$I \equiv \frac{dq}{dt}$ $\left(\frac{C}{s}\right)$ Ampere, A $\Delta V = 0 \rightarrow$ no current flow

$I \equiv \frac{\Delta Q}{\Delta t}$ Base SI

Conventional current
+ is w/ the dir that + charges would flow
e^- flow, + opp of +

P.833) $I = \frac{\Delta Q}{\Delta t}$ ΔQ = # of charge carriers x charge/carrier

$I = \frac{NAV_d \Delta t q}{\Delta t}$ N = # of charge carriers/volume
charge carrier density

$\boxed{I = NAV_d q}$

$\Delta Q = N \forall q = NA\Delta x q$

$\Delta Q = NAV_d \Delta t q$

$\forall = A \Delta x$

$V = \frac{\Delta x}{\Delta t}$

$\Delta x = V_d \Delta t$

V_d = drift velocity ≈ 0.1 mm/s

$J = \frac{I}{A} = \frac{N\cancel{A}V_d q}{\cancel{A}} \rightarrow J = N V_d q$

Current Density

$J = \sigma E$

Ohmic, or follow Ohm's law

σ = conductivity

$\rho = \frac{1}{\sigma}$ resistivity

$\|\Delta V\| = E d$

$\Delta V = E \ell$

$E = \frac{\Delta V}{\ell}$

$J = \sigma \frac{\Delta V}{\ell}$

$\Delta V = \frac{J \ell}{\sigma}$

$\Delta V = \frac{I \ell}{A \sigma}$

$\Delta V = \underbrace{\left(\frac{\ell}{A\sigma}\right)}_{\text{resistance}} I$

$\Delta V = \underbrace{\frac{\rho \ell}{A}}_{\text{resistance}} I$

$R = \frac{\rho \ell}{A}$

$\Delta V = I R$

$R = \frac{\Delta V}{I} \left(\frac{V}{A}\right)$

$\frac{V}{A} = \Omega$ ohms

Resistance $\Delta V = IR$ + $R = \frac{\rho \ell}{A}$ → object-specific

Resistivity ρ → material property

$\rho = \frac{RA}{\ell} \rightarrow \frac{\Omega m^2}{m} = \Omega m$

Ex) $\Delta V = 12 V$

$\ell = 25$ m

dia = 0.001 m

Aluminum

R = ? I = ?

$R = \frac{\rho \ell}{A}$

$R = \frac{(2.82\times10^{-8})(25)}{\pi\left(\frac{0.001}{2}\right)^2}$

$\boxed{R = 0.898\ \Omega}$

$\rho = 2.82\times10^{-8}\ \Omega m$

$A = \pi r^2$

$\Delta V = I R$

$I = \frac{\Delta V}{R} = \frac{12}{0.898} = \boxed{13.4 A}$

$P = \frac{dU}{dt} = \frac{d}{dt}(q\Delta V)$

$P = \frac{dq}{dt}\Delta V$

$P = I\Delta V$

$= I(IR) = I^2R$

$= \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2}{R}$

$P = I\Delta V = I^2R = \frac{\Delta V^2}{R}$

Ex | $\Delta V = 120$ V (standard)

100 watt + 60 watt

$R_{100} = ?$ $R_{60} = ?$

$P = \frac{\Delta V^2}{R}$

$R = \frac{\Delta V^2}{P}$

$R_{100} = \frac{120^2}{100} = 144\ \Omega$

$R_{60} = \frac{120^2}{60} = 240\ \Omega$

my monitor

$I = 1.5$ A

10¢/kWhr

8 hrs/day

$\Delta V = 120$ V

$P = I\Delta V = 1.5(120) = 180\ W = 0.180$ kW

$0.180\ kW(8\ hr) = 1.44$ kWhr

$1.44\ \cancel{kWhr}\left(\frac{10¢}{\cancel{kWhr}}\right) = 14.4¢$

$\cancel{k}Whr\left(\frac{1000\ W}{1\ \cancel{kW}}\right) = 1000$ W/hr $= 1000\ \frac{J\,hr}{s}$

$1000\ \frac{J}{\cancel{s}}\cancel{hr}\left(\frac{3600\ \cancel{s}}{1\ \cancel{hr}}\right) = \boxed{3.6\times10^6\ J}$ kWhr is energy

Thank You, Emily Rencsok, for these notes.

