Flipping Physics Lecture Notes:
3 States of Matter – Solid, Liquid, Gas
http://www.flippingphysics.com/solid-liquid-gas.html

On Earth, the three most common states, or phases, of matter are solid, liquid, and gas.

This table describes the differences between them:

		Fluid	
	Solid	**Liquid**	**Gas**
Fixed Shape?	Yes	No	No
Fixed Volume?	Yes	Yes	No
Distance between particles	Small	Almost as Small as a Solid	Large
Force of attraction between particles	Large	Medium	Small
Kinetic Energy of Particles	Small	Medium	Large
Example: H_2O	Ice	Water	Water Vapor

Flipping Physics Lecture Notes:
Density
http://www.flippingphysics.com/density.html

Density is a material property of any pure substance. For example, the density of pure copper is 8.96 g/cm^3. And, any object made of pure copper, regardless of size, will have that same density of 8.96 g/cm^3.

The symbol for density is ρ. Which is the lowercase, Greek letter "rho".

The equation for density is: $\rho = \frac{\text{mass}}{\text{volume}}$

Let's determine the densities of two, equal diameter spheres. One steel and one wood:

$$\text{diameter} = 50.7mm \Rightarrow r = \frac{\text{diameter}}{2} = \frac{50.7mm}{2} = 25.35mm\left(\frac{1cm}{10mm}\right) = 2.535cm$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2.535cm)^3 = 68.2374cm^3 = V_{\text{steel}} = V_{\text{wood}}$$

$$m_{\text{wood}} = ??g \ \& \ m_{\text{steel}} = 535g$$

$$\rho_{\text{steel}} = \frac{m_{\text{steel}}}{V_{\text{steel}}} = \frac{535g}{68.2374cm^3} = 7.84027 \Rightarrow \rho_{\text{steel}} \approx 7.84\frac{g}{cm^3} \text{ (observed value)}$$

$$\rho_{\text{wood}} = \frac{m_{\text{wood}}}{V_{\text{wood}}} = \frac{45g}{68.2374cm^3} = 0.65946 \Rightarrow \rho_{\text{wood}} \approx 0.66\frac{g}{cm^3} \text{ (observed value)}$$

The accepted value for the density of steel is roughly 7.7 – 8.0 g/cm^3.♥
The accepted value for the density of birch wood is roughly 0.5 – 0.8 g/cm^3.♣
So, both of our observed values are within the range of their accepted values.

We have just shown that steel is more dense than wood, and steel has a larger mass per unit volume than wood.

♥ https://hypertextbook.com/facts/2004/KarenSutherland.shtml
♣ https://www.engineeringtoolbox.com/wood-density-d_40.html

Flipping Physics Lecture Notes:
Pressure – Billy's Physics Dream
http://www.flippingphysics.com/pressure.html

If you pound a hammer against a nail with the nail oriented with the flat head of the nail against a wood board, the nail will not pierce the wood board. The reason why has to do with pressure!

Pressure equals Force over Area: Pressure, $P = \frac{F}{A}$

Example: You can determine the pressure caused by your feet pressing against the ground. For me the numbers are:

The area of one foot is roughly a 21 cm by 7.0 cm rectangle:

$$A_{\text{foot}} = 21cm \times 7.0cm \approx 147cm^2$$

However, let's have all our numbers in base S.I. units. So, let's convert to square meters.

$$A_{\text{foot}} = 147cm^2 \times \frac{1^2m^2}{100^2cm^2} = 0.0147m^2 \Rightarrow A_{\text{2 feet}} = 2 \times 0.0147 = 0.0294m^2$$

And I actually have two feet, not just one.

The weight of my body in newtons: $\text{weight} = 170\text{pounds} \times \frac{4.448N}{1\text{pound}} = 756.16N = F_g$

And we can determine the pressure on my two feet caused by the weight of my body while I am standing at rest.

$$P_{\text{2 feet}} = \frac{F_g}{A_{\text{2 feet}}} = \frac{756.16}{0.0294} = 25720\frac{N}{m^2} = 25720Pa \times \frac{1kPa}{1000Pa} \approx 26kPa$$

Note: Pressure is measured in newtons per square meter and those are called pascals in honor of Blaise Pascal, a 17th century French physicist, mathematician, and inventor.

