The range of an object in projectile motion means something very specific. It is the displacement in the x direction of an object whose displacement in the y direction is zero.

Students often get confused by the statement “displacement in the y direction is zero” or $\Delta y = 0$. This does not mean that the object does not move up or down, it simply means that it ends at the same height it started as: $\Delta y = y_f - y_i = 0$

The Range Equation is $R = \frac{v_i^2 \sin(2\theta_i)}{g}$ & the variables in the range equation are:

- $\Delta x = \text{Range} = R$ (in other words, “R”, stands for Range. Needs to be in meters.)
- $v_i \Rightarrow \|v_i\|$ (the magnitude of the initial velocity. Needs to be in meters per second.)
- $\theta_i \Rightarrow$ (the initial angle or launch angle. Usually in degrees & has to match your calculator mode.)
- $g_{\text{Earth}} = +9.81 \frac{m}{s^2}$ (remember $g$ is a positive number)

We can determine the angle that will give the largest range with the same magnitude initial velocity by remembering that the maximum value for the sine of any angle is 1:

Max Value = 1 = $\sin(2\theta_i) \Rightarrow 2\theta_i = \sin^{-1}(1) = 90^\circ \Rightarrow \theta_i = \frac{90^\circ}{2} = 45^\circ$

Therefore the maximum range is when $\theta_i$ is 45°

Also, remember the shape of the sin($\theta$) curve:
However, because this is $\sin(2\theta)$, the curve is a bit different:

This means that $\sin(2\theta_{ii}) = \sin\left(2\left(90 - \theta_{ii}\right)\right)$. Which means that there are two different angles that will have the same range. For example, $\theta_{ii} = 30^\circ$ & $\theta_{ii} = 90 - \theta_{ii} = 60^\circ$, because both have the same value for $\sin(2\theta_{ii})$:

You can also see that $\sin(2\theta_{ii}) = \sin\left(2\left(90 - \theta_{ii}\right)\right)$ or $\sin(2\theta_{ii}) = \sin\left(2\left(90 - \theta_{ii}\right)\right)$ when I add a horizontal line to the above graph. And you can see that the y-value on the graph is the same for both 30° and 60°.

Just so you know, $\theta_{ii}$ & $\theta_{ii}$ are complimentary angles because they add up to 90°. So two launch angles that are complimentary will result in the same range.
Now, $\theta_1$ & $\theta_2$ will both result in the same range, however, $\theta_2 = 60^\circ$ with go higher and be in the air longer than $\theta_1 = 30^\circ$. Which can be seen in the figure below. (By “air” or course, I mean “the vacuum you can breathe”.)