27-4 $r = 5.29\times10^{-11}\ m$

$$\Sigma F_{in} = F_e = ma_c$$

$$\frac{kq_1q_2}{r^2} = m\frac{v^2}{r}$$

$$\frac{kq^2}{r} = mv^2$$

$$V = \sqrt{\frac{kq^2}{rm}} = \sqrt{\frac{(8.99\times10^9)(1.60\times10^{-19})^2}{(5.29\times10^{-11})(9.11\times10^{-31})}} = \boxed{2.19\times10^6\ m/s}$$

$v = r\omega$

$\omega = \frac{v}{r}$

$$I = \frac{\Delta Q}{\Delta t} = \frac{q}{2\pi/\omega} = \frac{q}{2\pi/v/r} = \frac{qv}{2\pi r} = \frac{(1.6\times10^{-19})(2.19\times10^6)}{2\pi(5.29\times10^{-11})}$$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

$I = 0.001054\ A$

$\boxed{I = 1.05\ mA}$

Electromotive force, emf, $\mathcal{E}$

Ideal, max voltage

$\Delta V_t \rightarrow$ terminal voltage

(voltage across the terminal voltage)

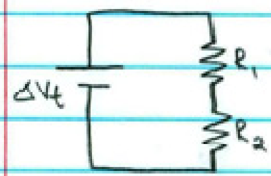
All real batteries have an internal resistance, r

p.860 | $\Delta V_t = \mathcal{E} - \Delta V_r$

$\Delta V_t = \mathcal{E} - Ir$ As $I \uparrow$, $\Delta V_t \downarrow$

$\Delta V_t = \mathcal{E}$ if $I = 0$

2 resistors in series



$I_1 = I_2 = I_t$ $\quad\quad \Delta V = IR$

$\Delta V_t = \Delta V_1 + \Delta V_2$

$\cancel{I_t} R_{eq} = \cancel{I_1} R_1 + \cancel{I_2} R_2$

$\boxed{R_S = R_1 + R_2 + \ldots}$

2 resistors in parallel

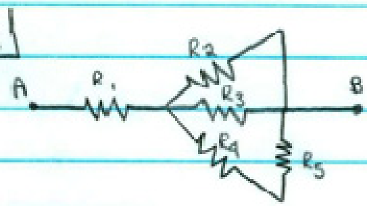
$I_t = I_1 + I_2$ $\quad\quad I = \frac{\Delta V}{R}$

$\Delta V_t = \Delta V_1 = \Delta V_2$

$\frac{\Delta V_t}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$

$\boxed{R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}}$

Ex)



$R_{45} = R_4 + R_5 = 4 + 5 = 9\,\Omega$
S

$R_{2345} = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{45}}\right)^{-1} = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{9}\right)^{-1} = 1.0588$
∥

$\Delta V_{AB} = 25V$

$R_{eq} = ?$

$I_3 = ?$

$R_{eq} = R_1 + R_{2345} = 1 + 1.0588 = 2.0588\,\Omega$

$\Delta V = IR$

$25 = I_{eq}(2.0588)$

$I_{eq} = 12.143\,A = I_1 = I_{2345}$

$\Delta V = IR$ (2345)

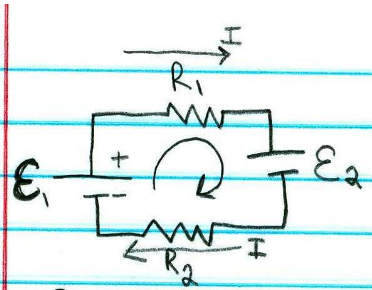
$\Delta V = 12.143(1.0588)$

$\Delta V = 12.857 = \Delta V_2 = \Delta V_3 = \Delta V_{45}$

$\Delta V = IR$

$12.857 = I(3)$

$\boxed{I_3 = 4.29A}$

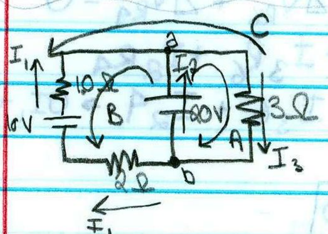


Kirchoffs Rules

1) $\Delta V_{loop} = 0$
2) $\Sigma I_{in} = \Sigma I_{out}$ of a junction

$\mathcal{E}_1 = 12V$
$\mathcal{E}_2 = 8$ V
$R_1 = 4\ \Omega$
$R_2 = 6\ \Omega$
Charging 2nd battery

