F_e is a conservative force

$F_x = \frac{-dU}{dx}$ $\qquad E = \frac{F_e}{q}$

$F_e = -\frac{dU}{dr}$ $\qquad F_e = qE$

$\int dU = -\int F_e \, dr$

$\Delta U_{ele} = -\int_A^B F_e \cdot ds$

$\Delta U_{ele} = -\int_A^B qE \cdot ds$

$\boxed{\Delta U_{ele} = -q\int_A^B E \cdot ds}$ Change in electrical potential energy

Electric potential difference

$\boxed{\Delta V = \frac{\Delta U_{ele}}{q}} = \frac{J}{C}$

$\Delta V = -\int_A^B E \cdot ds = V_B - V_A$ Independent of test charge

often set $V_A = 0$

same — scalar

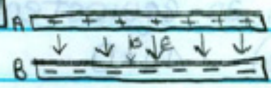
$E = -\frac{dV}{dr}$

$E \rightarrow \frac{N}{C} \cdot \frac{m}{m} = \frac{N \cdot m}{C \cdot m} = \frac{J}{C} \cdot \frac{1}{m} = \frac{V}{m}$ $\qquad \frac{N}{C} = \frac{V}{m}$

$1\,eV = 1.60 \times 10^{-19}\,J$ Electron volt is unit of energy (used for small energies)

Derivation of Electric Potential Difference in a Constant Electric Field
2
AP Physics C – Video Lecture Notes
Chapter 25-26
Thank You, Emily Rencsok, for these notes.

Ex



$\Delta V = 12\ V$

$d = 1.0\ cm$

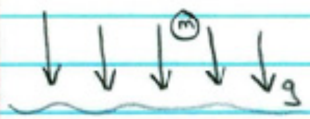
$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$
$$= -\int_A^B E \cos\theta\, ds$$
$$= -\int_A^B E \cos 0\, ds$$
$$= -E \int_A^B ds$$

$\boxed{\Delta V = -Ed}$ constant Efield

MEMORIZE + DERIVE

$\Delta V = -Ed$

What does the negative mean?



E field line is always in direction w/ a decreasing V

Ex: b Release a proton at plate A... $v_B = ?$

PE_g for atomic particles is negligible

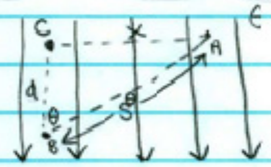
$$ME_i = ME_f$$
$$U_{ele_i} = KE_f + U_{ele_f}$$
$$-KE_f = \Delta U_{ele}$$
$$-\tfrac{1}{2} m v_f^2 = q\,\Delta V$$
$$v_f = \sqrt{\frac{-2q\,\Delta V}{m}}$$
$$v_f = \sqrt{\frac{-2(1.6\times10^{-19})(-12)}{1.67\times10^{-27}}}$$
$$v_f = 47952\ \tfrac{m}{s}\left(\frac{3600\ s}{hr}\right)\left(\frac{1\ mi}{1609\ m}\right)$$
$$v_f = 107288$$
$$\boxed{v_f = 107000\ mi/hr}$$

$\Delta V = \frac{\Delta U_{ele}}{q}$

$\Delta U_{ele} = q\,\Delta V$

$\underline{\underline{\Delta V = -12V}}$

Derivation of Equipotential Surface and Electric Potential due to a Point Charge
3
AP Physics C – Video Lecture Notes
Chapter 25-26
Thank You, Emily Rencsok, for these notes.



$$\Delta V_{AB} = -\int_A^B E \cdot ds = -E\int_A^B \cos\theta \, ds$$

$$= -E\int_A^B \frac{d}{s}\, ds = -\frac{Ed}{s}\int_A^B ds$$

$$= -\frac{Ed}{s} s$$

$$\Delta V_{AB} = -Ed$$

$$\Delta V_{AC} = -\int_A^C E \cdot ds \qquad \theta = 90$$

$$= -E\int_A^C ds \cos\theta$$

$$\Delta V_{AC} = 0$$

$$\Delta V_{AB} = \Delta V_{CB}$$

- A + C are on an equipotential surface (line) $\Delta V = 0$
- it takes zero work to move a charged particle along an equipotential surface
- Always ⊥ to Efield

Thank You, Emily Rencsok, for these notes.

