-1}\left(\frac{a}{y}\right)$

$\cos\theta = \frac{A}{H} = \frac{y}{\sqrt{a^2+y^2}}$

if $y >> a$, $a^2+y^2 \approx y^2$

$E_t = \frac{2kqy}{(y^2)^{3/2}}$

$H^2 = a^2+y^2$

$H = \sqrt{a^2+y^2}$

$E_t = \frac{2kqy}{y^3} = \boxed{\frac{2kq}{y^2} = E_t}$

E field of a continuous charge distribution
(not a point charge)

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$dE = \frac{k\,dq}{r^2}\hat{r}$$

$$\int dE = \int \frac{k\,dq}{r^2}\hat{r}$$

$$\boxed{E = k\int \frac{dq}{r^2}\hat{r}}$$ not on eqn sheet

$\rho = \frac{Q}{V}$ Volumetric-charge density C/m^3

$\sigma = \frac{Q}{A}$ Surface-charge density C/m^2

$\lambda = \frac{Q}{L}$ Linear-charge density C/m

Example - Electric Field from Charged Rod
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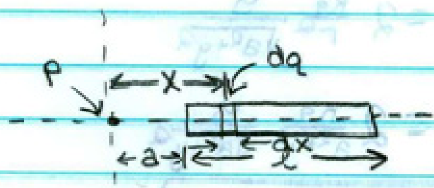
Ex) Uniformly, positively-charged rod

$E = ?$ @ P



$E = k\int \frac{dq}{r^2}\hat{r}$

$E = k\int \frac{dq}{x^2}(-\hat{i})$

$\lambda = \frac{Q}{\ell} = \frac{dq}{dx}$

$dq = \lambda dx$

$E = -k\hat{i}\int \frac{\lambda dx}{x^2}$

$E = -k\lambda\hat{i}\int_a^{a+\ell} \frac{1}{x^2}\,dx$

$E = -k\lambda\hat{i}\int_a^{a+\ell} x^{-2}\,dx$

$E = \cancel{-}k\lambda\hat{i}\left[\frac{x^{-1}}{\cancel{-}1}\right]_a^{a+\ell}$

$E = k\lambda\hat{i}\left[\frac{1}{x}\right]_a^{a+\ell}$

$E = k\lambda\hat{i}\left[\frac{1}{a+\ell} - \frac{1}{a}\right]$

$E = \frac{k\lambda\hat{i}\,-\ell}{a(a+\ell)} = \frac{-k\lambda\ell}{a(a+\ell)}\hat{i}$

$\lambda\ell = Q$

$\boxed{E = \frac{-kQ}{a(a+\ell)}\hat{i}}$

if $a \gg \ell$, then $\ell + a \approx a$

$\boxed{E = \frac{-kQ}{a^2}\hat{i}}$

Example - E Field from Positively Charged Ring

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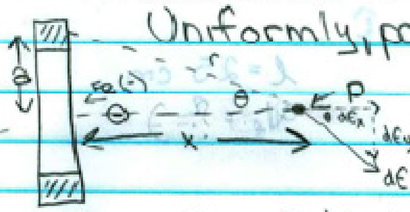
$E = ?\ @\ P$

Ex | Uniformly, positively-charged ring $(+Q)$



Known: $a, x, +Q$

All dE_y's cancel due to symmetry

$$E = k\int \frac{dq}{r^2}\hat{r}$$

$$dE = \frac{k\,dq}{r^2}\hat{r}$$

$$dE = \frac{k\,dq}{a^2+x^2}\hat{r}$$

$$dE_t = dE_x = dE\cos\theta$$

$$dE_t = \frac{k\,dq}{a^2+x^2}\hat{r}\cos\theta$$

$\uparrow \hat{\imath}$

$$E_t = \int \frac{k}{a^2+x^2}\cos\theta\,\hat{\imath}\,dq$$

$$\cos\theta = \frac{A}{H} = \frac{x}{r} = \frac{x}{\sqrt{a^2+x^2}}$$

$$E_t = \int \frac{k}{a^2+x^2}\frac{x}{\sqrt{a^2+x^2}}\hat{\imath}\,dq$$

$$E_t = \int \frac{kx}{(a^2+x^2)^{3/2}}\hat{\imath}\,dq$$

$$E_t = \frac{kx}{(a^2+x^2)^{3/2}}\hat{\imath}\int dq$$

$$\boxed{E_t = \frac{kQx}{(a^2+x^2)^{3/2}}\hat{\imath}}\ @\ P$$

if $x >> a$, then $x^2 + a^2 \approx x^2$

$$E_t = \frac{kQx}{(x^2)^{3/2}}\hat{\imath} = \frac{kQx}{x^3}\hat{\imath} = \boxed{\frac{kQ}{x^2}\hat{\imath}}$$