$\Delta V_{loop} = 0 = +\mathcal{E}_1 - \Delta V_{R1} - \mathcal{E}_2 - \Delta V_{R2}$
$0 = \mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2$
$0 = 12 - I4 - 8 - I(6)$
$0 = 4 - 10I$
$I = 4/10 = \boxed{0.40\ A}$



Junction a: $\Sigma I_{in} = \Sigma I_{out}$
$I_1 + I_2 = I_3$

Junction b: $\Sigma I_{in} = \Sigma I_{out}$
$I_3 = I_1 + I_2$

$\Delta V_A = \mathcal{E}_{20} - \Delta V_{R3} = 0$
$0 = \mathcal{E}_{20} - I_3 R_3$
$0 = 20 - I_3(3)$
$I_3 = \frac{20}{3}$ A

$\Delta V_B = 0 = \mathcal{E}_{20} + \Delta V_{R_1} - \mathcal{E}_{10} + \Delta V_{R2}$
$0 = \mathcal{E}_{20} + I_1 R_1 - \mathcal{E}_{10} + I_1 R_2$
$0 = 20 + I_1 1 - 10 + I_1 2$
$0 = 10 + 3I_1$
$I_1 = -10/3$ A

$I_1 + I_2 = I_3$
$-\frac{10}{3} + I_2 = \frac{20}{3}$
$I_2 = \frac{30}{3} = 10$ A

$I_1 = \frac{10}{3}$ A down through R_1
$I_2 = 10$ A up through $\mathcal{E}_{20}$
$I_3 = \frac{20}{3}$A down through R_3

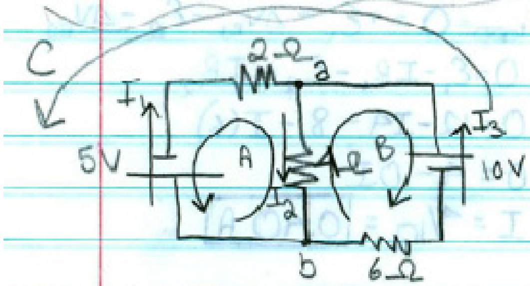
$$\Delta V_C = 0 = -\mathcal{E}_{10} + \Delta V_{R2} + \Delta V_{R3} + \Delta V_{R1}$$

$$0 = -\mathcal{E}_{10} + I_1 R_2 + I_3 R_3 + I_1 R_1$$

$$0 = -10 + -\frac{10}{3}(2) + \frac{20}{\cancel{3}}(\cancel{3}) + -\frac{10}{3}(1)$$

$$0 = -10 - \frac{20}{3} + 20 - \frac{10}{3}$$

$$0 = 0$$



$$\Delta V_A = +\mathcal{E}_5 + \Delta V_{R_2} + \Delta V_{R_4} = 0$$
$$0 = +\mathcal{E}_5 + I_1 R_2 + I_2 R_4$$
$$0 = I_2 4 + I_1 2 + 5 \rightarrow 2I_1 + 4I_2 + 0I_3 = -5$$

$$\Delta V_B = -\mathcal{E}_{10} + \Delta V_{R_6} + \Delta V_{R_4} = 0$$
$$0 = -\mathcal{E}_{10} + I_3 R_6 + I_2 R_4$$
$$0 = -10 + I_3 6 + I_2 4 \rightarrow 0I_1 + 4I_2 + 6I_3 = 10$$

Junction a $\Sigma I_{in} = \Sigma I_{out}$

$$I_1 + I_3 = I_2$$
$$I_1 - I_2 + I_3 = 0$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & 4 & 0 & -5 \\ 0 & 4 & 6 & 10 \end{bmatrix}$$

$$\Downarrow \text{rref}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2.045 \\ 0 & 1 & 0 & -0.2\overline{27} \\ 0 & 0 & 1 & 1.\overline{81} \end{bmatrix}$$