ΔV due to a point charge

$$\Delta V = V_B - V_A = -\int_A^B \mathcal{E} \cdot ds$$

$$= -\int_A^B \frac{kq}{r^2}\hat{r} \cdot ds$$

$$\hat{r} \cdot ds = \hat{r}\, ds \cos\theta = (1)\, ds \frac{dr}{ds} = dr$$

$$\Delta V = -\int_A^B \frac{kq}{r^2}\, dr$$

$$= -kq\int_A^B r^{-2}\, dr$$

$$= -kq\left[\frac{-1}{r}\right]_A^B$$

$$= kq\left[\frac{1}{r}\right]_A^B$$

$$\Delta V = kq\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad \text{take } r_A \approx \infty \quad \frac{1}{r_A} \approx 0$$

$$\boxed{\Delta V = \frac{kq}{r}} \text{ due to a point charge}$$

$$\Delta V = \frac{\Delta U_{ele}}{q}$$

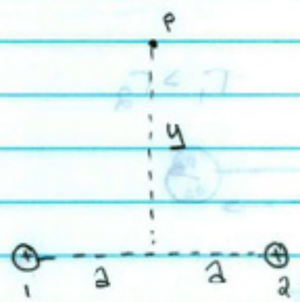
$$\Delta V q = \Delta U_{ele}$$

$$\frac{kq}{r} q = \Delta U_{ele} \qquad \boxed{\Delta U_{ele} = \frac{kq_1 q_2}{r}}$$

Ex

$$\mathcal{E}_r = -\frac{dV}{dr}$$

V = ? + $\mathcal{E}$ = ? @ P



$$V_p = V_1 + V_2$$

$$= \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$

$$= \frac{2kq}{r} = \boxed{\frac{2kq}{\sqrt{a^2+y^2}} = V_p}$$

$$\mathcal{E} = \frac{-dV}{dr} = -\frac{d}{dy}\,\frac{2kq}{\sqrt{a^2+y^2}}$$

$$\mathcal{E} = -2kq\,\frac{d}{dy}\left(a^2+y^2\right)^{-1/2}$$

$$\mathcal{E} = -2kq\left(-\frac{1}{2}\right)\left(a^2+y^2\right)^{-3/2}(2y)$$

$$\boxed{\mathcal{E} = \frac{2kqy}{(a^2+y^2)^{3/2}}}$$

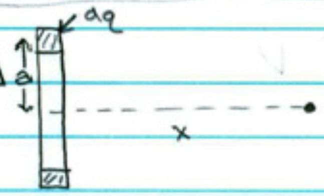
Thank You, Emily Rencsok, for these notes.

$\mathcal{E} \neq$ constant due to a charge distribution

$V = k\int \frac{dq}{r}$

$\left(V = \frac{kq}{r} \rightarrow dV = \frac{k\,dq}{r}\right)$

Ex)



V due to a uniformly, positively-charged ring $+Q$

$\mathcal{E} = ?$ @ P

$V = k\int \frac{dq}{r}$

$V = \frac{k}{r}\int dq$

$V = \frac{kq}{r} = \frac{kq}{\sqrt{a^2+x^2}}$

$\mathcal{E} = -\frac{dV}{dr} = -\frac{d}{dx}\frac{kq}{\sqrt{a^2+x^2}}$

$\mathcal{E} = -kq\frac{d}{dx}(a^2+x^2)^{-1/2}$

$\mathcal{E} = -kq\left(-\frac{1}{2}\right)(a^2+x^2)^{-3/2}(2x)$

$\mathcal{E} = \frac{kqx}{(a^2+x^2)^{3/2}}$

Thank You, Emily Rencsok, for these notes.