- Pascal, $1\,Pa = 1\frac{N}{m^2}$

Also, pressure is often measured in kilopascals, kPa, just like kilometers and kilograms.

$$P_{\text{1 foot}} = \frac{F_g}{A_{\text{2 feet}}/2} = (2)\frac{F_g}{A_{\text{2 feet}}} = (2)P_{\text{2 feet}} = (2)(25720) = 51440\frac{N}{m^2} \approx 51kPa$$

And going back to the nail example, if you turn the nail around so the pointy side is on the wood board, the area in contact with the wood board is significantly decreased, which means, using the same force from the hammer, the pressure from the nail on the wood board is significantly increased, and the nail will pierce the wood board!

A few additional tidbits:

- The equation definition for pressure actually has the force perpendicular to the surface in it, not just the force. Any force component of the force which is parallel to the surface does not cause any pressure on the surface. $P \equiv \frac{F_\perp}{A}$
- Pressure is a scalar. Pressure does not have direction, it has only magnitude.

Flipping Physics Lecture Notes:
Fluid Pressure - Billy's Still Dreaming about Physics
http://www.flippingphysics.com/fluid-pressure.html

Example: We can determine the pressure exerted by water in a rectangular fish tank on the bottom of the tank. For this tadpole tank, our numbers are:

The area of the bottom of the tank in square meters:

$$A_{\text{bottom}} = 50.2cm \times 25.4cm = 1275cm^2 \times \frac{1^2 m^2}{100^2 cm^2} = 0.1275cm^2$$

The height (or depth) of the water in meters:

$$h_{\text{water}} = 18.5cm \times \frac{1m}{100cm} = 0.185m$$

And we can solve for the pressure on the bottom of the tadpole tank caused by the weight of the water pushing down on the bottom of the tank:

$$P_{\text{bottom}} = \frac{F_g}{A_{\text{bottom}}} \ \& \ F_g = mg$$

We need to determine the weight, or force of gravity, of the water.
For that we need to use the density equation:

$$\rho = \frac{m}{V} \Rightarrow m = \rho V = \rho A h \Rightarrow F_g = mg = \rho A_{\text{bottom}} h g$$

And we can substitute that back into our pressure equation:

$$P_{\text{bottom}} = \frac{F_g}{A_{\text{bottom}}} = \frac{\rho A_{\text{bottom}} h g}{A_{\text{bottom}}} \Rightarrow P_{\text{bottom}} = \rho g h$$

Which means we need the density of water at room temperature:

$$\rho_{\text{water}} = 998 \frac{kg}{m^3}$$

We can solve for the pressure caused by the water on the bottom of the tadpole tank:

$$\Rightarrow P_{\text{bottom}} = \rho g h = (998)(9.81)(0.185) = 1793Pa \times \frac{1kPa}{1000Pa} \approx 1.79kPa$$

Notice the area of the tank cancelled out of the equation. In other words, the pressure caused by a fluid, $P_{\text{fluid}} = \rho g h$, depends on:

- ρ, the density of the fluid.
- g, the gravitational field of the planet.
- h, the depth of the fluid.
    - *It does **not** depend on the area of the fluid*

In other words, the deeper you dive into water, the larger the pressure from the water. The weight of all the water above you pushes down on you causing this pressure.

The same is true for all the air in the atmosphere above you. This is why there is pressure all around you when you are standing on the surface of the Earth. The miles and miles of air above you is pushing down and causing the pressure you currently experience.♥ This is called atmospheric pressure and is a typical unit.

- $1.00atm = 1.01 \times 10^5 Pa$.
  - In other words, 1 atmospheres of pressure is 1.01 x 10^5 Pa.
- This is the pressure referred to in Standard Temperature and Pressure or STP.
  - Standard Pressure is 1.00 atm.
  - Standard Temperature is 0°C or 32°F or 273.15 Kelvins.

This atmospheric pressure is pushing down on the tank of water. That means the total pressure at the bottom of the tank is the addition of the atmospheric pressure and the pressure caused by the water.