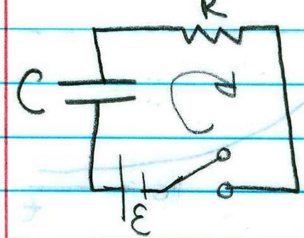
$$\Delta V_C = \mathcal{E}_{10} + \Delta V_{R_2} + \mathcal{E}_5 - \Delta V_{R_6} = 0$$
$$0 = \mathcal{E}_{10} + I_1 R_2 + \mathcal{E}_5 - I_3 R_6$$
$$0 = 10 + (-2.045)(2) + 5 - (1.\overline{81})(6)$$
$$0 = 0$$

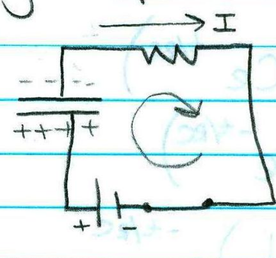
$I_1 = 2.05$ A down through $\mathcal{E}_5$

$I_2 = 0.227$ A up through R_4

$I_3 = 1.81$ A up through $\mathcal{E}_{10}$

RC Circuit | Charging a capacitor through a resistor





$C = \frac{Q}{\Delta V}$ $\quad q_i = 0$

$\Delta V = \frac{q}{C}$ $\quad t = 0$

$\Delta V_{loop} = 0 = +\mathcal{E} - \Delta V_C - \Delta V_R$

$0 = \mathcal{E} - \frac{q}{C} - IR$

$IR = \mathcal{E} - \frac{q}{C}$

$I = \frac{\mathcal{E}}{R} - \frac{q}{RC}$

$\frac{dq}{dt} = \frac{\mathcal{E}C}{RC} - \frac{q}{RC}$

$\frac{dq}{dt} = \frac{1}{RC}(\mathcal{E}C - q)$

$\frac{dq}{dt} = -\frac{1}{RC}(q - \mathcal{E}C)$

$\int_0^q \frac{1}{q - \mathcal{E}C}\, dq = -\int_0^t \frac{1}{RC}\, dt$

$\ln(q - \mathcal{E}C)\Big]_0^q = -\frac{1}{RC}\int_0^t dt$

$\ln(q - \mathcal{E}C) - \ln(0 - \mathcal{E}C) = -\frac{t}{RC}$

$\ln\left(\frac{q - \mathcal{E}C}{-\mathcal{E}C}\right) = -\frac{t}{RC}$

$e \quad e$

$\frac{q - \mathcal{E}C}{-\mathcal{E}C} = e^{-\frac{t}{RC}}$

$q - \mathcal{E}C = e^{-\frac{t}{RC}}(-\mathcal{E}C)$

$q = -\mathcal{E}Ce^{-\frac{t}{RC}} + \mathcal{E}C$

$$\boxed{q(t) = \mathcal{E}C\left(1 - e^{-t/RC}\right)}$$

$0 = \mathcal{E} - IR$

$IR = \mathcal{E}$

$I_i = \frac{\mathcal{E}}{R} = I_{max}$

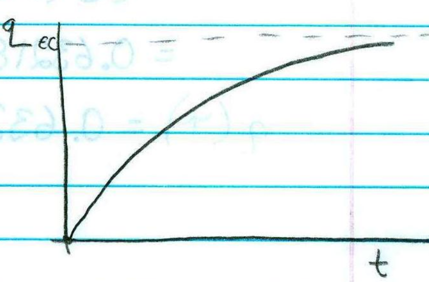
$@\ t \approx \infty \quad I_f = 0$

$q_f = Q$

$0 = \mathcal{E} - \frac{Q}{C} = 0$

$\frac{Q}{C} = \mathcal{E}$

$Q = \mathcal{E}C = Q_{max}$



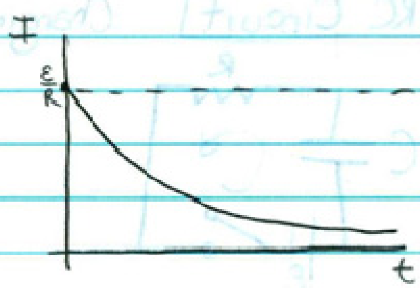
$I(t) = \; ? = \frac{dq}{dt}$

$I = \frac{d}{dt}\left(\mathcal{E}C - \mathcal{E}Ce^{-t/RC}\right)$