Charged conductor in ES equilibrium

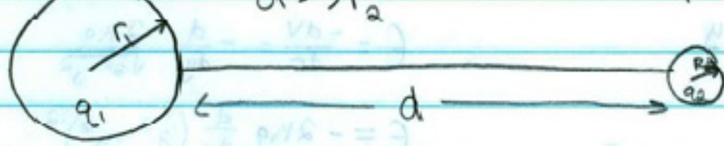
$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{E} \cdot d\vec{s} = E\,ds\cos\theta$$
$$= E\,ds\cos 90$$
$$\vec{E} \cdot d\vec{s} = 0$$
$$\Delta V = 0$$

Surface of a conductor in ES equil is an equipotential surface

Ex) in ES equil

$d >> r_2$ $\qquad r_1 > r_2$



$$V_1 = V_2 \rightarrow \frac{kq_1}{r_1} = \frac{kq_2}{r^2}$$

$$\frac{\cancel{k}q_1}{r_1} = \frac{\cancel{k}q_2}{r_2}$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$

$$q_1 = \frac{r_1}{r_2} q_2 \rightarrow r_1/r_2 > 1 \rightarrow q_1 > q_2$$

$$E_1 = \frac{kq_1}{r_1^2} \qquad E_2 = \frac{kq_2}{r_2^2}$$

$$\frac{E_1}{E_2} = \frac{\frac{kq_1}{r_1^2}}{\frac{kq_2}{r_2^2}} = \frac{\cancel{k}q_1}{r_1^2} \cdot \frac{r_2^2}{\cancel{k}q_2}$$

$$= \frac{q_1(r_2)^2}{q_2(r_1)^2} = \frac{\left(\frac{r_1}{r_2}\right)q_2(r_2)^2}{q_2(r_1)^2}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1} \qquad \frac{r_2}{r_1} < 1$$

$$E_1 = \frac{r_2}{r_1} E_2$$

$$E_1 < E_2$$

A smaller radius means a larger E field

OR

σ is ↑ where radius of curvature is ↓

Thank You, Emily Rencsok, for these notes.

25-59 $W = ? = \Delta U_{ele} = q\Delta V$

spherical shell $\rightarrow R, Q$

$q_i = 0$

$q_f = Q$

$W = qV$ ← V is changing

$$W = \int_0^Q V\,dq$$

$$W = \int_0^Q \frac{kq}{R}\,dq$$

$$W = \frac{k}{R}\int_0^Q q\,dq$$

$$W = \frac{k}{R}\left[\frac{q^2}{2}\right]_0^Q$$

$$W = \frac{kQ^2}{2R}$$

Problem 25-60 Electric Field between 2 Parallel Plates

9

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Chapter 25-26

Thank You, Emily Rencsok, for these notes.

25-60

$d = 0.12$ cm

$\sigma = \pm 36.0 \frac{nC}{m^2}$

Proton

a) ΔV

b) $KE = ?$

c) $V_f = ?$

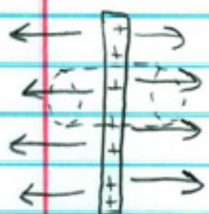
d) $a = ?$

e) $F_E = ?$

f) $E = ?$

$\Delta V = \frac{kq}{r}$ ← point charge

$\Delta V = -Ed$ ← constant E field



$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$\oint_{2\,ends} E\, dA \cos\theta = \frac{q_{in}}{\epsilon_0}$ Ends, $\theta = 0°$; Side, $\theta = 90°$

$\oint E\, dA = \frac{q_{in}}{\epsilon_0}$

$\sigma = \frac{Q}{A} = \frac{q_{in}}{A_{end}}$ $E \oint dA = \frac{q_{in}}{\epsilon_0}$

$q_{in} = \sigma A_{end}$ $EA = \frac{q_{in}}{\epsilon_0}$

$E\,2\cancel{A_{end}} = \frac{\sigma \cancel{A_{end}}}{\epsilon_0}$

$E = \frac{\sigma}{2\epsilon_0}$

$E = \frac{\cancel{2}\sigma}{\cancel{2}\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{36 \times 10^{-9}}{8.85 \times 10^{-12}} = \boxed{4070 \frac{N}{C} \text{ or } \frac{V}{m}}$