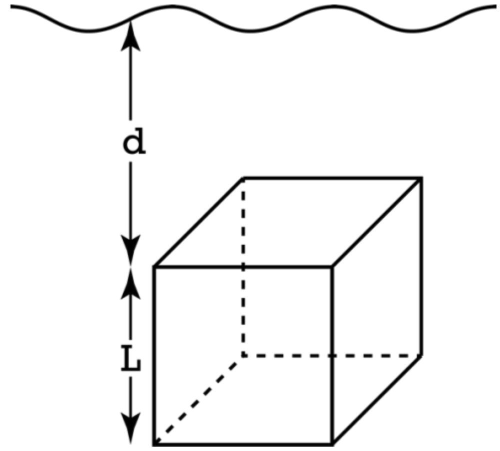
- $P_{\text{total}} = P_{\text{atm}} + \rho g h$
- The pressure caused by a vertical column of fluid is called *gauge* pressure.
  - $P_{\text{gauge}} = \rho g h$
- The total pressure is called the *absolute* pressure.
  - $P_{\text{absolute}} = P_{\text{atm}} + P_{\text{gauge}}$
- The absolute pressure at the bottom of the tank is:
  - $P_{\text{absolute}} = P_{\text{atm}} + P_{\text{gauge}} = 1.01 \times 10^5 + 1793 = 102793Pa \times \frac{1kPa}{1000Pa} \approx 103kPa$

A few extra tidbits:

- What we've been talking about here is called Fluid Pressure. The pressure from the water and the air on objects is fluid pressure.
- The particles in fluids are constantly moving around, colliding with one another, and colliding with the surface bordering the fluid. The pressure exerted by a fluid is caused by the particles in the fluid colliding with the surfaces next to the fluid.
- Often liquids are considered to be incompressible. The volumes and densities of incompressible fluids do not change regardless of the pressure applied to them.

♥ Yes, I am assuming you are reading this while on the surface of planet Earth and not underwater.

Flipping Physics Lecture Notes:
Buoyant Force Equation: Step-by-Step Derivation
http://www.flippingphysics.com/buoyant-force-derivation.html



Let's start by imagining a cube as shown with sides of width, L, at a depth, d, in a fluid of known density, ρ_f. Now let's determine the net force acting on this cube.

Knowns: Top of Cube Depth $= d$, Sides $= L$, ρ_f; $\sum F = ?$

In our lesson about fluid pressure, we determined that with increased depth in a fluid comes increased pressure caused by the weight of the fluid above. The net pressure from the fluid acting on the cube comes from the pressure acting inward on all six sides of the cube. Therefore, the force caused by this pressure on each side of the cube is:

- On the left side of the cube, the force acts to the right, which is positive.
- On the right side of the cube, the force acts to the left, which is negative.
- On the back of the cube, the force acts forward, which is positive.
- On the front of the cube, the force acts backward, which is negative.
- On the bottom of the cube, the force acts upward, which is positive.
- On the top of the cube, the force acts downward, which is negative.

$$\sum F = F_{\text{left}} - F_{\text{right}} + F_{\text{back}} - F_{\text{front}} + F_{\text{bottom}} - F_{\text{top}}$$

Notice the force acting on all four vertical sides of the cube are equal in magnitude because they are at equal depths in the fluid, therefore, the forces from the left and right sides cancel one another out and the forces from the back and front sides cancel one another out. That means the net force from the fluid acting on the cube comes from just the bottom and top sides of the cube.

$$F_{\text{left}} - F_{\text{right}} = 0 = F_{\text{back}} - F_{\text{front}} \Rightarrow \sum F = F_{\text{bottom}} - F_{\text{top}}$$

(Technically, because pressure increases with depth, the force also increases with depth, therefore the force is not constant on the vertical sides; it is larger near the bottom of each vertical side than near the top. However, due to symmetry, the forces at the same depth on each mirrored side have the same magnitude, and therefore cancel out.)

From Billy's first dream, we know the equation for pressure: $P = \frac{F_\perp}{A} \Rightarrow F_\perp = PA$

We can substitute that into the net force equation:

$$\Rightarrow \sum F = P_{\text{bottom}} A_{\text{bottom}} - P_{\text{top}} A_{\text{top}}$$

In the same lesson about fluid pressure, we derived the equation for the absolute pressure at a depth in a fluid. And we know the area of the bottom and top of the cube are the same. Let's identify that as the area of a side.

$$A_{\text{bottom}} = A_{\text{top}} = A_{\text{side}} \;\&\; P_{\text{absolute}} = P_0 + \rho g h$$

Where P_0 is the pressure pushing down on the top surface of the fluid and ρgh is the gauge pressure caused by the weight of the fluid above. We can substitute this equation in for the pressure at the bottom and top of the cube.