if $a >> x$, then $x^2 + a^2 \approx a^2$

$$E_t = \frac{kQx}{(a^2)^{3/2}}\hat{\imath} = \frac{kQx}{a^3}\hat{\imath}$$

$$\Sigma F_x = -F_e = ma_x$$

$$-qE = ma_x$$

$$-q\left(\frac{kQx}{a^3}\right) = ma_x$$

$$a_x = \frac{-qkQ}{ma^3}x$$

$$\frac{d^2x}{dt^2} = -\left(\frac{qkQ}{ma^3}\right)x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega = \sqrt{\frac{qkQ}{ma^3}} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

$$\boxed{T = 2\pi\sqrt{\frac{ma^3}{qkQ}}}$$

Thank You, Emily Rencsok, for these notes.

Electric field lines
- in direction of E field -more lines = more charge
- lines/unit area $\propto$ field strength
- Start + $\perp$ to $\oplus$ + end $\ominus$
(more one charge than another, E fields $\rightarrow \infty$
- E field lines never cross

Example - Charge Moving between 2 Parallel Plates
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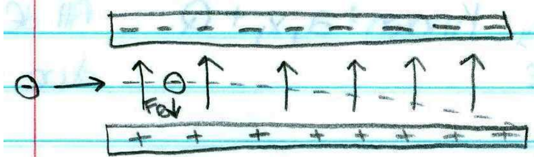
Ex) e^- $V_i = 1\times10^6$ m/s $\hat{i}$

$E = 25$ N/C $\quad\quad \ell = 25$ cm

$V_f = ?$



$$\frac{F_e}{F_g} = \frac{qE}{mg} = \frac{(1.6\times10^{-19})(25)}{(9.11\times10^{-31})(9.8)}$$

$$\frac{F_e}{F_g} = 4.48\times10^{11} \rightarrow F_e = 4.48\times10^{11}\,F_g$$

$F_e = 448000000000\,F_g$

F_g is negligible for atomic particles

$\Sigma F_y = -F_e = ma_y$

$-qE = ma_y$

$a_y = -\frac{qE}{m}$

x-dir

$V_x = \frac{\Delta x}{\Delta t}$

$1.0\times10^6 = \frac{0.25}{\Delta t}$

$\Delta t = 2.5\times10^{-7}$ s

$V_{fy} = V_{iy} + a_t\,\Delta t$

$V_{fy} = 0 + \left(\frac{-qE}{m}\right)\Delta t$

$$V_{fy} = \frac{-(1.6\times10^{-19})(25)(2.5\times10^{-7})}{9.11\times10^{-31}}$$

$V_{fy} = -1.098\times10^6$ m/s

$$\boxed{V_{fy} = (1.00\,\hat{i} - 1.10\,\hat{j})\times10^6\ \frac{m}{s}}$$

Introduction to Electric Flux
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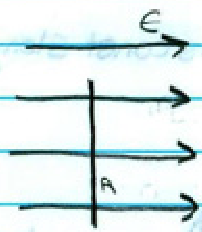
Electric flux = # lines that pass through an area