$\Delta V = -E d$

$\Delta V = -(4067.8)(0.12)$

$\Delta V = -488.14\ V$

$\boxed{\Delta V = 488\ V}$

b) $ME_i = ME_f$

$PE_{elei} = KE_f + PE_{elef}$

$KE = -\Delta PE$

$KE = -q\Delta V$

$KE = -(1.6\times10^{-19})(-488.14)$

$\boxed{KE = 7.81\times10^{-17}\ J}$

c) $KE = \frac{1}{2}mv_f^2$

$7.81\times10^{-17} = \frac{1}{2}(1.67\times10^{-27})\ v_f^2$

$\boxed{v_f = 3.058\times10^5\ m/s}$

d) $v_f^2 = \cancel{v_i^2} + 2a\Delta x$

$(3.058\times10^5)^2 = 2a(0.12)$

$\boxed{a = 3.90\times10^{11}\ m/s^2}$

e) $\Sigma\vec{F} = m\vec{a}$

$F_e = ma$

$F_e = (1.67\times10^{-27})(3.90\times10^{11})$

$\boxed{F_e = 6.51\times10^{-16}\ N}$

f) $E = \frac{F_e}{q}$

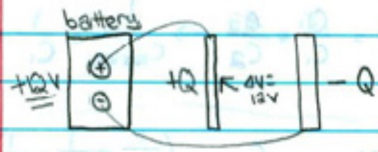
$\boxed{E = 4070\ N/C \text{ or } \frac{V}{m}}$

- Capacitor

Q → charge on + plate

$q_{net} = +Q - Q = 0$

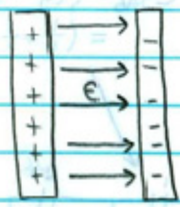
$C = \frac{Q}{\Delta V}$ ($\frac{C}{V}$ = farad) *always positive



charge → Coulombs, C

capacitance, c → Farad, F

Ex)



$C_{\text{parallel plate capacitor}} = ?$

$C = \frac{Q}{\Delta V}$ $\quad \Delta V = -Ed$ ← constant E fields

$E = ?$

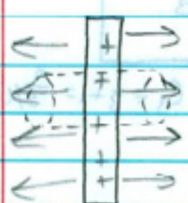
$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$\oint E\, dA \cos\theta = \frac{q_{in}}{\epsilon_0}$

$\theta = 0°$ for ends

$\theta = 90°$ for sides

$\cos 90° = 0$



$\sigma = \frac{Q}{A} = \frac{q_{in}}{A_{end}}$

$q_{in} = \sigma A_{end}$

$\oint_{ends} E\, dA = \frac{q_{in}}{\epsilon_0}$

$E \oint dA = \frac{\sigma A_{end}}{\epsilon_0}$

$E\, 2 A_{end} = \frac{\sigma A_{end}}{\epsilon_0}$

$E = \frac{\sigma}{2\epsilon_0}$

$E_t = \frac{2\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$

$\Delta V = -Ed$

$||\Delta V|| = Ed$

$\Delta V = \frac{\sigma d}{\epsilon_0}$

$C = \frac{Q}{\Delta V} = Q\left(\frac{\epsilon_0}{\sigma d}\right)$

$C = Q\left(\frac{\epsilon_0}{(\frac{Q}{A})d}\right)$ $\quad \boxed{C = \frac{A\epsilon_0}{d}}$ parallel plate capacitors

Derivation of Capacitors in Parallel and Series
11
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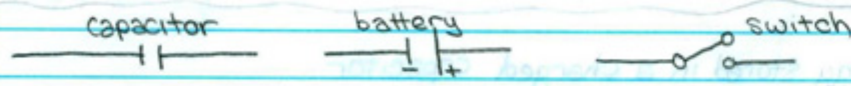
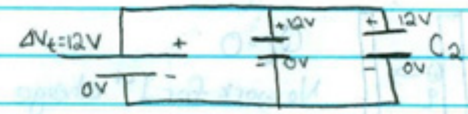
Thank You, Emily Rencsok, for these notes.