$$\Rightarrow \sum F = (P_0 + \rho_f g h_{\text{bottom}}) A_{\text{side}} - \left(P_0 + \rho_f g h_{\text{top}}\right) A_{\text{side}}$$

$$\Rightarrow \sum F = P_0 A_{\text{side}} + \rho_f g h_{\text{bottom}} A_{\text{side}} - P_0 A_{\text{side}} - \rho_f g h_{\text{top}} A_{\text{side}}$$

$$\Rightarrow \sum F = \rho_f g A_{\text{side}} \left(h_{\text{bottom}} - h_{\text{top}}\right) = \rho_f g A_{\text{side}} \left((d + L) - d\right)$$

$$\Rightarrow \sum F = \rho_f g A_{\text{side}} L = \rho_f g V_{\text{cube}} = \rho_f g V_f$$

Realize the volume of the cube equals the length of one side, L, times the area of one side. And remember the density in this pressure equation is the density of the fluid the cube is submerged in. We typically refer to that as the "fluid displaced" by the cube. And, from our lesson on density, we can recall that density equals mass divided by volume. So, we can substitute the mass of the fluid displaced by the cube in for the density of the fluid displaced by the cube times the volume of the fluid displaced by the cube.

$$\rho = \frac{m}{V} \Rightarrow m_f = \rho_f V_f \Rightarrow \sum F = m_f g = F_{\text{buoyant}}$$

And we get an equation for the net force acting on the cube caused by the fact that the cube is displacing the fluid. That net force acts upward on the cube and is called the buoyant force, F_B.

Buoyant Force:

- Upward force acting on an object in fluid.
- Equal in magnitude to the weight of the fluid displaced by the object.
  - The upward buoyant force acting on a hypothetical cube of water is equal in magnitude to the force of gravity acting on the hypothetical cube of water and causes our hypothetical cube of water to remain at rest.
- Sum of all the forces applied by the fluid surrounding the object.
- The composition of the object submerged in the fluid does not affect the buoyant force, only the mass of the fluid displaced by the object and the gravitational field.

Flipping Physics Lecture Notes:
Buoyant Force Explained: Submerged Objects in Fluids
http://www.flippingphysics.com/buoyant-force.html

In our previous lesson we derived the Buoyant Force, the upward force acting on an object which is in a fluid whose magnitude equals the weight of the fluid displaced by the object.

$$F_{\text{Buoyant}} = m_{\text{fluid displaced}}g \Rightarrow F_B = m_f g$$

The Buoyant Force is often called Archimedes' Principle, because it was discovered by the Greek physician, engineer, and mathematician Archimedes.

Often this equation is used in terms of density and volume instead of mass. We know the equation for density is density equals mass divided by volume, which we can rearrange to get mass equals density times volume.

$$\rho = \frac{m}{V} \Rightarrow m = \rho V$$

Which we can substitute back into the equation for Buoyant Force.

$$F_B = m_f g \Rightarrow F_B = \rho_f V_f g$$

Let's look at 3 examples of different objects submerged in water. But first realize, when an object is submerged[1] in a fluid, the volume of the fluid displaced by the object is the same as the volume of the object: When object is submerged: $V_f = V_o$

1) A wood sphere.
    a. The density of wood is less than the density of water.
    b. The density of the object is less than the density of the fluid it is displacing.
    c. The magnitude of the upward Buoyant Force acting on the object is greater than the magnitude of the downward force of gravity acting on the object.
        i. When released, the object will accelerate upward.

$$\sum F_y = F_B - F_g = m_o a_y \ \& \ F_B = m_f g = \rho_f V_f g \ \& \ F_g = m_o g = \rho_o V_o g$$

$$\rho_o < \rho_f \Rightarrow F_B > F_g \Rightarrow a_y > 0$$

2) A rubber sphere.
    a. The density of this rubber is more than the density of water.
    b. The density of the object is more than the density of the fluid it is displacing.
    c. The magnitude of the upward Buoyant Force acting on the object is less than the magnitude of the downward force of gravity acting on the object.
        i. When released, the object will accelerate downward.