$\Phi_E = EA\cos\theta$ $\quad\theta$ is angle btwn E + A

$= E \cdot A \quad \left(\frac{N\cdot m^2}{C}\right)$

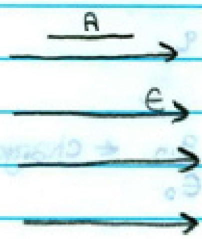
Constant Efield



$\Phi = EA\cos\theta$

$= EA$

$\Phi = EA\cos\theta$



$\Phi = EA\cos 90$

$= 0$

Example - Net Electric Flux through a Closed Surface
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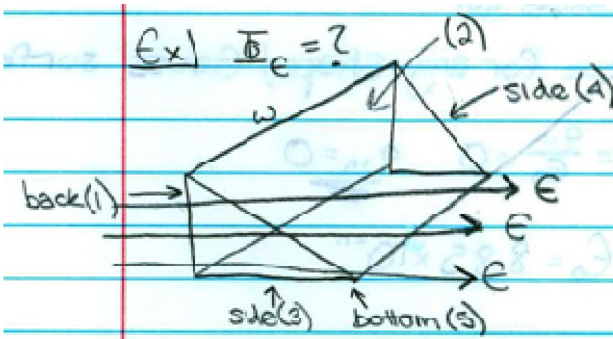
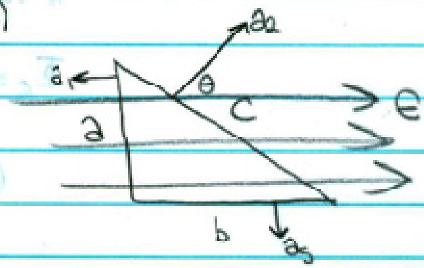
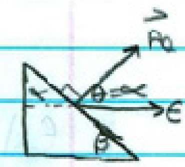
Ex) $\Phi_E = ?$







$$\Phi_{E5} = EA\cos\theta$$
$$= EA\cos 90$$
$$= 0$$

$$\Phi_{E2} = EA\cos\theta$$
$$= Ecw\cos\alpha$$
$$= Ecw\frac{a}{c}$$
$$= Eaw$$

$$\Phi_{E3} = \Phi_{E4} = EA\cos 90$$
$$= 0$$

$$\Phi_{E} = -Eaw + Eaw = 0$$

$$\Phi_{E1} = EA\cos 180$$
$$= -Eaw$$

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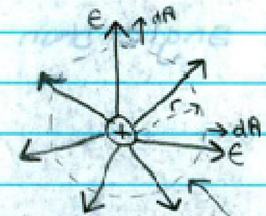
* Draw Gaussian surface

Gauss' Law:



Gaussian surface (sphere)

Nonconstant E fields

$$\Phi_E = \oint E \cdot dA$$
$$= \oint E\, dA \cos\theta$$
$$= \oint E\, dA \cos 0$$
$$= \oint E\, dA \quad E \text{ is const along Gauss sur}$$
$$= E \oint dA$$
$$= EA$$

$$\Phi_E = \left(\frac{kq}{r^2}\right)(4\pi r^2) \qquad k = \frac{1}{4\pi\epsilon_0}$$
$$= 4\pi k q$$
$$= 4\pi \left(\frac{1}{4\pi\epsilon_0}\right) q$$
$$= q/\epsilon_0$$

$$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0} \leftarrow \text{charge in closed gauss sur}$$

total E field @ Gauss sur

* True for any-shaped Gauss surface

$$\Phi_E = \frac{q_{in}}{\epsilon_0} = 0 \qquad q_{in} = 0$$

$\epsilon_o = 8.85 \times 10^{-12}\ C^2/N\cdot m^2$

Example - E Field Outside a Solid, Uniform Insulating Sphere
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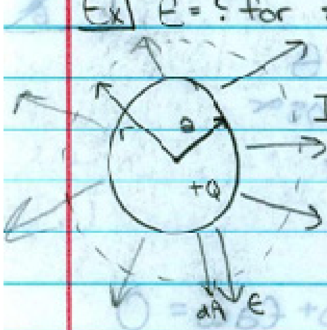
Ex) $E = ?$ for solid, uniform sphere w/ charge Q, $R = a$ insulator



$$\Phi_e = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$r > a$

$$\Phi_e = \oint E\,dA\cos\theta = \frac{q_{in}}{\epsilon_0}$$

$$\Phi_e = \oint E\,dA\cos 0 = \frac{q_{in}}{\epsilon_0}$$

$$E \oint dA = \frac{q_{in}}{\epsilon_0}$$

$$EA = \frac{Q}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$\boxed{E = \frac{kQ}{r^2}}$$