ΔV is constant in parallel

2 capacitors in parallel
ΔV_t (terminal voltage)



$C = \frac{Q}{\Delta V} \rightarrow Q = C\Delta V$

$\Delta V_t = \Delta V_1 = \Delta V_2$

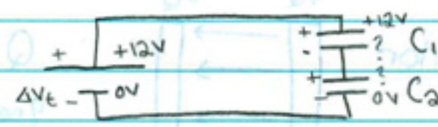
$Q_t = Q_1 + Q_2$

$C_t \Delta V_t = C_1 \Delta V_1 + C_2 \Delta V_2$

$$C_{parallel} = C_1 + C_2 + \ldots$$

Q + current are constant in series

2 capacitors in series



$\Delta V = \frac{Q}{C}$

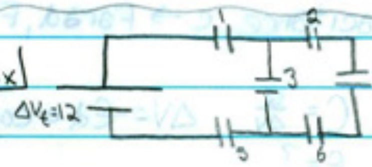
$\Delta V_t \neq \Delta V_1 \neq \Delta V_2$

$\Delta V_t = \Delta V_1 + \Delta V_2$

$$C_{series} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right)^{-1}$$

$\frac{Q_t}{C_t} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \rightarrow \frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}$

Ex



$\Delta V_t = 12\,V$

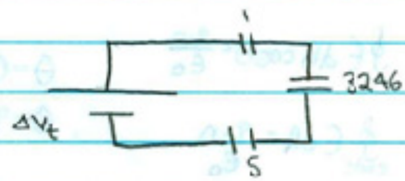
$C = 1.0\,\mu F$

$C_{eq} = ?$

$\Delta V_3 = ?$

$$C_{246} = \left(\frac{1}{C_2} + \frac{1}{C_4} + \frac{1}{C_6}\right)^{-1} = \left(\frac{1}{1} + \frac{1}{1} + \frac{1}{1}\right)^{-1} = \frac{1}{3}\,\mu F$$

$$C_{3246} = C_{246} + C_3 = \frac{1}{3} + 1 = \frac{4}{3}\,\mu F$$



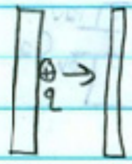
$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_{3246}} + \frac{1}{C_5}\right)^{-1} = \left(\frac{1}{1} + \frac{1}{(4/3)} + \frac{1}{1}\right)^{-1} = 0.\overline{36} = \boxed{0.36\,\mu F}$$

$$C_{eq} = \frac{Q_t}{\Delta V_t} \rightarrow Q_t = C_{eq}\Delta V_t = (0.\overline{36}\times10^{-6})(12) = 4.\overline{36}\times10^{-6}\,C = Q = Q_{3246} = Q_5$$

$$C = \frac{Q}{\Delta V} \rightarrow \Delta V_{3246} = \frac{Q_{3246}}{C_{3246}} = \frac{4.\overline{36}\times10^{-6}}{(\frac{4}{3})\times10^{-6}}$$

$$\Delta V_{3246} = 3.\overline{27}\,V = \Delta V_3 = \Delta V_{246} \quad \boxed{\Delta V_3 = 3.3\,V}$$

Energy stored in a charged capacitor



$Q_i = 0$

No work for 1st charge

$W = \Delta U_{ele} = q\Delta V$

$dW = \Delta V \, dq$

$\Delta V = \frac{Q}{C}$

$dW = \frac{Q}{C} dq$

$W = \int_0^Q \frac{q}{C} dq = \left[\frac{q^2}{2C} \right]_0^Q \quad W = \frac{Q^2}{2C}$

$U_C = \frac{Q^2}{2C} = \frac{Q^2}{2\frac{Q}{\Delta V}} = \frac{1}{2} Q\Delta V$

$= \frac{1}{2}(C\Delta V)\Delta V = \frac{1}{2}C(\Delta V)^2$

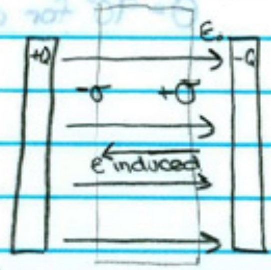
$$\boxed{U_C = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2}$$

Energy is stored in Efield

$\mu = \frac{U}{V}$ energy density

Thank You, Emily Rencsok, for these notes.