$I = \frac{d}{dt}\left(-\mathcal{E}Ce^{-t/RC}\right)$

$I = -\mathcal{E}\cancel{C}\left(-\frac{1}{R\cancel{C}}\right)e^{-t/RC}$

$\boxed{I(t) = \frac{\mathcal{E}}{R}e^{-t/RC}}$



Time constant, τ

$$\tau = RC \rightarrow \_\Omega F$$
$$= \frac{\cancel{V}}{A}\left(\frac{C}{\cancel{V}}\right)$$
$$= \frac{C}{A}$$
$$= \frac{C}{C/s} = \text{seconds}$$

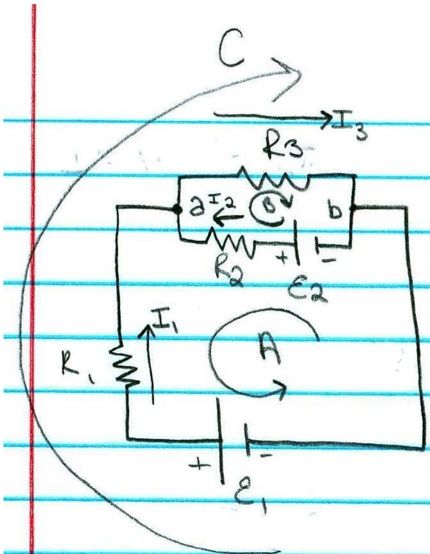
$$q(t) = \mathcal{E}C(1-e^{-t/RC})$$
$$q(\tau) = \mathcal{E}C(1-e^{-RC/RC})$$
$$= \mathcal{E}C(1-e^{-1})$$

KNOW THIS #!!

$$= \mathcal{E}C(1-0.3678)$$
$$= 0.6321\mathcal{E}C$$
$$q(\tau) = 0.632\,Q_{max}$$

Time for a 63.2% change

Another Kirchhoff's Rules Example Problem
11
AP Physics C – Video Lecture Notes
Chapter 27-28
Thank You, Emily Rencsok, for these notes.



$R_1 = 4.50\ \Omega$

$R_2 = 3.00\ \Omega$

$R_3 = 6.00\ \Omega$

$\varepsilon_1 = 9.00\ V$

$\varepsilon_2 = 12.0\ V$

$I_1 = ?\quad I_2 = ?\quad I_3 = ?$

Junction A: $\Sigma I_{in} = \Sigma I_{out}$

$I_1 + I_2 = I_3$

$I_1 + I_2 - I_3 = 0$

$4.5 I_1 + 3 I_2 - 0 I_3 = -3$

$0 I_1 - 3 I_2 - 6 I_3 = -12$

$\Delta V_A = 0 = +\Delta V_{R1} - \varepsilon_1 + \varepsilon_2 - \Delta V_{R2}$

$0 = I_1 R_1 - \varepsilon_1 + \varepsilon_2 - I_2 R_2$

$0 = I_1(4.5) - 9 + 12 - I_2(3)$

$\Delta V_B = 0 = -\Delta V_{R3} + \varepsilon_2 - \Delta V_{R2}$

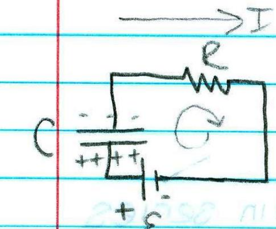
$0 = \varepsilon_2 - I_2 R_2 - I_3 R_3$

$0 = 12 - I_2 3 - I_3 6$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4.5 & -3 & 0 & -3 \\ 0 & -3 & -6 & -12 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0.153846 \\ 0 & 1 & 0 & 1.23076 \\ 0 & 0 & 1 & 1.38461 \end{bmatrix} \Rightarrow \boxed{\begin{array}{ll} I_1 = 1.54\,mA & \text{up through } R_1 \\ I_2 = 1.23\,A & \text{left through } R_2 \\ I_3 = 1.38\,A & \text{right through } R_3 \end{array}}$$