Acts like a point particle

Outside a sphere w/ const charge, assume point particle

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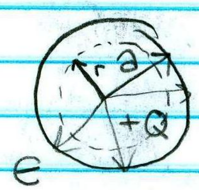
$r < a$



$q_{in} \neq +Q$

$q_{in} < +Q$

$\rho = \frac{Q_t}{V_t} = \frac{q_{in}}{V_{in}}$

$q_{in} = \frac{V_{in} Q}{V_t}$

$q_{in} = \frac{\cancel{\frac{4}{3}\pi} r^3 Q}{\cancel{\frac{4}{3}\pi} a^3}$

$q_{in} = \frac{Q r^3}{a^3}$

$\Phi_e = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$\oint E\, dA \cos\theta = \frac{q_{in}}{\epsilon_0}$

$\oint E\, dA \cos 0 = \frac{q_{in}}{\epsilon_0}$

$\oint E\, dA = \frac{q_{in}}{\epsilon_0}$

$E \oint dA = \frac{q_{in}}{\epsilon_0}$

$EA = \frac{q_{in}}{\epsilon_0}$

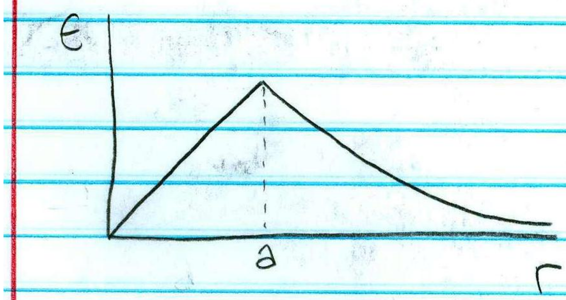
$E(4\pi \cancel{r^2}) = \frac{Q r^{\cancel{3}}}{a^3 \epsilon_0}$

$4\pi E = \frac{Qr}{a^3 \epsilon_0}$

$E = \frac{Qr}{4\pi \epsilon_0 a^3}$

$E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{a^3}$

$\boxed{E = \frac{kQr}{a^3}}$



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Ex A thin spherical uniformly charged shell, +Q

$\Phi_e = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

E = ? where $r > a$

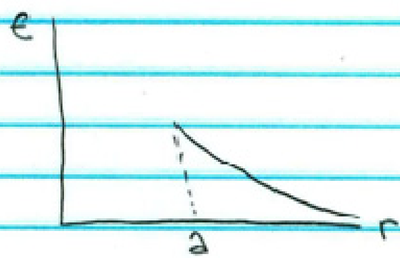
$\boxed{E = \frac{kq}{r^2}}$ ⟵ acts like a point charge

E = ? where $r < a$

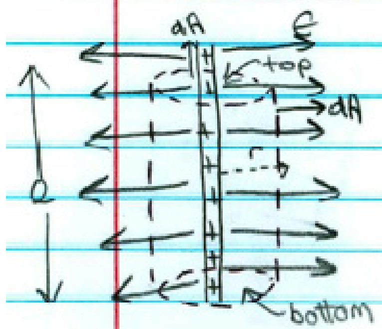
$q_{in} = 0$

$\oint E \cdot da = 0$

$\boxed{E = 0}$



Ex| $E = ?$ @ r from ∞ long line of positive charge w/ uniform λ



Gaussian surface cylinder

$\oint E\,dA\cos 90 = 0$ for top + bottom

$$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E\,dA\cos\theta = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E\,dA\cos 0 = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E\,dA = \frac{q_{in}}{\varepsilon_0}$$

$$E\oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$E A_{side} = \frac{q_{in}}{\varepsilon_0}$$

$$E A_{side} = \frac{\lambda \ell}{\varepsilon_0}$$

$$\lambda = \frac{Q}{L} = \frac{q_{in}}{\ell}$$

$$q_{in} = \lambda \ell$$

$$E(2\pi r\cancel{\ell}) = \frac{\lambda \cancel{\ell}}{\varepsilon_0}$$

$$E 2\pi r = \frac{\lambda}{\varepsilon_0}$$

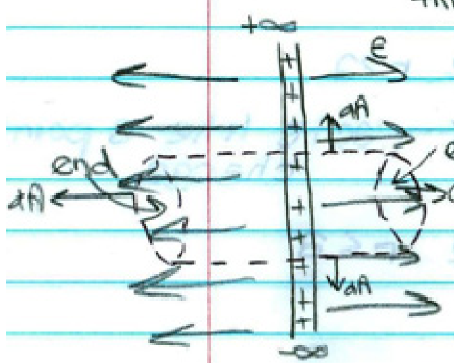
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\lambda}{r}$$