Dielectrics p.817 are insulators



$C = \frac{Q}{\Delta V}$

Q stays the same

C is increased

- Insulator

$E_{total} < E_0$ (in dielectric)

- Conductor

$E_{total} = 0$

Adding a dielectric reduces the E field

$\Delta V = -Ed$

it also reduces ΔV

$\Delta V = \frac{\Delta V_0}{K}$

↑ dielectric constant

$C = \frac{Q_0}{\Delta V}$

$C = \frac{Q_0}{\left(\frac{\Delta V_0}{K}\right)}$

$C = \frac{KQ_0}{\Delta V_0} = KC_0$

$C_0 = \frac{\epsilon_0 A}{d}$ parallel plate

$$\boxed{C = \frac{K\epsilon_0 A}{d}}$$

Example - Capacitance of a Spherical Conductor
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Chapter 25-26
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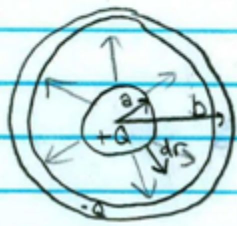
Ex| Spherical conductor



$a < b$

$C = \frac{Q}{\Delta V} = ?$

$\Delta V = -\int_a^b E \cdot dr$

$\Delta V = -\int_a^b \frac{kq}{r^2} dr \cos\theta \quad \theta = 0°$

$\Delta V = -\int_a^b \frac{kq}{r^2} dr$

$\Delta V = -kq \int_a^b r^{-2} dr$

$\Delta V = -kq \left[\frac{r^{-1}}{-1}\right]_a^b$

$\Delta V = \left[\frac{kq}{r}\right]_a^b$

$\Delta V = \frac{kq}{b} - \frac{kq}{a}$

$\Delta V = kq\left(\frac{1}{b} - \frac{1}{a}\right)$

$\Delta V = kq\left(\frac{a-b}{ba}\right)$

$\Delta V = \frac{kq(a-b)}{ba}$

$C = \frac{Q}{\Delta V} = Q\left(\frac{ba}{kq(a-b)}\right) = \frac{ba}{k(a-b)}$

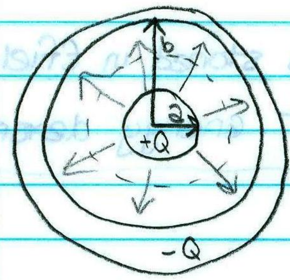
negative #1 C is always positive

$C = \frac{Q}{\Delta V} = -Q\left(\frac{ba}{kq(a-b)}\right)$

$C = \frac{-ba}{k(a-b)} = \frac{ba}{k(b-a)}$

Ex] Cylindrical conductor

coaxial cable $\qquad \ell >> a$ or b



$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$\oint E\, dA \cos\theta = \frac{q_{in}}{\epsilon_0}$ $\qquad \theta = 0°$ for side, $\theta = 90°$ for ends

$C = ? = \frac{Q}{\Delta V}$ $\qquad \oint_{side} E\, dA = \frac{q_{in}}{\epsilon_0}$

$\Delta V = ? = -\int_a^b E \cdot dr$ $\qquad E \oint dA = \frac{q_{in}}{\epsilon_0}$

$\lambda = \frac{Q}{L} = \frac{q_{in}}{\ell}$ $\qquad E A_{side} = \frac{q_{in}}{\epsilon_0}$

$q_{in} = \lambda \ell$ $\qquad E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$

$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{r}$

$E = \frac{2k\lambda}{r}$

$\Delta V = -\int_a^b E \cdot dr = -\int_a^b \frac{2k\lambda}{r}\, dr \cos\theta \qquad \theta = 0°$