[1] Please realize that adding the word "completely" to submerged is not necessary. The word "submerged" means to be completely surrounded by water. So, yeah, I will do my best to not say "completely submerged" and instead just say "submerged". [I'd add a footnote to this footnote if I could because "completely" is also unnecessary on "completely surrounded", but, I'll stop here.]

$$\rho_o > \rho_f \Rightarrow F_B < F_g \Rightarrow a_y < 0$$

3) A water balloon.
    a. The density of water balloon is the same as the density of water.
    b. The density of the object is the same as the density of the fluid it is displacing.
    c. The magnitude of the upward Buoyant Force acting on the object is the same as the magnitude of the downward force of gravity acting on the object.
        i. When released, the object will have zero acceleration and float in the water.[2]

$$\rho_o = \rho_f \Rightarrow F_B = F_g \Rightarrow a_y = 0$$

We can see that it is only the relative densities of the object and fluid displaced by the object which determine the direction of the acceleration of an object submerged in a fluid.

- $\rho_o < \rho_f \Rightarrow a_y > 0$
- $\rho_o > \rho_f \Rightarrow a_y < 0$
- $\rho_o = \rho_f \Rightarrow a_y = 0$

But how does a steel boat float? The air inside the hull of the boat has such a low density that it makes it so the average density of the boat is lower than the density of water, which makes it possible for steel boats to float.

Notice the Buoyant Force is independent of depth in the fluid. Typically, we deal with ideal fluids which are incompressible. That means the density of the fluid will not change with depth. This is similar to assuming the acceleration due to gravity is constant near the surface of the planet. It’s not quite true, however, under normal circumstances the error is negligible.

[2] I regret to inform you that I did have to add a little salt to the water inside the water balloon. This is because, no matter how hard I tried, there were little pockets of air trapped inside the water balloon which decreased the average density of the water balloon. Saltwater has slightly higher density than water. By adding saltwater to the balloon, I was able to achieve a water balloon with the same density as water.

Flipping Physics Lecture Notes:
Buoyant Force Calculation: A Submerged Wood Cylinder
http://www.flippingphysics.com/buoyant-force-wood.html

In our previous lesson we discussed the Buoyant Force which acts on submerged objects. Now we are going to walk through a specific example and then demonstrate it.

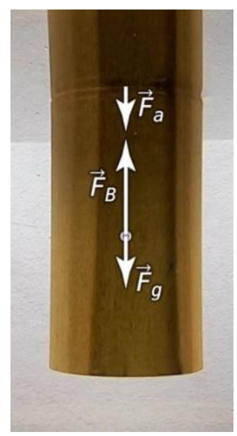
Example: This wood cylinder has a diameter of 50.7 mm and a height of 86.9 mm. If the density of this wood is 540 kg/m^3 and the density of water is 1.00 x 10^3 kg/m^3, what buoyant force acts on the wood cylinder when it is submerged in water?

Knowns: $D = 50.7mm\left(\frac{1m}{1000mm}\right) = 0.0507m \Rightarrow R = \frac{D}{2} = \frac{0.0507}{2} = 0.02535m;$
$H = 86.9mm\left(\frac{1m}{1000mm}\right) = 0.0869m; \rho_{\text{wood}} = \rho_o = 540\frac{kg}{m^3};$
$\rho_{\text{water}} = \rho_f = 1.00 \times 10^3\frac{kg}{m^3}; F_B = ?$

$$F_B = m_f g \;\&\; \rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow F_B = \rho_f V_f g$$

Sumberged: $V_f = V_o = \pi R^2 H = \pi(0.02535)^2(0.0869) = 1.75439\times10^{-4}m^3$

$$\Rightarrow F_B = \rho_f V_f g = (1000)\left(1.75439 \times 10^{-4}\right)(9.81) = 1.72105 \approx 1.7N$$



But notice 1.7 N is not the force we measure when the wood cylinder is submerged in the water. What we measure is the downward force applied necessary to hold the cylinder under the water. To check if our answer is correct we need to draw a free body diagram, sum the forces, and solve for the force applied.