$$E = \frac{2k\lambda}{r}$$

$E = ?$ @ r from an

Ex) Infinite thin plane of + charges, uniform



$$\Phi_E = \oint_{ends} E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$\sigma = \frac{Q}{A} = \frac{q_{in}}{A_{end}}$$

$$q_{in} = \sigma A_{end}$$

$$\oint E\, dA \cos 0 = \frac{q_{in}}{\epsilon_0}$$

$$E \oint_{ends} dA = \frac{\sigma A_{END}}{\epsilon_0}$$

$$E\, 2 \cancel{A_{end}} = \frac{\sigma \cancel{A_{END}}}{\epsilon_0}$$

Gaussian surface cylinder

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

$$\oint_{side} E\, dA \cos 90 = 0$$

Example - Electric Field caused by 2 Infinitely Long Plates of Charges
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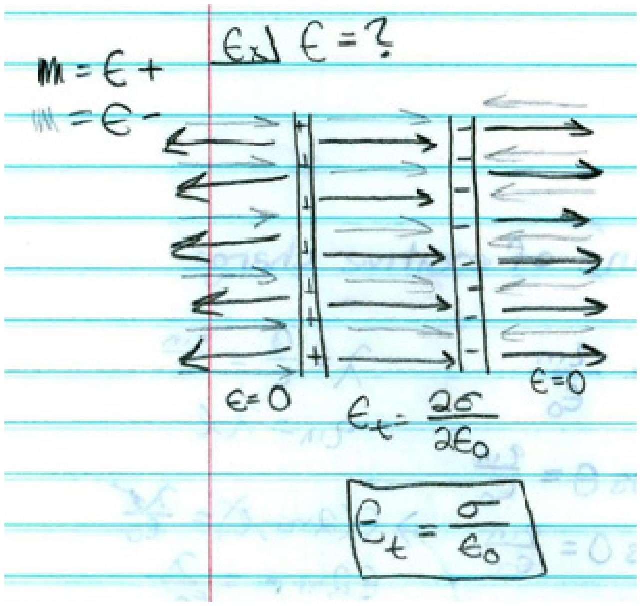


no net motion of charge

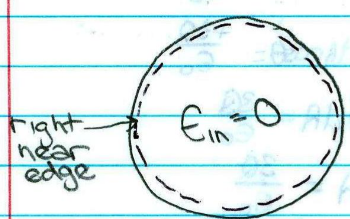
charges don't move

* Conductors in electrostatic equilibrium
(Assume if not stated)

1) $\epsilon_{inside} = 0$

if $\epsilon \neq 0 \rightarrow F_e = q\epsilon$, so charges would move

2) All excess charge is on surface ($Q_\epsilon \neq 0$)

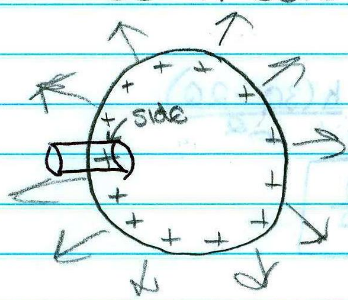


$$\Phi_\epsilon = \oint \epsilon \cdot dA = \frac{q_{in}}{\epsilon_0}$$
$$0 = \frac{q_{in}}{\epsilon_0}$$
$$q_{in} = 0$$

3) $\epsilon_{just\ outside} = \frac{\sigma}{\epsilon_0}$ ← at that location + ⊥ to surface

$\sigma = \frac{dq}{dA}$

if ϵ_{field} has a component // to surface, it wouldn't be in estatic equilibrium



$$\Phi_\epsilon = \oint \epsilon \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$\oint_{side} \epsilon\, dA \cos 90 = 0$ $\quad \oint_{right\ end} \epsilon \cdot dA = 0 \quad \epsilon = 0$