$\Delta V = -2k\lambda \int_a^b \frac{1}{r}\, dr$

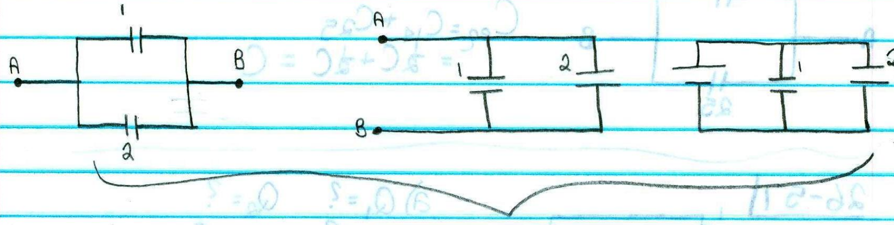
$= -2k\lambda \left[\ln r\right]_a^b$

$= -2k\lambda(\ln b - \ln a)$

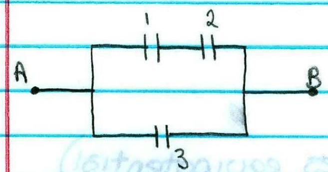
$= \boxed{-2k\lambda \ln\frac{b}{a}}$

$C = \frac{Q}{\Delta V} = \frac{Q}{2k\lambda \ln\frac{b}{a}} = \frac{\lambda \ell}{2k\lambda \ln\frac{b}{a}}$

$$\boxed{C = \frac{\ell}{2k\ln\left(\frac{b}{a}\right)}}$$

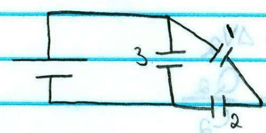


Shape of wire is irrelevant

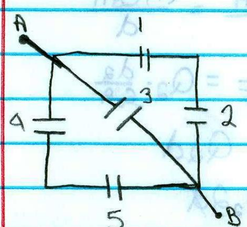


1 + 2 are in series

C_{12} is // C_3



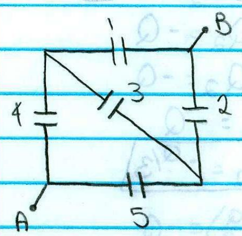
Same as above ↑



1 + 2 in series

4 + 5 in series

C_{12} is // C_3 + C_{45}



Nothing in parallel or series

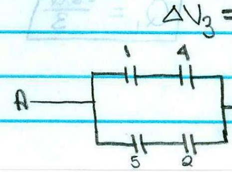
*If all capacitors have capacitance C

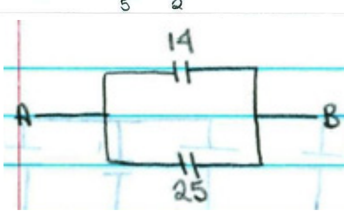
$\Delta V_4 = \Delta V_5$

$\Delta V_1 = \Delta V_2 = \Delta V_4 = \Delta V_5$

$\Delta V_3 = 0$

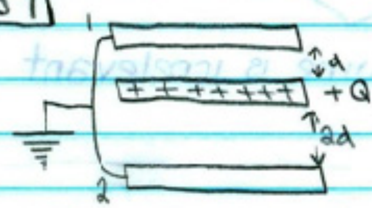
$C_{14} = \left(\frac{1}{C_1} + \frac{1}{C_4}\right)^{-1} = \left(\frac{1}{C} + \frac{1}{C}\right)^{-1} = \frac{1}{2}C = C_{25}$





$C_{eq} = C_{14} + C_{25}$
$= \frac{1}{2}C + \frac{1}{2}C = C$

26-57



a) $Q_1 = ?$ $Q_2 = ?$
$\Delta V_1 = ?$ $\Delta V_2 = ?$

$$Q_t = Q_1 + Q_2 = -Q$$

$\Delta V_{12} = 0$ (with wire, it becomes equipotential)