Row Reduced Echelon Form

Discharging a capacitor through a resistor



$\Delta V_{loop} = 0 = \mathcal{E} - \Delta V_C - \Delta V_R$

$0 = \mathcal{E} - \frac{q}{C} - IR$

$q(t) = ?$

$I(t) = ?$

Removed battery

$C = \frac{Q}{\Delta V}$

$\Delta V = \frac{Q}{C}$

$0 = -\frac{q}{C} - IR$

$IR = -\frac{q}{C}$

$I = -\frac{q}{RC}$

$\frac{dq}{dt} = \frac{-q}{RC}$

$\int_Q^q \frac{dq}{q} = \int_0^t -\frac{1}{RC}\, dt$

$\left[\ln q\right]_Q^q = -\frac{1}{RC}\int_0^t dt$

$\ln q - \ln Q = -\frac{t}{RC}$

$e^{\ln \frac{q}{Q}} = e^{-\frac{t}{RC}}$

$\frac{q}{Q} = e^{-t/RC}$

$\boxed{q(t) = Qe^{-t/RC}}$

@ $t=0$ close switch

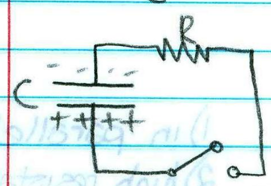
$q_i = Q_{max}$

$I_i = I_{max}$

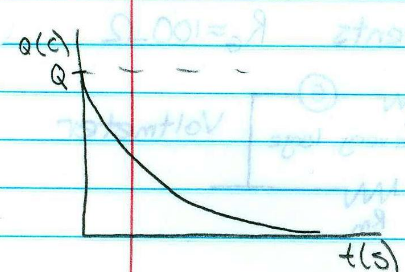
@ $t \approx \infty$

$q_f = 0$

$I_f = 0$



$t=0$
close switch



$I = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right)$

$I(t) = -\frac{Q}{RC} e^{-t/RC}$

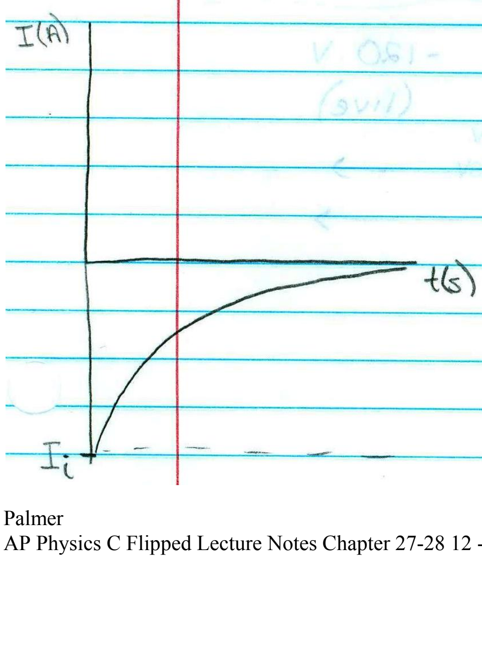
$Q_i = \mathcal{E}C$

$I(t) = -\frac{\mathcal{E}\cancel{C}}{R\cancel{C}} e^{-t/RC}$

$I(t) = -\frac{\mathcal{E}}{R} e^{-t/RC}$

$\boxed{I(t) = -I_i e^{-t/RC}}$

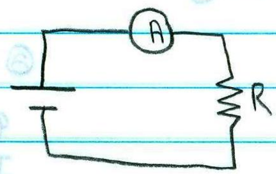
discharging

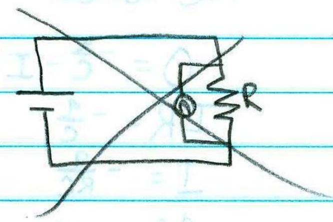


Thank You, Emily Rencsok, for these notes.