$\oint_{left\ end} \epsilon\, dA \cos 0 = \frac{q_{in}}{\epsilon_0}$

$$\epsilon \oint dA = \frac{q_{in}}{\epsilon_0}$$

$\sigma = \frac{Q}{A} = \frac{q_{in}}{A_{end}}$

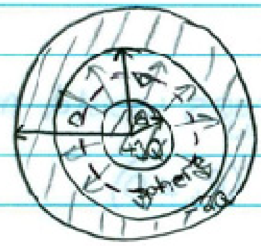
$q_{in} = \sigma A_{end}$

$$\epsilon \cancel{A_{end}} = \frac{\sigma \cancel{A_{end}}}{\epsilon_0}$$

$$\boxed{\epsilon = \frac{\sigma}{\epsilon_0}}$$

4) For an irregular shape, σ_{max} @ radius of curvature min

Ex) $E = ?$ Conductors in ES equil



$r < a$ $\boxed{E = 0}$

$a < r < b$

$$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$\oint E\,dA\cos\theta = \frac{+3Q}{\epsilon_0}$$

$$E \oint dA = \frac{3Q}{\epsilon_0}$$

$$EA = \frac{3Q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{3Q}{\epsilon_0}$$

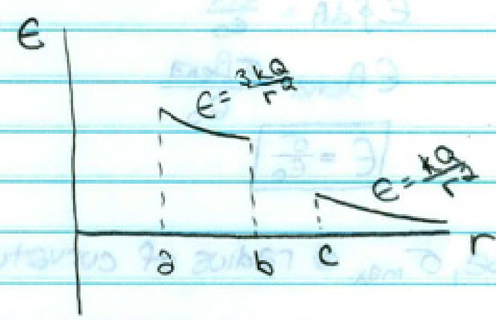
$$E = \frac{3Q}{4\pi\epsilon_0 r^2} = \boxed{\frac{3kQ}{r^2}}$$

$b < r < c$ $\boxed{E = 0}$

$r > c$

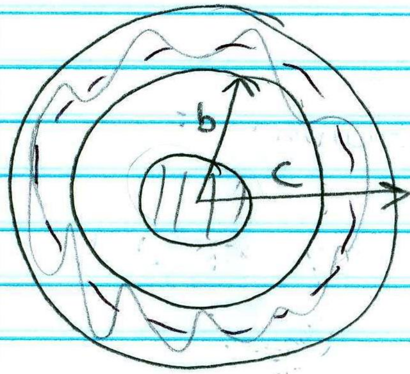
$$E = \frac{kq}{r^2} = \frac{k(3Q - 2Q)}{r^2}$$

$$\boxed{E = \frac{kQ}{r^2}}$$



Charge on surfaces?

$\{$surface of $+3Q = +3Q$



$q_b = ?$ $q_c = ?$

$\Phi_e = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$0 = \frac{q_{in}}{\epsilon_0}$

$q_{in} = 0$

$q_{in} = q_a + q_b = 0$

$q_b = -q_a = \boxed{-3Q}$

$q_{3R} + q_{2R} = +q$

$q_c + q_B = q_t = -2Q$

$q_c - 3Q = -2Q$

$\boxed{q_c = Q}$

Law of Charges

$Q = ne$

$F_e = \frac{kq_1q_2}{r^2}$ Coulomb's

Uniform charge distribution

$\int dE = \int \frac{k\,dq}{r^2}$

$E = k\int \frac{dq}{r^2}\hat{r}$

$\rho = \frac{Q}{V}$ $\sigma = \frac{Q}{A}$ $\lambda = \frac{Q}{L}$

capital phi $\Phi_e = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$ ← in Gaussian surface GAUSSIAN SURFACE

SA of Gaussian surface

Shape of Gaussian surface
- dA ⊥ or // to E field lines
- Efield must be constant on Gaussian Surface

	GS
* inside + outside sphere (C + I) →	Sphere
* inside + outside shell →	sphere
* infinitely long wire →	vert cylinder As
* infinitely long plate →	horiz cylinder Aends
* 2 infinitely large // plates →	horiz cyl for 1 + logic

Conductors in ES equil

Outside sphere of uniform charge, acts like point charge
so $E = \frac{kq}{r^2}$ ← q inside sphere