Free body diagram: Buoyant Force, up; Force of Gravity, down; Force Applied, down.

$$\sum F_y = F_B - F_a - F_g = m_o a_y = m_o(0) = 0$$

$$\Rightarrow F_g = \rho_o V_o g = (540)\left(1.75439 \times 10^{-4}\right)(9.81) = 0.92937 \approx 0.93N$$

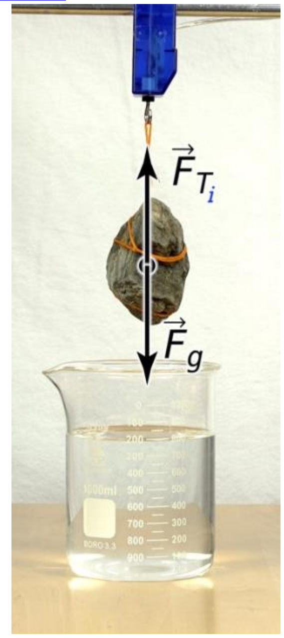
$$\Rightarrow F_a = F_b - F_g = 1.72105 - 0.92937 = 0.79168 \approx 0.79N$$

And the measured Force Applied is really close to 0.79 N. The Physics Works!!

Two common mistakes made by students when using the buoyant force equation:

- The density in the buoyant force equation is the density of the fluid displaced by the object, not the density of the object itself.
- The volume in the buoyant force equation is the volume of the fluid displaced by the object. Only when the object is submerged in a fluid is this the same as the volume of the object.

Flipping Physics Lecture Notes:
Buoyant Force in Action: Weight of Displaced Water!
http://www.flippingphysics.com/buoyant-force-weight.html



Several times we have referred to the buoyant force as the "weight of the fluid displaced by the object". Today we are going to demonstrate that.

We have an object hanging by a string attached to a force sensor. Before we lower the object into water in a beaker, you can see the force measured by the force sensor, the force of tension in the string, is equal in magnitude to the force of gravity, or weight, of the object.

Free body diagram of all the forces acting on the object: initial tension force is up; force of gravity is down.

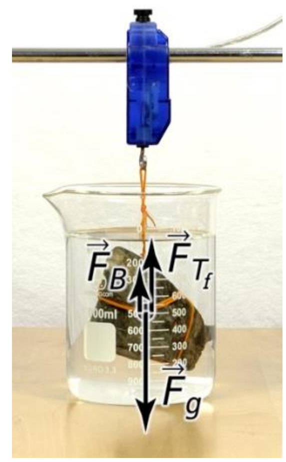
$$\sum F_y = F_{T_i} - F_g = m_o a_y = m_o (0) = 0 \Rightarrow F_{T_i} = F_g$$

When we lower the object into the water, the upward buoyant force is added to the free body diagram and the tension force is now the final tension force.

$$\sum F_y = F_{T_f} + F_B - F_g = m_o a_y = m_o (0) = 0 \Rightarrow F_{T_f} = F_g - F_B$$

And we can solve for the change in the tension force:

$$\Delta F_T = F_{T_f} - F_{T_i} = \left(F_g - F_B\right) - F_g = -F_B$$



The change in the tension force equals the negative of the buoyant force.

Therefore, if we zero the force sensor before we lower the object into the water, and then lower the object into the water, we have now measured the buoyant force acting on the object.

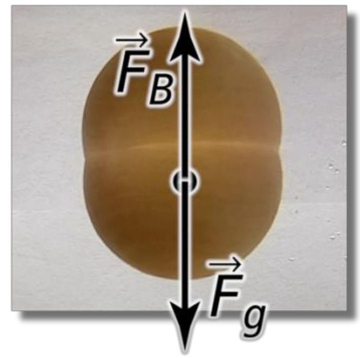
Now, instead of using a beaker full of water, we are going to use a measuring cup with a spout for pouring out the water. And, before we lower the object into the water, we are going to fill the measuring cup to above the spout and allow enough time for the excess water to flow out of the measuring cup.

And we are going to place the beaker under the spout of the measuring cup to collect all the water which leaves the measuring cup as we lower the object into the water in the beaker. All the water in the beaker will then be the water displaced by the object. And we are going to place the beaker on a force sensor to measure the weight of the water displaced by the object. When we zero the force sensor under the beaker and then lower the object into the water, you can see that the buoyant force acting on the object is equal in magnitude to the weight of the fluid displaced by the object. The physics works!!

Flipping Physics Lecture Notes:
Buoyant Force Explained: Objects Floating on Fluids!
http://www.flippingphysics.com/buoyant-force-float.html

We've talked a lot about the buoyant force acting on objects submerged in fluids. Today we are going to look at the buoyant force acting on an object floating in a fluid.

Example: A wooden sphere floats on water. Determine the percentage of the volume of the wood sphere which is below the surface of the water. The density of water is 1.00 x 10^3 kg/m^3. The density of this wood is 660 kg/m^3.