* Ammeter + voltmeter

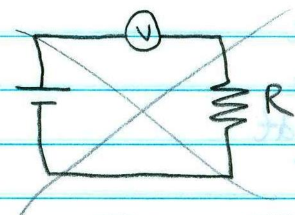
Ammeter





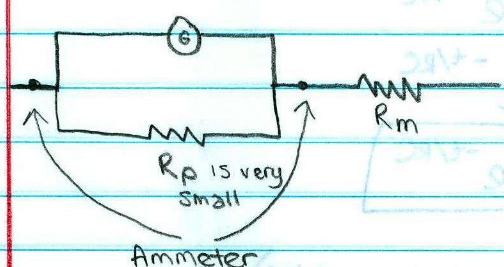
1) in series
2) small resistance

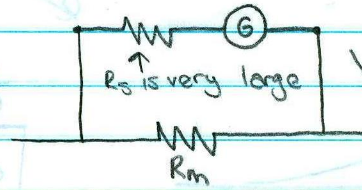
Voltmeter



1) in parallel
2) high resistance

Galvonometer – measures small currents $R_G = 100\,\Omega$





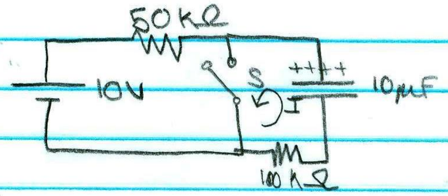
Voltmeter

+120 V	0 V	−120 V
(live)	(neutral) ground	(live)

$\longleftarrow \Delta V\ 120\text{V} \longrightarrow$ $\longleftarrow \Delta V\ 120\text{V} \longrightarrow$

$\longleftarrow 240\text{V} \longrightarrow$

28-36



$t = \infty$ switch open

a) $\tau = ?$ before switch is closed + after

b) $I(t) = ?$ in switch after $t=0$ when switch is closed

$\tau_{before} = RC$
$= (150\times10^3)(10\times10^{-6})$
$\boxed{\tau_b = 1.5 \text{ sec}}$

$R_{before} = R_{50} + R_{100}$
$= 50 + 100 = 15 \text{ k}\Omega$

$\tau_{after} = RC$
$= (100\times10^3)(10\times10^{-6})$
$\boxed{\tau_a = 1.0 \text{ sec}}$

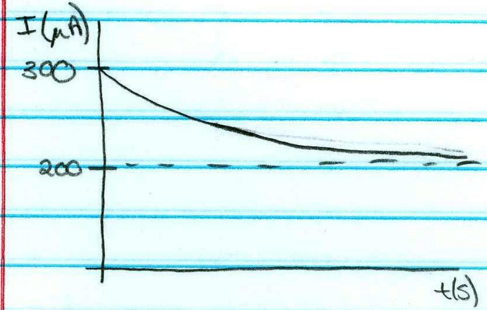
$\Delta V = IR$
$I_\varepsilon = \frac{\Delta V_t}{R} = \frac{10}{50\times10^3}$
$I_\varepsilon = 2\times10^{-4} \text{ A}$
$= 200\ \mu\text{A}$

$I_{switch} = I_\varepsilon + I_c$

$I_c = I_0 e^{-t/RC}$ from τ_A
$= (100 e^{-t/(1)})\ \mu\text{A}$

$\Delta V_t = \Delta V_{ci} = I_i R_{100}$
$I_i = \Delta V_{ci} / R_{100}$
$I_i = \frac{10}{100\times10^3} = 100\times10^{-6} = 100\ \mu\text{A}$

$I_{switch} = (200 + 100e^{-t})\ \mu\text{A}$
$= (0.200 + 0.100 e^{-t})\ \text{mA}$



$J = \frac{I}{A}$

ΔV_t + emf, $\mathcal{E}$
(measured) (ideal)

$\Delta V_t = \mathcal{E} - Ir$

Kirchhoff's Rules
1) $\Delta V_{loop} = 0$
2) $\Sigma I_{in} = \Sigma I_{out}$

Equations
↓
matrix
↓
rref
↓
matrix answer
↓
Ans w/ I dir
↓
Check w/ extra loop

RC circuit

$q(t)$ + $I(t)$ charging + discharging

1) Derive
2) Limits
3) Graphs
4) Equations

Ammeter + voltmeter
- where they go, ↑/↓ resistance