$$\Delta V_1 = \Delta V_2$$

$$C = \frac{Q}{\Delta V} \longrightarrow \Delta V = \frac{Q}{C}$$

$$\Delta V_1 = \Delta V_2$$

$$\frac{Q_1}{C_2} = \frac{Q_2}{C_2}$$

$$C = \frac{K\epsilon_0 A}{d} = \frac{(1)\epsilon_0 A}{d}$$

$$Q_1 \frac{d_1}{\epsilon_0 A} = Q_2 \cdot \frac{d_2}{\epsilon_0 A}$$

$$Q_1 d = Q_2 d$$

$$Q_1 d = Q_2 2d$$

$$Q_1 = 2Q_2$$

$$Q_1 + Q_2 = -Q$$

$$2Q_2 + Q_2 = -Q$$

$$3Q_2 = -Q$$

$$\boxed{Q_2 = -Q/3}$$

$$Q_1 + \left(-\frac{Q}{3}\right) = -Q$$

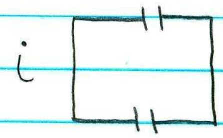
$$\boxed{Q_1 = \frac{-2Q}{3}}$$

$$\Delta V_1 = \Delta V_2$$

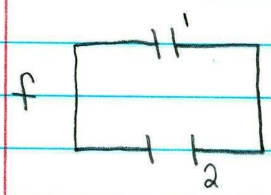
$$\Delta V_1 = \frac{Q_1}{C} = \frac{-\frac{2Q}{3}}{\epsilon_0 A/d}$$

$$\boxed{\Delta V_1 = -\frac{2dQ}{3\epsilon_0 A}}$$

26-40) $C_{1i} = C_{2i} = C_i$ ΔV known



a) $U_{ti} = ?$
b) $\Delta V_{1f} = ?$ $\Delta V_{2f} = ?$
c) $U_{tf} = ?$
d) (a) = (c) ?



$d_f = 2d_i$
$\Delta V_{1i} = \Delta V_{2i}$

a) $U_{ti} = U_{1i} + U_{2f}$
$= \frac{1}{2} C_{1i} \Delta V_i^2 + \frac{1}{2} C_{2i} \Delta V_i^2$
$\boxed{U_{ti} = C_i \Delta V_i^2}$

b) $\Delta V_{1f} = \Delta V_{2f} = \Delta V_f = ?$
$Q_{ti} = Q_{tf}$
$Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$
$C_{1i}\Delta V_{1i} + C_{2i}\Delta V_{2i} = C_{1f}\Delta V_{1f} + C_{2f}\Delta V_{2f}$
$C_i \Delta V_i + C_i \Delta V_i = C_i \Delta V_f + \left(\frac{C_i}{2}\right)\Delta V_f$
$2\Delta V_i = \frac{3}{2}\Delta V_f$
$\boxed{\Delta V_f = \frac{4}{3}\Delta V_i}$

$C_{2f} = \frac{k\epsilon_0 A}{d_f} = \frac{k\epsilon_0 A}{2d_i}$
$C_{2f} = \frac{1}{2} k\epsilon_0 A / d_i$
$C_{2f} = \frac{1}{2} C_i$
$C = \frac{Q}{\Delta V} \rightarrow Q = C\Delta V$

c) $U_{tf} = U_{2f} + U_{1f}$
$= \frac{1}{2} C_{1f} \Delta V_{1f}^2 + \frac{1}{2} C_{2f} \Delta V_{2f}^2$
$= \frac{1}{2} C_i \left(\frac{4\Delta V_i}{3}\right)^2 + \frac{1}{2}\left(\frac{C_i}{2}\right)\left(\frac{4\Delta V_i}{3}\right)^2$
$= C_i \Delta V_i^2 \left(\frac{16}{18} + \frac{16}{36}\right)$
$= C_i \Delta V_i^2 \left(\frac{32}{36} + \frac{16}{36}\right)$
$= C_i \Delta V_i^2 \left(\frac{48}{36}\right)$
$\boxed{U_{tf} = \frac{4}{3} C_i \Delta V_i^2}$

d) Energy was put in to move plates apart

$E = \frac{-dV}{dr}$

$\Delta V = -Ed$ → memorized + prove

Equipotential surface ($\Delta V = 0$)

No work to move a charge @ CV

⊥ to Efield lines

$C = \frac{Q}{\Delta V}$ always positive

→ $\Delta V = ?$, $E = ?$, Gauss' Law

$C_{\parallel} = C_1 + C_2 + C_3 + \ldots$

$\Delta V_1 = \Delta V_2$

$Q_t = Q_1 + Q_2$

$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\right)^{-1}$

$\Delta V_t = \Delta V_1 + \Delta V_2 + \Delta V_3$

$Q_t = Q_1 = Q_2$

$U_c = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{Q^2}{2C}$

$C = kC_0$

$E \downarrow \rightarrow \Delta V \downarrow \rightarrow C \uparrow$