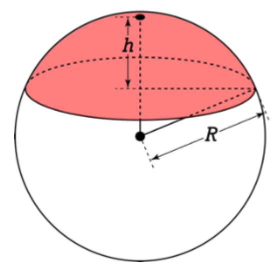


Knowns: $\rho_{water} = \rho_f = 1.00\times10^3 \frac{kg}{m^3}; \rho_{wood} = \rho_o = 660\frac{kg}{m^3}; \frac{V_f}{V_o} = ?\ (\%)$

$$\sum F_y = F_B - F_g = m_o a_y = m_o(0) = 0 \Rightarrow F_B = F_g$$

$$\Rightarrow m_f g = m_o g \Rightarrow m_f = m_o \ \&\ \rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow \rho_f V_f = \rho_o V_o$$

$$\&\ \text{not submerged} \rightarrow V_f \neq V_o \Rightarrow \frac{V_f}{V_o} = \frac{\rho_o}{\rho_f} = \frac{660}{1000} = 0.66$$



So, we have determined that 66% of the wooden sphere is below the surface of the water. Incidentally, that means 34% of the wooden sphere is above the surface of the water.

In order to test this, let's use an equation from Wolfram MathWorld.[1]

The equation is for a spherical cap: $V_{cap} = \frac{1}{3}\pi h^2(3R - h)$

Knowns: $D = 50.7mm\left(\frac{1m}{1000mm}\right) = 0.0507m \Rightarrow R = \frac{D}{2} = \frac{0.0507}{2} = 0.02535m$

$$h = 2.0cm\left(\frac{1m}{100cm}\right) = 0.02m$$

$$V_o = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.02535)^3 = 6.82374\times10^{-5}m^3$$

$$\Rightarrow V_{cap} = \frac{1}{3}\pi(0.02)^2((3)(0.02535) - 0.02) = 2.34782\times10^{-5}m^3$$

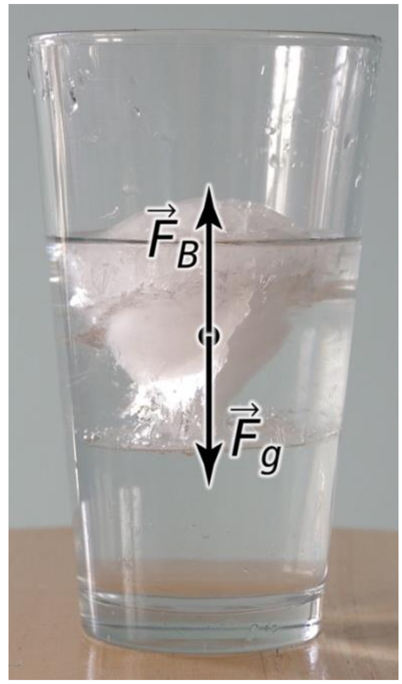
$$\frac{V_{cap}}{V_o} = \frac{2.34782\times10^{-5}}{6.82374\times10^{-5}} = 0.3441 \approx 0.34 \Rightarrow 34\%$$

We just showed that 34% of the wood sphere is above the surface of the water. That matches our prediction. The physics works!

[1] Wolfram MathWorld – Spherical Cap

Flipping Physics Lecture Notes:
Ice Melting in Water: Does the Water Level Change?
http://www.flippingphysics.com/buoyant-force-ice.html

We've talked a lot about the buoyant force. Today we are going to look at a classic buoyancy question.



Example: A chunk of ice floats in a glass of water. As the ice melts, does the water level in the glass go up, down, or remain the same?

Start with the free body diagram of the forces acting on the chunk of ice. The buoyant force is up and the force of gravity is down. And then we sum the forces:

$$\sum F_y = F_B - F_g = m_o a_y = m_o(0) = 0$$
$$\Rightarrow F_B = F_g \Rightarrow m_f g = m_o g \Rightarrow m_f = m_o$$

In other words, the mass of the fluid displaced by the ice is the same as the mass of the ice. And we know the equation for density:

$$\rho = \frac{m}{V} \Rightarrow m = \rho V \Rightarrow \rho_f V_f = m_o$$

In other words, if we were to remove the chunk of ice and replace it with water, the volume of the water we would need to use would equal the empty space below the waterline where the chunk of ice used to be. As the ice melts, that's exactly what happens and the water level remains the same.